

# Modeling of a new acceleration sensor as part of a 2D sensor array in VHDL-AMS

E. Markert, M. Schlegel, M. Dienel, G. Herrmann and D. Müller

Department of Electrical Engineering and Information Technology,  
Chemnitz University of Technology, Reichenhainer Str. 70, 09126 Chemnitz, Germany  
e-mail: erik.markert@etit.tu-chemnitz.de

## ABSTRACT

In this paper we present an approach for modeling and simulation of a new acceleration sensor using VHDL-AMS.

The sensor consists of four capacitive segments and one mass segment, aligned in a semicircle. Each capacitive segment includes fixed and movable combs. Any acceleration causes a rotation and a displacement of the seismic mass coupled with moveable combs. The capacity altered with these movements is measurable.

Modeling of the sensor is achieved using analytic equations for capacity changes, currents and torques derived from the sensor geometry. So the development of digital hardware and mixed-signal simulation could start before the sensor is available in silicon. Besides rotational movements also displacements in  $x$ - and  $y$ -direction are considered for capacity calculation.

**Keywords:** acceleration sensor, modeling, VHDL-AMS, geometry

## 1 INTRODUCTION

Micro accelerometers belong to the most important silicon sensors. Usually they use measurements of capacity changes for acceleration detection. This sensor [1] consists of four capacity areas which are aligned in a semicircle as

shown in figure 1. One segment is prepared as seismic mass. Three springs force the segments to a rotational movement around the reference point.

The capacity changes are measured by applying voltages to the segments. The analog sensor outputs will be converted to digital values for further signal processing. VHDL-AMS offers the facility to simulate mechanical, electrical and digital parts in one tool [2]. Different abstraction levels allow changes of model accuracy and therewith acceleration of simulation for noncritical components. Interchangeability of component models of different abstraction levels can be simplified by using "Multi Architecture Modelling" [3].

Section 2 shows the derivation of the sensor's modeling equations. The simulation results are compared to FEM simulation in section 3. The last section gives a summary and an outlook for further work with this sensor.

## 2 MODELING

### 2.1 Partition of Sensor Structure

The sensor consists of one seismic mass and four comb segments as shown in figure 1. For simplification issues the mechanical behaviour of the segments is concentrated in their centres of gravity. Figure 2 shows the structure's simplification into 11 points and the static reference coordinate system. The sensor thickness  $D$  is assumed to be constant.

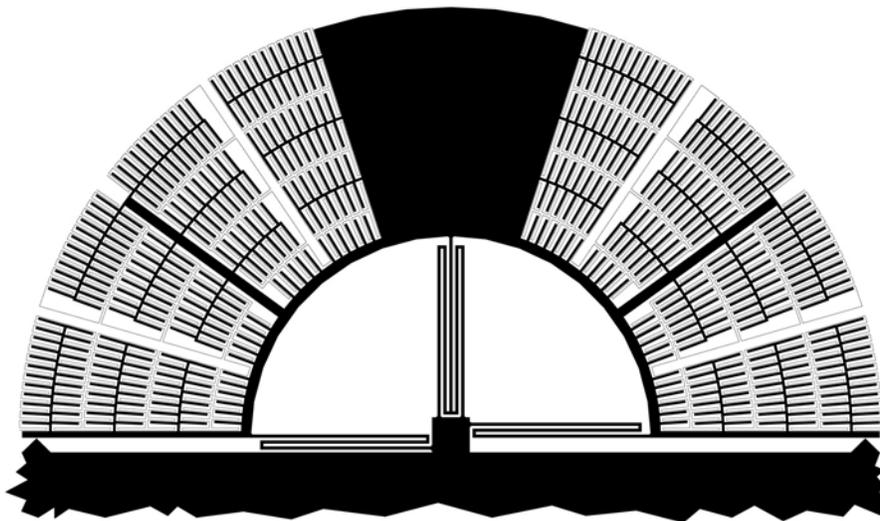


Figure 1: Picture of sensor structure [1]

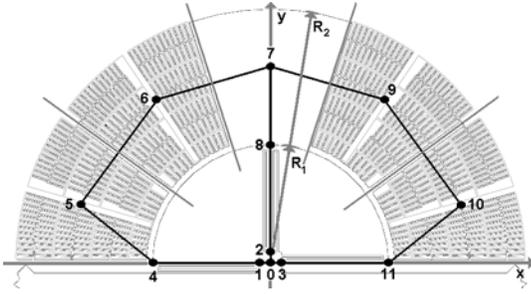


Figure 2: Simplification of sensor structure

Flexible connections (black lines) are used to describe the behaviour between the 11 points. The resting position of points 5, 6, 7, 9 and 10 is calculated by the equations for circle segments (limited by angles  $\beta$  and  $\gamma$ ). Mass distribution is assumed to be homogenous inside a segment.

$$x_S = \frac{2}{3} \cdot \frac{(R_2^3 - R_1^3) \cdot (\sin \gamma - \sin \beta)}{(R_2^2 - R_1^2) \cdot (\gamma - \beta)} \quad (1)$$

$$y_S = \frac{2}{3} \cdot \frac{(R_2^3 - R_1^3) \cdot (\cos \gamma - \cos \beta)}{(R_2^2 - R_1^2) \cdot (\gamma - \beta)} \quad (2)$$

The segment's resting position angle is defined as:

$$\beta_S = \frac{\gamma - \beta}{2} \quad (3)$$

The segment's mass is determined by its volume:

$$m_S = \rho_{Si} \cdot (V_{Finger} \cdot n_{Fingers} + V_{Branches}) \quad (4)$$

The spring connections between points 1 and 4, 2 and 8, 3 and 11 have flexible length, all other connections may only change their direction but not their length.

## 2.2 Derivation of modeling equations

The sensor's behaviour can be expressed using the motion equation for rotation:

$$M = J \cdot \frac{d^2 \alpha}{dt^2} + k \cdot \frac{d\alpha}{dt} + c \cdot \alpha \quad (5)$$

The moment of inertia of each segment is calculated as following:

$$J_{Segment} = \int \int \int \rho \cdot r^3 dz dr d\phi \quad (6)$$

$$J_{Segment} = \frac{1}{4} \cdot \rho \cdot D \cdot (R_2^4 - R_1^4) \cdot (\gamma - \beta) \quad (7)$$

$$\rho = \frac{m}{V} = \frac{2 \cdot m}{(R_2^2 - R_1^2) \cdot D \cdot (\gamma - \beta)} \quad (8)$$

$$J_{Segment} = \frac{1}{2} \cdot m_{Segment} \cdot \frac{R_2^4 - R_1^4}{R_2^2 - R_1^2} \quad (9)$$

For spring constant calculation, springs are assumed as series of beams with different effective lengths  $l_{eff}$ . So formulas (10) and (11) can be used.

$$c_{Beam} = \frac{E_{Si} \cdot w \cdot h^3}{4 \cdot l_{eff}^3} \quad (10)$$

$$\frac{1}{c_{Spring}} = \sum \frac{1}{c_{Beam}} \quad (11)$$

For small displacements the rotational spring constant  $c_{rot}$  can be derived from translational spring constant  $c_{Spring}$  with  $l$  as cantilever arm length:

$$c_{rot} = c_{Spring} \cdot l^2 \quad (12)$$

An additional rotatable coordinate system is introduced with the same origin of ordinates as the static system. The rotation of the axes is described by the angle  $\alpha$ . Hence rotational coordinates may be derived from static coordinates:

$$x_{rot} = x \cdot \cos \alpha + y \cdot \sin \alpha \quad (13)$$

$$y_{rot} = y \cdot \cos \alpha - x \cdot \sin \alpha \quad (14)$$

The displacement in x- and y-direction can be expressed in the following equations:

$$\Delta x = \frac{(x_{rot} - x \cdot \cos \alpha - y \cdot \sin \alpha - \Delta y \cdot \sin \alpha)}{\cos \alpha} \quad (15)$$

$$\Delta y = \frac{y_{rot} - y \cdot \cos \alpha + x_{rot} \cdot \tan \alpha - y \cdot \frac{(\sin \alpha)^2}{\cos \alpha}}{\frac{(\sin \alpha)^2}{\cos \alpha} + \cos \alpha} \quad (16)$$

A single comb finger is characterized by its dimensions as shown in figure 3.

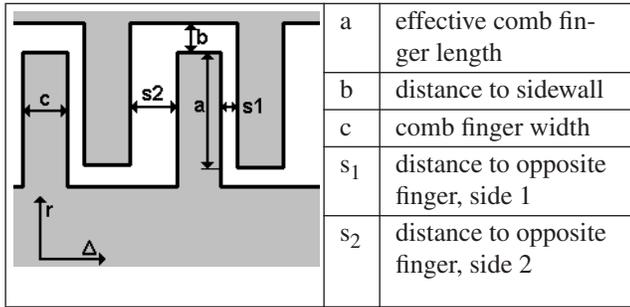


Figure 3: Part of capacitive segment (comb structure)

The structure capacity depends on its distance to the origin  $r$  and the absolute displacement  $\Delta$ . It consists of three homogeneous parts

$$C_F = \varepsilon \cdot D \cdot \left( \frac{c}{b} + \frac{a}{s_2 + \Delta + \alpha \cdot r} + \frac{a}{s_1 - \Delta - \alpha \cdot r} \right) \quad (17)$$

where  $\Delta$  is calculated from the displacements and depends on the segment's resting position angle  $\beta_S$ :

$$\Delta = \Delta x \cdot \cos(\alpha + \beta_S) + \Delta y \cdot \sin(\alpha + \beta_S) \quad (18)$$

Inhomogeneous fields on the edges are ignored.

The torque of the sensor is calculated by summation of the segment's torques which can be described as shown in formulas (19) and (20).

$$a_S = a_x \cdot \sin(\alpha + \beta_S) + a_y \cdot \cos(\alpha + \beta_S) \quad (19)$$

$$M_S = m_S \cdot a_S \cdot \sqrt{x_S^2 + y_S^2} \quad (20)$$

Acceleration causes a change in capacity. The change leads to a displacement current due to applied voltage  $U_S$ .

$$I_S = U_S \cdot \frac{dC_S}{dt} + C_S \cdot \frac{dU_S}{dt} \quad (21)$$

The measurement voltage itself causes a reset torque on capacity changes:

$$M_R = \frac{1}{2} \cdot U_S^2 \cdot \sqrt{x_S^2 + y_S^2} \cdot \left( \frac{dC_{s1}}{ds} + \frac{dC_{s2}}{ds} \right) \quad (22)$$

Therewith the motion equation for rotation (5) can be precisised:

$$\sum M_S - \sum M_R = \sum J_S \cdot \frac{d^2 \alpha}{dt^2} + k \cdot \frac{d\alpha}{dt} + \sum c_{rot} \cdot \alpha \quad (23)$$

After implementation of all these formulas in VHDL-AMS and setting up a testbench with environment values, simulation may start.

### 3 SIMULATION RESULTS

The developed model is verified using results from FEM simulation. For a first static test the sensor is stimulated with a slowly rotating acceleration vector of 1g length. Figure 4 shows the sensor's rotation for a semicircle stimulation. A difference of up to 5% for the capacity of a comb finger and for the rotation angle of the whole sensor compared to FEM model was measured. Reasons for these deviations seem to be the nonlinear electric field on edges, the capacity to the substrate and the unexact estimation of the spring constants.

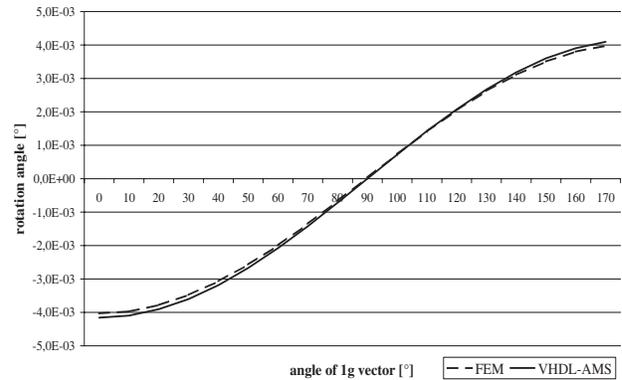


Figure 4: Sensor's rotation on 1g rotating acceleration

Furthermore a frequency domain simulation was done. The sensor's resonance frequency in VHDL-AMS simulation differs little (up to 5%) from the FEM model. The reason is the unexact analytic model of the spring. Especially the behaviour of the beam in the middle can not be expressed exactly with formula (10). The FEM model shows a higher spring constant for this part.

For a dynamic test the sensor is stimulated with a step as shown in figure 5. The stimulated acceleration is displayed in the first diagram. The second diagram shows the rotation angle. The last two diagrams plot the capacities of the two left-hand side comb segments.

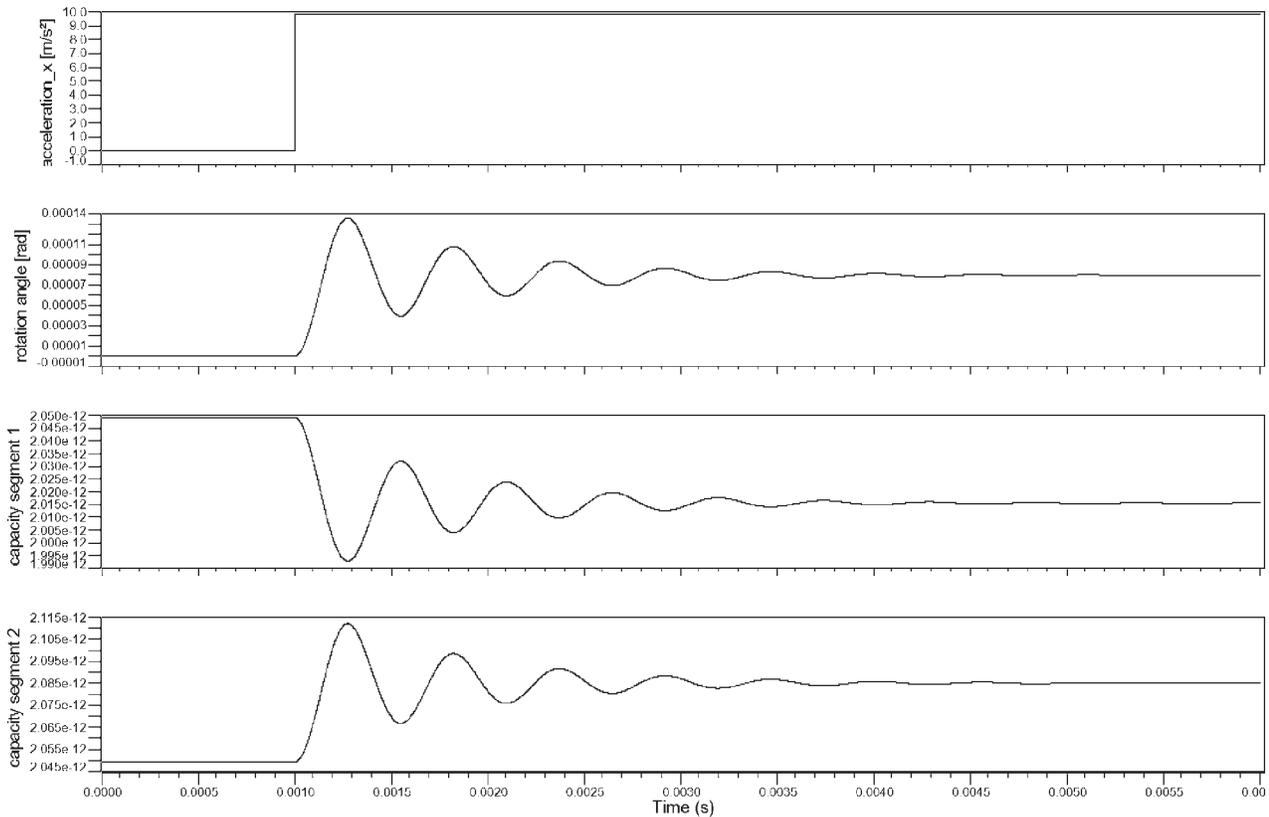


Figure 5: Simulation of a step response

## 4 CONCLUSION AND OUTLOOK

This paper presents a description of an acceleration sensor using VHDL-AMS. Modeling equations were derived from sensor geometry by simplification of sensor structure into 11 points and a sum of comb finger capacities. The sensor model was simulated and compared to a FEM model showing differences up to 5%. For further usage sensor parameters (especially spring constants) are corrected based on FEM simulation results.

The sensor structure as presented is part of a 2D acceleration sensor array. Several single sensors are arranged on one chip. The sensors have an equal basic structure, the detection axis is changed by different seismic mass positions. An optional correction voltage keeps the sensor's displacement in the linear region.

After analog and digital error correction the measured accelerations are used as a base for inertial navigation. The analysis of navigation contains software components. To achieve a global system model it is planned to convert the presented sensor model to SystemC-AMS [4], a mixed signal extension library of SystemC.

## ACKNOWLEDGEMENTS

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