Noise Considerations for Closed Loop Digital Accelerometers

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ABSTRACT

This paper investigates the noise shaping properties of a sigma-delta modulator type control system applied to a micromachined accelerometer. Three noise sources are present in such an electromechanical closed loop system: mechanical noise due to Brownian motion, electronic noise introduced by the interface circuit due to thermal noise sources in the electronic devices and quantisation noise due to the analog to digital conversion process.

It is well known that the sigma-delta modulator system shapes the quantisation noise in an advantageous way by attenuating it in the signalband. It is shown here that it also shapes the electronic noise favorably. However, this property is less pronounced. The Brownian noise remains unaffected in the signal band. With the outlined simulation procedures it is possible to predict the dominant noise contributions for different sampling frequencies of the sigma-delta control system and hence design an optimized control system.

Keywords: Brownian noise, quantisation noise, micromachined accelerometers, sigma-delta modulator

1 INTRODUCTION

Micromachined sensing elements for high precision capacitive accelerometers are often incorporated in a closed loop, force feedback control structure based on a sigma-delta modulator ($\Sigma\Delta$M) [i,ii]. This has many advantages such as a direct digital output signal in form of a pulse density modulated bitstream suitable for digital signal processing, improved system stability since it basically rules out electrostatic pull-in of the proof mass, and an inherent self-test based on the limit cycling property of a $\Sigma\Delta$M control system [iii]. The resolution and dynamic range of such a sensor are mainly determined by the signal to noise ratio of the system. A closed loop, digital accelerometer suffers from three noise sources: 1) Brownian mechanical noise originating from the constant bombardment of air-molecules on the proof mass; 2) thermal noise from the electronic position measurement interface and 3) quantisation noise from the analogue to digital conversion process. The $\Sigma\Delta$M control system should be designed in such a way that the quantisation noise is much less significant than the other two noise sources since it is relatively easy to reduce the quantisation noise by increasing the sampling frequency of the modulator.

In this paper system level simulations in Matlab/Simulink are used to investigate the significance of the three noise sources and a design procedure is outlined for the digital control system. Furthermore, the noise-shaping property of the modulator is studied which is different for the three noise sources. Similar studies have been undertaken for purely electronic sigma-delta A/D converters [iv], however, not for digital, force-feedback accelerometers.

2 MODELLING AND THEORETICAL ANALYSIS

2.1 SIMULINK Model

The Simulink model used in this work, as shown in fig. 1, is based on previous work [v] and simulates a typical bulk-micromachined, capacitive accelerometer with a critically damped sensing element having a resonant frequency of f_0=0.4kHz. The transfer function of the sensing element relating force to proof mass displacement is a second order mass-damper-spring system (parameters used here: m=1.5x10^{-6}kg, b=0.0055Nm/s and k=10N/m, respectively).

The proof mass displacement is sensed capacitively; the differential change in capacitance is given by: $\Delta C= \varepsilon_0 A(d_0-x)^{-1}-(d_0+x)^{-1}$, where ε_0 is the dielectric constant in vacuum, A the capacitor electrode area, d_0 the nominal gap between the proof mass and the electrodes and x the proof mass displacement. For small mass displacements this becomes a constant $k_C=2\varepsilon_0 A/d_0^2 x$. The electronic capacitive measurement circuit was assumed to be a simple gain block, k_{po} relating the change in capacitance to a voltage. Further, an electronic compensator is required to stabilize the control loop by introducing a zero to give some phase lead at higher frequencies. The feedback arrangement applies a feedback voltage to the electrode the proof mass is further away from, thus generating an electrostatic feedback force pulling the mass back to its nominal position. The magnitude of the feedback force is given by $F_{fb}=\varepsilon_0 A V_{fb}^2(d_0+sgn(x))^{-2}$ where V_{fb} is the feedback voltage. For small mass displacements this simplifies again to a constant $k_{fb}= \varepsilon_0 A V_{fb}^2/d_0^2$.

A Brownian and an electronic noise source have been added to the model. The former can be accounted for by adding a random force component acting on the proof mass with a standard deviation of $F_B=(k_B T b f_0)^{-}$ where k_B is the Boltzman constant, T the temperature, b the damping noise originating from the position interface and Brownian noise.

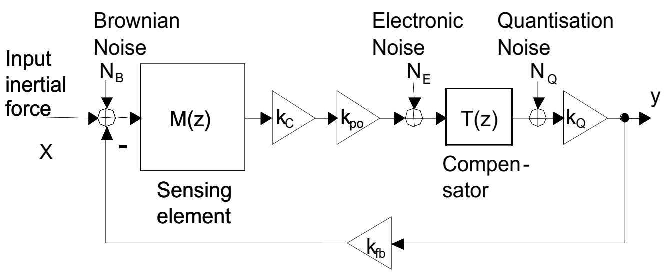

Fig. 2: Linearised mathematical model of the closed loop sensor.

constant. The standard deviation of the electronic noise introduced by the position measurement interface was derived from the input referred power spectral noise density of a standard low-noise amplifier given in its datasheet and its closed loop bandwidth. Due to the model structure the noise was referred to the output of the amplifier. Both noises sources were assumed to be white. The electronic noise obviously also has a flicker noise component which was neglected here for simplicity. This assumption is justified since the capacitive position interface operates at a high frequency (1MHz). However, the synchronous amplitude demodulation is not considered in the Simulink model to keep simulation time reasonable.

2.2 Noise Shaping

Quantisation noise, Brownian noise and noise from the electronic interface circuitry are shaped differently by the modulator. To derive analytical expressions for the noise shaping, a linearised block diagram model was derived from the Simulink model shown in fig.1. It makes the usual approximation that the quantiser is replaced by white quantisation noise; furthermore small proof mass displacements are assumed so that the feedback path can be assumed as a simple gain block. The transfer function of the sensing element was transformed to the z-domain. The model is shown in fig. 2.

It can be easily shown that the signal transfer function, $S_S(z)$ is given by:

$$S_S(z) = \frac{Y}{X} = \frac{T(z)M(z)k_C k_{po}}{1 + k_{fb}k_C k_{po}T(z)M(z)} \qquad (1)$$

The Brownian noise transfer function is the same as the signal transfer function. The electronic noise transfer function, S_{NE} is given by:

$$S_{NE}(z) = \frac{Y}{N_E} = \frac{T(z)k_{po}}{1 + k_{fb}k_C k_{po}T(z)M(z)} \qquad (2)$$

and the quantisation noise, S_{NQ} transfer function by:

$$S_{NQ}(z) = \frac{Y}{N_Q} = \frac{1}{1 + k_{fb}k_C k_{po}T(z)M(z)} \qquad (3)$$

Magnitude plots of the three transfer functions are shown in fig. 3. The top graph shows the input signal transfer function which is identical to the Brownian noise transfer function. The input signal is not shaped in the signal band, which was assumed to be B=1kHz. The bottom graph shows the quantisation noise transfer function which exhibits the typical noise shaping characteristic for a ΣΔM system, i.e. noise attenuation in the signal band since the transfer function is basically the inverse of the mechanical sensing element transfer function. The middle graph shows the transfer function of electronic noise introduced by the position measurement interface which also exhibits a favorable noise shaping property as noise is attenuated a lower frequencies and swept to higher frequencies, out of

Fig. 3: Signal and Brownian noise transfer function (top graph), electronic noise transfer function (middle graph) and quantisation noise transfer function (bottom graph). A sampling frequency of 130kHz was chosen.

the signal band. However, the noise shaping is not as pronounced as for the quantisation noise since the electronic noise is influenced by the compensator and the inverse of the sensing element transfer function. Nevertheless, this property is another significant advantage of incorporating a micromachined sensing element in a closed loop ΣΔM type control structure.

For the magnitude plots a sampling frequency, fs of 130kHz was assumed, thus the oversampling ratio is M=fs/2B=64. Standard ΣΔM theory for electronic analog-to-digital converters predicts that a doubling of the oversampling ratio increases the signal to quantisation noise ratio (SQNR) by 12dB [vi]. In the case of an electromechanical ΣΔM the role of the double electronic integrator is performed by the mechanical sensing element and leads to the same increase of SQNR of 12dB per doubling of the oversampling ratio. However, in the presence of other noise sources this will be only the case if quantisation noise is dominant. Eventually, for higher oversampling ratios, quantisation noise will become negligible and the system's overall signal to noise ratio (SNR) will be mainly determined by the other noise sources. Consequently, increasing the oversampling ratio beyond a certain value will not result in a further increase in SNR.

3 SIMULATION RESULTS

As a general simulation procedure, the oversampling ratio, M was varied and the ratio of input signal power to noise in the signal bandwidth was calculated for each simulation run. This was done by taking a FFT of the sensor's output signal, summing up all noise contributions up to 1kHz and calculating the ratio to the signal power. Fig.4 shows the simulation result for a variation of the oversampling ratio from M=64 to M=2048. The first curve assumes neither Brownian nor electronic noise and exhibits an increase of approximately 12dB per doubling of the oversampling ratio, which agrees with the theoretically predicted value. The second curve assumes only electronic noise, the third curve considers only Brownian noise and the forth curve has all noise sources included. Quantisation noise is intrinsically present in all simulations by modeling the sampling action and the comparator, which acts as a one bit quantiser. The simulation results clearly shows that above a certain value of M a further increase of the oversampling ratio does not significantly improve the overall signal to noise ratio. Furthermore, it confirms the

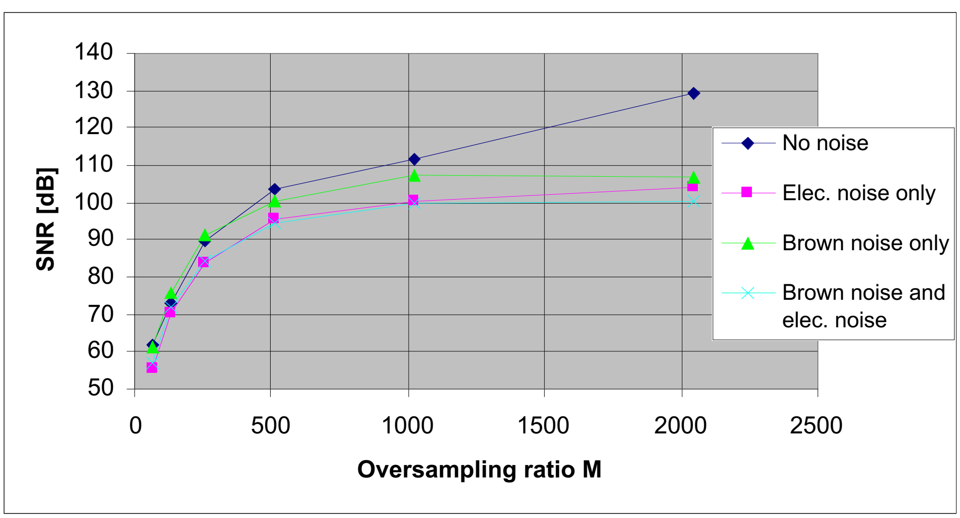

Fig. 4: Simulations results showing the SNR as a function of the oversampling ratio.

(a) (b)

Fig. 5: Power spectral densities of a simulation without (a) and with Brownian and electronic noise (b). As expected the noise floor is higher and the noise shaping is less pronounced for (b).

behavior predicted in the previous section that if only Brownian and quantisation noise are considered, the SNR stays constant above a certain oversampling ratio since the Brownian noise is not shaped by the modulator and eventually dominates. When electronic noise and quantisation noise only are simulated, the SNR continues to increase, even for high values of M, but much slower than 12dB per decade.

To further illustrate above considerations fig.5 shows the power spectrum of a simulation for an input signal with f_{in}=128Hz and M=512 with quantisation noise only (a) and with all noise sources considered (b). Clearly, the noise floor in the latter case is higher, hence the system SNR is lower.

The above considerations may lead to an argument to design an over-damped mechanical sensing element as this makes the compensator obsolete. Consequently, the electronic noise will be shaped in the same way as the quantisation noise. However, previous studies showed that the SQNR is much lower for over-damped systems [5]. This problem may be alleviated by a cascaded ΣΔM structure in which the quantisation noise is subjected to a second, purely electronic ΣΔM.

CONCLUSIONS

The procedure derived in this work allows a structured approach to the design of a ΣΔM control system for micromachined inertial sensors. Electronic thermal, Brownian and quantisation noise are shaped differently by the ΣΔM. Also, this work will allow to predict which noise source is dominant for different operating conditions. Consequently, the most important design parameter for the ΣΔM, the sampling frequency, can be chosen so that the quantisation noise level lies well below the other noise sources. For high oversampling ratios, Brownian noise will dominate as it is not shaped by the modulator. If the achieved SNR is not sufficient for the sensor specifications packaging at reduced pressure has to be considered as this lowers the Brownian noise level.

REFERENCES

[i] Lemkin, M.A. and Boser, B.E. A 3-axis micromachined accelerometer with a CMOS position-sense interface and digital offset-trim electrodes. *IEEE J. of Solid-State Circuits,* Vol. 34, No. 4, pp. 456-468, 1999.

[ii] Kraft, M., et.al. Closed loop silicon accelerometers. IEE Proceedings Circuits, Devices and Systems, Vol. 145, No. 5, pp. 325 – 331, 1998.

[iii] Kraft, M. Closed loop accelerometer employing oversampling conversion. Coventry University, Ph.D. dissertation, 1997.

[iv] Dias, V.G., et.al. Noise in mixed continuous-time switched-capacitor sigma-delta modulators. IEE Proceedings-G, Vol. 139, No. 6, pp. 680 – 684, 1992.

[v] Kraft, M., et.al. Closed loop micromachined sensors with higher order SD-Modulators. *Proc. 4^{rd} Conf. on Modeling and Simulation of Microsystems*, pp. 104-107, Hilton Head Island, March 2001.

[vi] Norsworthy, S.N., Schreier, R. and Temes, G.C. (editors) Delta-Sigma Data Converter. *IEEE Press*, 1996.