Precise 2D Compact Modeling of Nanoscale DG MOSFETs Based on Conformal Mapping Techniques

T. A. Fjeldly, S. Kolberg
University Graduate Center (UNIK)
Norwegian University of Science and Technology, Kjeller, Norway

B. Iñiguez
Departament d'Enginyeria Electronica, Electrica i Automatica,
Universitat Rovira i Virgili, Tarragona, Spain
Overview

Objective:
Establish framework for precise, compact 2D modeling of short-channel nanoscale MOSFETs

Here: Double Gate (DG) MOSFET

- **Part 1**: Subthreshold and near threshold conditions where capacitive coupling between contacts dominant electrostatics

- **Part 2**: Strong inversion where effects of electrons dominant electrostatics. Must be treated self-consistently.

- Determine 2D barrier topography
- Calculate electron distribution
- Find short-channel effects including DIBL
- Calculate current – drift-diffusion and ballistic
Modeling procedure

- Include gate oxide in extended device body: \( t'_{ox} = t_{ox} \varepsilon_{Si} / \varepsilon_{ox} \)

- Map body into upper half plane of transformed \((u, iv)\)-plane using **conformal mapping** (Schwartz-Christoffel transform).

- **Subthreshold**: Solve Laplace’s equation in \((u, iv)\)-plane (low-doped body) → 2D potential distribution for capacitive coupling. Map back to \((x, y)\)-plane

- **Near threshold**: Determine total potential distribution self-consistently by including electrostatic effects of electrons.

- **Strong inversion**: Use long-channel solution. Treat 2D capacitive coupling as perturbation.

- Calculate **current** (drift-diffusion and ballistic)

- **Verify** against 2D numerical simulations.

\[ z = x + i y = \frac{L}{2} \int \frac{F(k, u + iv)}{K(k)} \]
Mapping of the body boundary

\[ z = x + iy = \frac{L F(k, u)}{2} K(k) \]
Mapping along symmetry lines

Gate-to-gate symmetry line

Source-to-drain symmetry line
DG MOSFET and regimes of operation

SINANO template device:

- Double gate $n$-channel SOI MOSFET with aluminum gates
- Specifics: $L = 25$ nm, $t_{si} = 12$ nm, $t_{ox} = 1.6$ nm, $\varepsilon_{ox} = 7$, body doping $N_d = 10^{15}$ cm$^{-3}$, contact doping $10^{19}$ cm$^{-3}$, extended body thickness: $t_{si} + 2t'_{ox}$

Operating regimes:

$\rightarrow$ Part 1: In subthreshold, dopant and carrier charges can be neglected in electrostatics when $n$ or $N_d \leq 10^{18}$ cm$^{-3}$. Potential and electron distributions dominated by the 2D capacitive coupling between the contacts (source, drain and gates)

$\rightarrow$ Part 2: Above threshold requires self-consistent analysis
    - 2a) Near threshold: Use approximate barrier profile
    - 2b) Strong inversion: Electrons dominate, treat capacitive coupling as a perturbation
Part 1: Subthreshold – body potential

Solution of Laplace’s equation in (u,iv)-plane (thin oxide approx):

\[
\phi(u, v) = \frac{\nu}{\pi} \int_{-\infty}^{\infty} \frac{\phi(u')}{(u'-u)^2 + v^2} du'
\]

\[
= \frac{1}{\pi} \left\{ (V_{GS2} - V_{FB}) \left[ \pi - \tan^{-1} \left( \frac{1-ku}{kv} \right) - \tan^{-1} \left( \frac{1+ku}{kv} \right) \right] + (V_{GS1} - V_{FB}) \left[ \tan^{-1} \left( \frac{1-u}{v} \right) + \tan^{-1} \left( \frac{1+u}{v} \right) \right] + V_{bi} \left[ \tan^{-1} \left( \frac{1-ku}{kv} \right) - \tan^{-1} \left( \frac{1-u}{v} \right) \right] + (V_{bi} + V_{DS}) \left[ \tan^{-1} \left( \frac{1+ku}{kv} \right) - \tan^{-1} \left( \frac{1+u}{v} \right) \right] \right\}
\]
Comparison with numerical simulations

Subthreshold potential variation along the symmetry axes

\[ V_{GS} = -0.47 \text{ V}, \quad V_{DS} = 0 \text{ V} \]
Charge density distribution
– gate-to-gate symmetry line

• Classic electron distribution:

\[
n_e(y) = 2 \left( \frac{m_e k_B T}{2 \pi \hbar^2} \right)^{3/2} \exp \left( -\frac{E_g / 2 + q [\varphi_b - \varphi_{ss}(y)]}{k_B T} \right)
\]

• QM electron distribution (deep well):

\[
n_q(y) = \sum_{j=1}^{l} n_{sq} \left| \psi_j(y) \right|^2
\]

Combine subband states from parabolic well (lowest subbands) and square well potentials (higher subbands).

Electron concentration [M\textsuperscript{3}]

\[ V_G = -0.47 \text{ V, } V_D = 0 \text{ V} \]
Threshold voltage and DIBL

'Classical' threshold voltage definition:
Gate voltage $V_T$ where band bending at barrier is $2q\varphi_b$ at $V_{DS} = 0$ V

For symmetric gate bias: $V_T = -0.47$ V

Drain induced barrier lowering (DIBL):
Shift in $V_T$ with applied $V_{DS}$

Asymmetric gate biasing:
Apply fixed negative bias at Gate 2 relative to Gate 1. Pro/con effects:
  + reduced short channel effect
  + adjustable threshold voltage
  - effective channel width halved
  - added small-signal capacitance
Part 2: Strong inversion – self-consistency

Consider energy barrier at gate-to-gate symmetry axis

\( \varphi_1(y) \): 1D potential from electrons

\( \varphi_2(y) \): 2D capacitive coupling potential between contacts

Superposition principle: \( \varphi(y) = \varphi_1(y) + \varphi_2(y) \)

\[
\varphi_1(y) = -\int_0^y E_1(y) = -K_o \times \left\{ \begin{array}{l}
y \int_{t''_{ax}}^{(t_{si} + 2t''_{ax})/2} \exp\left(\frac{\varphi(y')}{V_{th}}\right) dy' \\
+ \int_{t''_{ax}}^{(t_{si} + 2t''_{ax})/2} dy' \int_{t''_{ax}}^{(t_{si} + 2t''_{ax})/2} \exp\left(\frac{\varphi(y'')}{V_{th}}\right) dy''
\end{array} \right. 
\text{ for } y < t'_{ox}
\]

\[
\varphi_2(y) = V_{GS} - V_{FB} + \frac{2}{\pi} \left[ \tan^{-1}\left(\frac{1}{kv}\right) - \tan^{-1}\left(\frac{1}{v}\right) \right] (V_{bi} - V_{GS} + V_{FB})
\]
2a Near operational threshold

Assume that the total gate-to-gate potential $\varphi(y)$ has a parabolic form, which agrees very well with numerical simulations (Atlas):

\[
\varphi(y) \approx \varphi_p(y) = V_{GS} - V_{FB} + \varphi_m \left[ 1 - \left( 1 - \frac{2y}{t_{Si} + 2t'_{ox}} \right)^2 \right]
\]

Parameter $\varphi_m$ is determined by:

• substituting $\varphi_p(y)$ into expression for $\varphi_1(y)$

• performing integrations in expression for $\varphi_1(y)$

• substituting $\varphi_1(y)$ into total potential $\varphi(y)$

• obtaining self-consistent expression for $\varphi_m$ by considering device center
**Parameter $\phi_m$**

\[
\phi_m = \left( V_{bi} - V_{GS} + V_{FB} \right) \left[ \frac{4}{\pi} \tan^{-1}\left( \frac{1}{\sqrt{k}} - 1 \right) \right] - \left( \frac{m_n k_B T}{2 \pi h^2} \right)^{3/2} \frac{q(t_{Si} + 2t'_{ox})^2}{4} \exp\left( \frac{V_{bi} - V_{GS} + V_{FB} - \phi_b - E_g / 2}{V_{th}} \right) \\
\times \left\{ \text{sign}(\phi_m) \sqrt{\frac{\pi V_{th}}{\phi_m}} \text{erf}\left( \frac{\phi_m}{V_{th}} \left[ 1 - \frac{2t'_{ox}}{t_{Si} + 2t'_{ox}} \right] \right) + \frac{V_{th}}{\phi_m} \exp\left( -\frac{\phi_m}{V_{th}} \left[ 1 - \frac{2t'_{ox}}{t_{Si} + 2t'_{ox}} \right] \right) - 1 \right\}
\]

\[V_{DS} = 0\ V\]
2b Strong inversion

Gate-to-gate barrier:

Use long-channel potential $\varphi_1(y)$ for electrons from 1D Poisson’s equation (Taur 2001), adjusted to include effects of body doping:

$$
\varphi_1(y) = \varphi_c - 2V_{th} \ln \left[ \cos \left( \frac{q n_i}{2 \varepsilon_s V_{th}} \right) \right] \left[ \exp \left( \frac{\varphi_c - \varphi_b}{2V_{th}} \right) (y - t'_{ox} - t_{Si}/2) \right]
$$

Treat capacitive coupling potential $\varphi_2(y)$ as a perturbation.
Find self-consistent total potential by first-order correction using 1D Poisson’s equation

Describes how DIBL-effect fades in strong inversion
Current modeling – subthreshold and near threshold

Drift-diffusion transport:

Use barrier shape derived self-consistently. Assume that the current is 'small'. Then we have:

\[ I_{DD} = -qW\mu_n n_s(x) \frac{dV_F(x)}{dx} = qW\mu_n V_{th} \left(1 - e^{-V_{DS}/V_{th}}\right) \int_0^L \frac{dx}{n_{so}(x)} \]

\( n_{so}(x) \): surface concentration of electrons in all cross-sections from source to drain.

Ballistic transport:

- Current consists of free-flight electrons with sufficient energy to fly over barrier
- Electrons 'filtered' through available quantum states at barrier position
- Current limited by ability of source and drain to supply high-energy carriers fast enough


\[ I_{BT} = q(F^+ - F^-) \quad \text{where} \quad F^\pm = \frac{(2k_B T)^{3/2}}{\pi^2 \hbar^2} \sum_{\text{valleys}} \sum_j \sqrt{m_i} F_{1/2} \left( \frac{E_{Fs} - E_{ij} - qV^\pm}{k_B T} \right) \]
Current modeling – strong inversion

Simulated (Atlas) strong inversion potential contour plot
$V_{GS} = 0.1 \text{ V}, V_{DS} = 0 \text{ V}$

Current flows primarily near the Si/SiO\(_2\) interfaces.
Use classical long-channel expressions for $I_{DD}$.

Strong inversion threshold voltage (Taur et al. 2002, adjusted for body doping):

$$V_T = V_0 + 2\varphi_B \quad \text{where} \quad V_0 = \phi_{ms} + \frac{2k_BT}{q} \ln \left( \frac{\epsilon_{ox} t_{Si}(V_{GS} - V_0)}{2t_{ox} q n_i} \right)$$
DD and ballistic current – comparison with simulations

Observations:
- Good correspondence between modeled and simulated currents
- Quite similar results for drift-diffusion and ballistic current
- Some more work needed to model 'knee' region
Summary

Compact 2D modeling framework established for short-channel, nanoscale DG MOSFETs by means of conformal mapping techniques

Precise description of:

• 2D device barrier topography
• Short-channel effects including DIBL
• Self-consistency at moderate and high electron densities
• Fading of DIBL in strong inversion
• Drift-diffusion and ballistic transport

Model verified by numerical simulations