Comparison and Insight into Long-Channel MOSFET Drain Current Models

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Abstract-- In this paper we provide an insight into the drain current model for long-channel MOSFET devices. A new method to perform the integral of the rigorous Pao-Sah dual integral current is derived. From it, we demonstrate the error of the traditional charge sheet models in predicting the drain current compared with Pao-Sah's dual integral model, also provide the reason that Brews' charge sheet model fails to pass the self consistency tests reported previously. Three charge-sheet approximation models are tested in order to find a simple yet accurate drain current model for surface potential-based compact models.

Keywords--MOSFET models, charge-sheet approximation, drain current model

I. INTRODUCTION

At present there are varieties of non Vth based compact models for MOSFETs which can be mainly divided into two groups: (1), The surface potential-based models, like PSP, HISIM, PUNSIM; and (2), The charge-based models, like EKV, ACM, BSIM5. The surface potential-based models are mostly based on Brews' charge sheet model [1]. It is said that the charge sheet model agrees well with Pao-Sah model [2]. But the model assumption that the current consists of drift and diffusion appears to be somewhat drastic while the exact mathematic link between Pao-Sah model and the charge sheet model is not evident so far. Recently it is proved that the predicted channel current by Brews's model is always less than that by Pao-Sah model [3]. Also, C.Galup-Montoro et al. test the consistency of the MOSFET models in [4] and they draw the conclusion that Brews' charge sheet model can not pass the consistency test although the reason is not pointed out clearly.

The purpose of this paper is to provide an insight into drain current model for the long-channel MOSFET. First we propose an analytical equivalent formulation of Pao-Sah's dual integral. It contains as an approximation Brews' charge sheet model. Based on the new formulation we obtain the exact drain current model, which is accurate and self-consistent naturally. Then we test three approximation methods to find a new current model, simple but enough accurate. At last we summarize the conclusions.

II. EQUIVALENT FORMULATION OF PAO-SAH DUAL INTEGRAL

In this paper we normalize various quantities, for example: $\phi_t = \dfrac{kT}{q}$ for voltage, $C_{ox}\phi_t = \dfrac{C_{ox}kT}{q}$ for charge, and $I = \dfrac{\mu W}{L} C_{ox}\phi_t^2$ for current.

The combination of the 1-D Poisson's equation along the normal direction to the channel and Gauss law yields to the surface potential equation:

$$U_g^{'} - U_s = \gamma[U_s - 1 + \exp(-U_s) + \exp(-2U_f - V)(\exp(U_s) - U_s - 1)]^{1/2} \quad (1)$$

where $U_g^{'} = U_g - U_{fb}$ means the effective gate voltage, $\gamma = \dfrac{\sqrt{2q\varepsilon_s N_A}}{C_{ox}\sqrt{\phi_t}}$ is the body factor after normalizing, and other symbols have their usual meanings. For simplicity we neglect some terms in the above expression and obtain the following general formula that is widely used in the compact model:

$$(U_g^{'} - U_s) = \gamma[U_s - 1 + \exp(U_s - 2U_f - V)]^{1/2} \quad (2)$$

Deriving the quasi Fermi-potential from (2) yields to:

$$V = U_s - 2U_f - \ln[\frac{(U_g^{'} - U_s)^2}{\gamma^2} - (U_s - 1)] \quad (3)$$

Notice that it is common to express the depletion charge density Q_B as

$$Q_B = \gamma\sqrt{U_s - 1} \tag{4}$$

and the inversion charge density Q_I is written as

$$Q_I = Q_T - Q_B = (U_g^{'} - U_s) - \gamma\sqrt{U_s - 1} \tag{5}$$

in a charge sheet based model. Q_T represents the total space charge density.

Then one can rewrite (3):

$$V = U_s - 2U_f - \ln[\frac{Q_T^2 - Q_B^2}{\gamma^2}] = U_s - 2U_f - \ln Q_I - \ln(Q_I + 2Q_B) + 2\ln\gamma \tag{6}$$

Differentiating (6) leads to the gradient of the quasi Fermi-potential that is useful in the drain current:

$$\frac{dV}{dy} = \frac{dU_s}{dy} - \frac{1}{Q_I}\frac{dQ_I}{dy} - \frac{1}{Q_I + 2Q_B}\frac{d(Q_I + 2Q_B)}{dy} \tag{7}$$

The Pao-Sah's current equation after normalizing is in the following form:

$$\frac{I}{L} = Q_I \frac{dV}{dy} \tag{8}$$

Substitute (7) into (8) one can obtain the following expression easily:

$$\frac{I}{L} = Q_I \frac{dU_s}{dy} - \frac{dQ_I}{dy} - \frac{Q_I}{Q_I + 2Q_B}\frac{d(Q_I + 2Q_B)}{dy} \tag{9}$$

Performing the above integral along the channel from the source edge to the drain edge, we get:

$$I = \int Q_I dU_s - \int dQ_I - \int \frac{Q_I}{Q_I + 2Q_B} d(Q_I + 2Q_B) \tag{10}$$

(10) is derived directly from the surface potential equation (2) and Pao-Sah's dual integral current equation (8), there is not doubt that the new model is self-consistent. Obviously Brews's model omitted the last term of the above current equation (9). The approximation in Brews's charge sheet model, neglecting a positive integral term, always causes the channel current being less than the exact value predicted by Pao-Sah's model [3]. Also, that is why the compact model based on Brews's one cannot pass the self-consistency test [4].

Further, when MOSFET works in the depletion mode or the weak inversion mode, the last term of (7) is relative small and the relationship between inversion charge and surface potential is about $\frac{d\ln Q_I}{dU_s} = 1$ for a constant quasi Fermi-potential along the channel. On the other hand, when strong inversion mode comes, the effect of the depletion charge is relative small and the relation $\frac{d\ln Q_I}{dU_s} = \frac{1}{2}$ holds. It agrees with the bench mark test on charge-sheet models performed in [3].

If integration of (10) is finished completely one can find the result will be the same with the model proposed by VAN DE WIELE in 1979 [5]. But (10) is more clear and direct than VAN's model.

Unfortunately, the complete integral seems too complex and is not suitable for compact model due to its high request in the computation time.

III. MODEL TEST AND COMPARISON

In order to apply the new insight of Eq.(10) to drain current model without introducing new parameters, we need to simplify the above obtained current expression from MOSFET device physics. In this section we will try to test some approximations in solving the integration (10) based on numeric calculation. The Pao-Sah's current model is used as the Golden Reference, calculated from Pierret's method of a single integration [6]. Also we assume $\mu_0 = 550cm^2/V\cdot S$, the constant electron mobility in the surface inversion channel.

A. The first approximation and result

The first one approximation to (10) can be obtained by approximating $Q_I + 2Q_B$ to $Q_T = Q_I + Q_B$ in the last integral, since it does not affect the general relationship of $Q_I vs U_s$ in [3]. Upon this approximation we can perform the integral in (10) and obtain the following current expression:

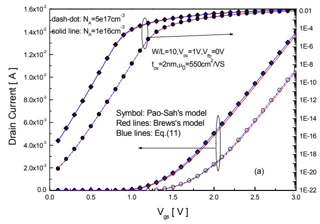
$$I = \int_{U_{s0}}^{U_{sl}} Q_I dU_s - \int_{Q_{I0}}^{Q_{Il}} dQ_I + \int_{U_{s0}}^{U_{sl}} \frac{Q_I}{Q_T} dU_s = f(U_{sl}) - f(U_{s0}) \tag{11}$$

The f function is the following form:

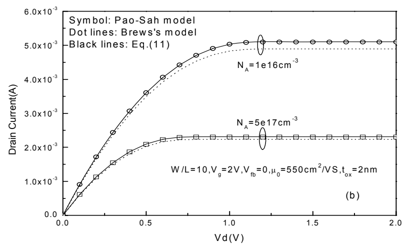
$$f(U_s) = (U_g^{'} + 2)U_s - \frac{1}{2}U_s^2 - \frac{2}{3}\gamma(U_s - 1)^{3/2} + 3\gamma(U_s - 1)^{1/2} - \gamma\sqrt{U_g^{'} - 1}\ln\left[\frac{(\sqrt{U_g^{'} - 1} + \sqrt{U_s - 1})}{(\sqrt{U_g^{'} - 1} - \sqrt{U_s - 1})}\right]$$

where U_{sl} and U_{s0} are surface potentials is the source and drain edges, evaluated from (2) for $V = V_{SB}$ and $V = V_{DB}$, respectively.

In Fig.1 we compare the result from (11) with Brews' charge sheet model and Pao-Sah model.



(a)



(b)

Fig.1 The comparison betwene Pao-Sah 's model, Brews's model and approximate one for (a) transfer characteristics (b) output characteristics.

It is shown that both Brews's model and Eq. (11) agree with Pao-Sah's model in depletion region and weak inversion region. But there are still some relative errors. We will explain it later. In strong inversion region, as can be seen in Fig.1 (b), the relative error brought by Brews model and our model is about +3% and -0.5%, for N_A=5e17cm^{-3}; about 4% and ±0.15%, for N_A=1e16cm^{-3}, respectively. The relative error is defined as: $\eta=(I_{pao}-I_{other})/I_{pao}$. A more light doping leads to a small body factor γ, the last term of (10) become more important, thus larger errors of Brews' charge sheet model are observed, identical with the result in [6].

B. The second approximation and result

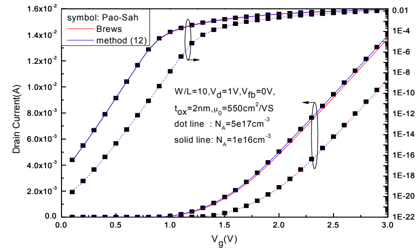
A more accurate one is to approximate Q_I+2Q_B to Q_I+Q_B in the last integral, then one can rewrite the above (11) in a new form:

$$I=\int_{U_{s0}}^{U_{sl}} Q_I dU_s-\int_{Q_{I0}}^{Q_{Il}} dQ_I+\int_{U_{s0}}^{U_{sl}} \frac{Q_I}{Q_T+Q_B} dU_s=g(U_{sl})-g(U_{s0}) \qquad (12)$$

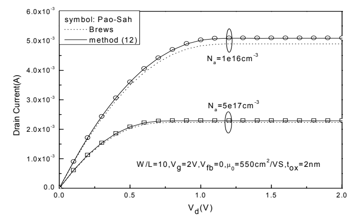
The g function is:

$$g(U_s)=(U_g^{'}+2)U_s-\frac{1}{2}U_s^2-\frac{2}{3}\gamma(U_s-1)^{3/2}+5\gamma(U_s-1)^{1/2}+2\gamma^2\ln[U_g^{'}-U_s+\gamma\sqrt{U_s-1}]$$

$$-\gamma\frac{2(U_g^{'}-1)+\gamma^2}{\sqrt{U_g^{'}-1+\frac{\gamma^2}{4}}}\ln\left[[\sqrt{U_g^{'}-1+\frac{\gamma^2}{4}}+(\sqrt{U_s-1}-\frac{\gamma}{2})]/[\sqrt{U_g^{'}-1+\frac{\gamma^2}{4}}-(\sqrt{U_s-1}-\frac{\gamma}{2})]\right]$$



(a)



(b)

Fig.2 The comparison betwene Pao-Sah 's model, Brews's model and approximate two for (a) transfer characteristics (b) output characteristics.

In Fig.2 we show the numerical calculation result from (12) compared with commonly used charge sheet model. (12) is more complicated in its form. Again our model shows good agreement with Pao-Sah model from depletion region to strong inversion region. In Fig.2 (b), the second approximation's relative errors remain about 0.5% and 0.25% for heavier doping N_A=5e17cm^{-3} and lighter doping N_A=1e16cm^{-3}, respectively. Though the improvement from (11) to (12) is not obvious, the comparison between their errors reveals that the predicted currents become smooth.

C. The third approximation and result

Based on the recognition of the fact that the depletion charge's effect in the last term of (9) is not significant in strong inversion, also the surface potential obtained rigorously from (2) and from the depletion approximation are almost the same in weak inversion [7], we approximate the depletion charge density $Q_B=\gamma\sqrt{U_{sd}-1}$ in the last term of (9) and obtain another more simple arithmetic of (10):

$$I=\int_{U_{s0}}^{U_{sl}} Q_I dU_s-\int_{Q_{I0}}^{Q_{Il}} dQ_I+\int_{U_{s0}}^{U_{sl}}[1-2\frac{\sqrt{U_{sd}-1}}{U_g^{'}-U_s+\sqrt{U_{sd}-1}}]dU_s=h(U_{sl})-h(U_{s0}) \qquad (13)$$

where h function is:

$$h=(U_g^{'}+2)U_s-\frac{1}{2}U_s^2-\frac{2}{3}\gamma(U_s-1)^{3/2}+\gamma(U_s-1)^{1/2}+2\gamma\sqrt{U_{sd}-1}\ln[U_g^{'}+\gamma\sqrt{U_{sd}-1}-U_s]$$

and the depletion approximation surface potential:

$$U_{sd}=U_g^{'}+\frac{\gamma^2}{2}-\gamma(\sqrt{U_g^{'}-1+\frac{\gamma^2}{4}}).$$

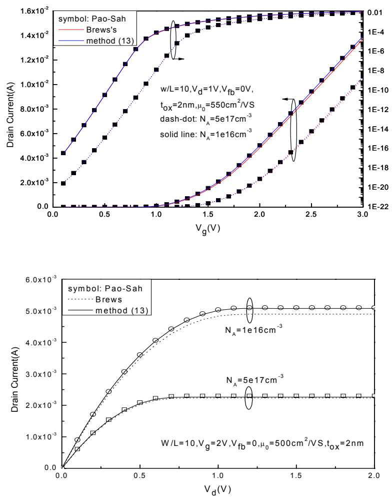


Fig.3The comparison betwene Pao-Sah 's model, Brews's model and approximate three for (a) transfer characteristics (b) output characteristics.

In Fig.3 we show the comparison between (13) and Brews's charge sheet model comparied with Pao-Sah's model. When MOSFET works in its strong inversion region as shown in Fig.3 (b), the relative error by this approximation is about 1.5% and 0.4% for heavier doping NA=5e17cm-3 and lighter doping NA=1e16cm-3, respectively. The simplification of (13) a bit overestimates the depletion charge's effect by the depletion approximation.

From the above test results we can conclude that if high accuracy is requested, a more complicated approximation, like B, should be taken. If not, one can consider a simpler one, like C. Yet all the three approximations predict the drain current more precise than Brews's charge sheet model in the strong inversion region and will success to pass the consistency test because of their direct derivation from the surface potential equation and Pao-Sah's model.

Here we would like to stress that both our present models and Brews's model neglect the holes terms in the surface potential and current expressions. As has been proved, the depletion charge (4) and the inversion charge (5) will cause some troubles in the inversion charge prediction [4] if the simplfied surface potential equation is used as Eq.(2). A more accurate current model should use the complete surface potential equation as shown in [3] and also include the majority carrier although it may be difficult for all present compact models.

IV. CONCLUSION

In this paper we provide an insight into the drain current model for long channel MOSFET. The reason of Brews' charge sheet model's error in predicting the drain current compared with Pao-Sah model and its disability to pass the self-consistency test is explained from the proposed method. Then we test three approximation methods to simplify the complex integral of the drain current formula. The conclusion is that to preserve current model's accuracy and self-consistency, we should take the complete form of the current formula (9). Some approximations must be needed to trade off between accuracy and complexity.

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