

# Calibrating Stiffness and Displacement of an Atomic Force Microscope using Self-Calibratable Micro-Electro-Mechanical Systems: Theoretical Study

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## ABSTRACT

We provide a theoretical study on the calibration of an atomic force microscope (AFM) using Micro-Electro-Mechanical Systems (MEMS). The method we propose uses our self-calibratable MEMS technology to traceably measure the AFM cantilever stiffness and displacement. The calibration of displacement includes measuring the change in optical sensor voltage per change in displacement, or optical level sensitivity (OLS), and the calibration of stiffness along with displacement yields an accurate measurement of force. Calibrating the AFM is important because the AFM has been a most important tool for nanotechnologists for over two decades, yet the accuracy of the AFM has been largely unknown. Previous efforts to calibrate the AFM such as a thermal vibration method, an added weight method, and a layout geometry method are about 10% uncertain. As a consequence such AFM measurements yield about 1 significant digit of accuracy. What is new and different about our method is that we use MEMS, with traceably-calibrated force, stiffness, and displacement, as a sensor to calibrate the displacement reading and cantilever stiffness of the AFM. And our method is practical, easy to use, and can be fabricated in a standard SOI process. In this study we propose a general MEMS design, and provide accuracy, sensitivity, and uncertainty analyses.

**Keywords:** AFM, stiffness, displacement, calibration, self-calibration, MEMS, uncertainty, EMM

## 1 INTRODUCTION

Due to the invention of the AFM [1], the field of nanotechnology has seen extraordinary growth. The AFM is used to apply and sense forces or displacements to better understand phenomena at the nanoscale, which is a key building block scale of matter.

The AFM consists of a cantilevered stylus for probing matter. Displacement is sensed by reflecting a beam of light off the cantilever onto a photodiode that detects the position of the light beam. Measurement of force is found by multiplying this deflection by the cantilever stiffness. The problem is that finding an accurate and practical way of

calibrating the AFM cantilever stiffness and its displacement has been difficult.

We briefly describe several common methods used to calibrate AFM as follows. In [2] an AFM calibration method that requires the accurate knowledge of cantilever geometry and material properties is proposed. Due to process variations, such properties must be measured; however, there has not been an accurate and practical means for such measurements. In [3] a calibration method that exploits thermally-induced vibration of the AFM cantilever is proposed. This method requires the accurate measurement of cantilever temperature and displacement; however, there has not been an accurate and practical means for such measurements. In [4] a mixed method that depends on geometry and dynamics is proposed. In [5] a traceable method that uses a series of extremely uniform cantilever calibrated by electrostatic force balance method as calibration references for AFM cantilever stiffness is presented. However, the method is impractical and therefore difficult for widespread use. The optical level sensitivity (OLS) of the AFM is the ratio of the change in photodiode voltage to the change in displacement. This calibration is typically done by pressing the cantilever tip onto a non-deformable surface [6]. It is assumed that a particular displacement can be prescribed by a piezoelectric positioning stage; however, calibrating the accuracy and precision of this positioning stage is difficult and impractical [7].

To address the above problems of inaccuracy, imprecision, and impracticality, we propose to calibrate the AFM's stiffness and displacement by using our self-calibratable MEMS. We achieve self-calibration through our technology called electro micro metrology (EMM), which capable of extracting accurate and precise mechanical properties in terms of electronic measurands [8]. Microfabrication of our microdevice can be done using a standard foundry process such as SOIMUMPs [9]. Once the force, displacement, and stiffness of our MEMS are accurately calibrated, we believe that the microdevice can be used calibrate the AFM by measuring its stiffness and deflection.

We organize the rest of this paper as follows. In Section 2, we describe our self-calibration method. In Section 3, we suggest how calibrated MEMS can be used to calibrate AFM stiffness and displacement. And we summarize our

discussions in Section 4. Our nomenclature is given in Table 1.

**TABLE 1: NOMENCLATURE**

$h$	Thickness of the device layer (unknown)
$g$	Gap between comb fingers (unknown)
$\varepsilon$	Permittivity of the medium (unknown)
$\beta$	Capacitance correction factor (unknown)
$L$	Initial finger overlap (unknown)
$C, C^P$	Capacitance (measured)
$\Delta, \delta$	Difference and uncertainty (measured)
$x$	Comb drive displacement (measured)
$F$	Comb drive force (measured)
$k$	System stiffness (measured)
$gap$	Gap stop size (measured)
$\Psi$	Comb drive constant (measured)
$\Delta gap$	Layout-to-fabrication (measured)
$V$	Applied voltage (known)
$N$	Number of comb fingers (known)
$n$	$n = gap_{2,layout} / gap_{1,layout} \neq 1$ (known)
$gap_{layout}$	Layout gap (known)

## 2 SELF-CALIBRATION OF MEMS

### 2.1 Change from layout to fabrication

Electro micro metrology (EMM) is an accurate, precise, and practical method for extracting effective mechanical measurements of MEMS [8]. The method of EMM that we implement here begins by using two unequal gaps to determine the difference in gap geometry between layout and fabrication. These gap stops establish a means of equating a well-defined distance in terms of change in capacitance. In subsequent sections, we use this gap-stop to facilitate measurements of comb drive displacement, force, and system stiffness.

We show our proposed self-calibratable MEMS in Figure 1. Two unequal gaps are identified, which are key. These two gaps are related by  $gap_{2,layout} = n gap_{1,layout}$ . They are needed to provide two necessary measurements to eliminate the unknown properties listed in Table 1 as follows.

Using differential capacitive sensing, measurements at zero-state and upon closing  $gap_1$  and  $gap_2$  by applying

enough actuation voltage may be expressed have the forms as

$$\Delta C_1 = \left( \left( \frac{2N\beta\varepsilon hL}{g} + C_+^P \right)_{left\ comb} - \left( \frac{2N\beta\varepsilon hL}{g} + C_-^P \right)_{right\ comb} \right) - \left( \left( \frac{2N\beta\varepsilon h(L - gap_1)}{g} + C_+^P \right)_{left\ comb} - \left( \frac{2N\beta\varepsilon h(L + gap_1)}{g} + C_-^P \right)_{right\ comb} \right) = \frac{-4N\beta\varepsilon h(gap_{1,layout} - \Delta gap)}{g}, \quad (1)$$

where define  $\Delta gap \equiv gap_1 - gap_{1,layout}$ , and parasitics cancel. Similarly, closing the second gap yields

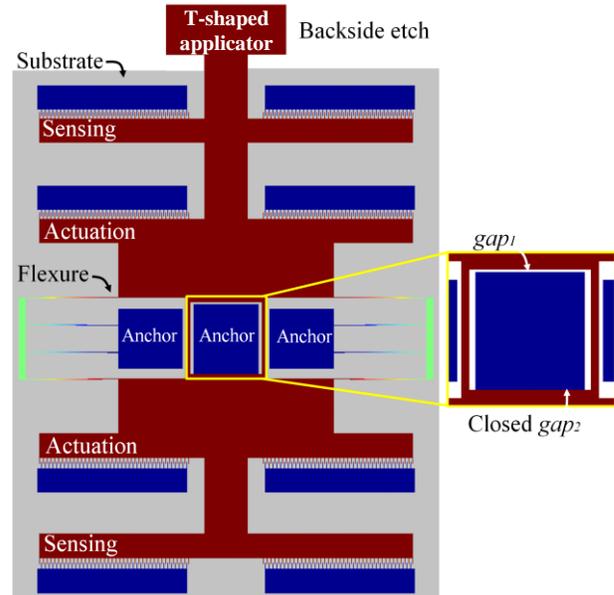
$$\Delta C_2 = \frac{4N\beta\varepsilon h(n gap_{1,layout} - \Delta gap)}{g}. \quad (2)$$

We eliminate the unknowns by taking the ratio

$$\frac{\Delta C_1}{\Delta C_2} = -\frac{gap_{1,layout} - \Delta gap}{n gap_{1,layout} - \Delta gap}, \quad (3)$$

which allows us to accurately *measure* the change in gap-stop from layout to fabrication as

$$\Delta gap = \frac{n\Delta C_1 + \Delta C_2}{\Delta C_1 + \Delta C_2} gap_{1,layout}. \quad (4)$$



**Figure 1: Self-calibratable force-displacement sensor.** Color mapped displacement is shown. Actuation comb drives have closed gap2. The substrate underneath the T-shape applicator is backside etched for sidewall interaction with the AFM cantilever.

## 2.2 Calibrating comb drive displacement

Once  $\Delta C_1$  and  $\Delta gap$  are measured, the comb drive is calibrated. We measure the comb drive constant as

$$\Psi \equiv \frac{\Delta C_1}{gap_{1,layout} - \Delta gap} = \frac{\Delta C_1}{gap_1}, \quad (5)$$

where  $\Psi$  is the unique quantity  $4N\beta\epsilon h/g$  expressed in the previous section.

That is,  $\Psi$  is the ratio of the change in capacitance to traverse a gap-stop distance to that distance. This ratio is applies to any intermediate displacement  $x \leq gap_1$  and corresponding change in capacitance  $\Delta C$ . We may therefore measure displacement as

$$\Psi \equiv \frac{\Delta C_1}{gap_1} = \frac{\Delta C}{\Delta x} \Rightarrow \Delta x = \Psi^{-1} \Delta C. \quad (6)$$

## 2.3 Calibrating comb drive force

Electrostatic force is defined as

$$F \equiv \frac{1}{2} \frac{\partial C}{\partial x} V^2. \quad (7)$$

When applied to comb drives within their large linear operating range, the partial derivatives in (7) can be replaced by differences,

$$F \equiv \frac{1}{2} \frac{\Delta C}{\Delta x} V^2 = \frac{1}{2} \Psi V^2 \quad (8)$$

where we have substituted our measured comb drive constant from (5). It is important to note that our force in (8) accounts for fringing fields and accommodates some non-ideal asymmetric geometries in the comb drive due to process variations.

## 2.4 Calibrating system stiffness

From measurements of comb drive displacement and force, system stiffness is defined as their ratio as

$$k \equiv \frac{F}{\Delta x} = \frac{1}{2} \Psi^2 \frac{V^2}{\Delta C} \quad (9)$$

which is able to account for large linear deflections. That is, the quantity  $V^2/\Delta C$  in (9) is nearly constant for small deflections, but is expected to increase for large deflections.

## 2.5 Measuring uncertainties

Uncertainties accompany all measurements, yet reporting uncertainties with measurements are noticeably lacking in micro and nanoscale peer-reviewed literature. Their absence is usually due to difficult or impractical metrological methods.

One method for measuring uncertainties is done by taking a multitude of measurements and computing the

standard deviation in measurement from the computed average. As the number of measurements increase, the smaller the standard deviation becomes. If taking a large number of measurements is impractical, we propose a more efficient method of measuring uncertainties due to a single measurement as follows.

With respect to the above analyses, electrical uncertainties in the measured capacitance  $\delta C$  and voltage  $\delta V$  produce corresponding mechanical uncertainties in displacement  $\delta x$ , force  $\delta F$ , and stiffness  $\delta k$ . To determine such uncertainties, we rewrite all quantities of capacitance and voltage in the above analyses as  $\Delta C \rightarrow \Delta C + \delta C$  and  $\Delta V \rightarrow \Delta V + \delta V$ . We then identify the first order terms of their multivariate Taylor expansions as the mechanical uncertainties. For instance, the uncertainty in displacement  $\delta x$  of a single measurement is the first order term of the Taylor expansion of (6) about  $\delta C$ . We obtain

$$\delta x = \left( gap_{1,layout} (1-n) \frac{\Delta C_1 + \Delta C_2 - 2\Delta C}{(\Delta C_1 + \Delta C_2)^2} \right) \delta C \quad (10)$$

where the parenthetical coefficient of  $\delta C$  is the sensitivity  $\partial \Delta x / \partial \delta C$ . Similarly, we can find the uncertainties in force  $\delta F$  and stiffness  $\delta k$  as

$$\delta F = \left( \frac{V^2}{gap_{1,layout} (1-n)} \right) \delta C + \left( \frac{(\Delta C_1 + \Delta C_2) V}{gap_{1,layout} (n-1)} \right) \delta V \quad (11)$$

and

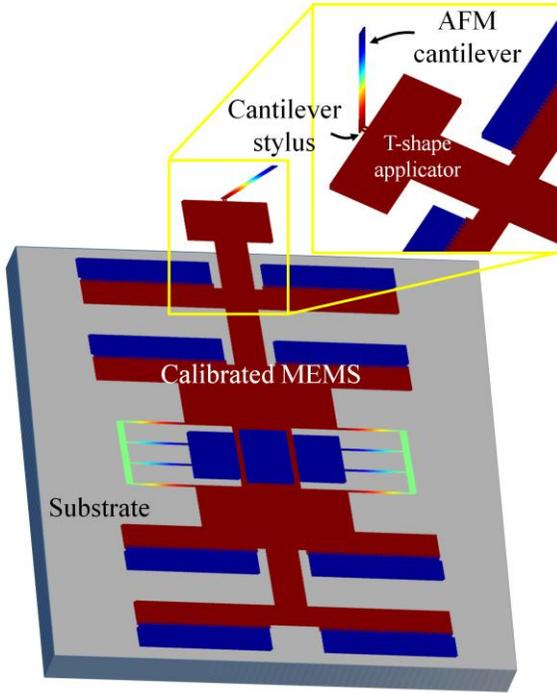
$$\delta k = \left( \frac{(\Delta C_1 + \Delta C_2)(\Delta C_1 + \Delta C_2 - 4\Delta C) V^2}{2(n-1)^2 \Delta C^2 gap_{1,layout}^2} \right) \delta C + \left( \frac{(\Delta C_1 + \Delta C_2)^2 V}{(n-1)^2 \Delta C gap_{1,layout}^2} \right) \delta V \quad (12)$$

where the parenthetical coefficients of  $\delta C$  and  $\delta V$  are the respective sensitivities.

## 3 AFM CALIBRATION WITH MEMS

### 3.1 AFM displacement

In Figure 2 we show a proposed application of our calibratable MEMS to calibrate the displacement and stiffness of an atomic force microscope. Since our MEMS is calibrated in plane (discussed above), it is necessary to position the sensor vertically underneath the AFM cantilever. In a vertical orientation, we use the thick sidewall of the SOI device layer as the surface with which the AFM cantilever stylus will physically interact with. It may also be necessary to create a backside etch to fully expose our MEMS T-shaped applicator (see Figure 2).



**Figure 2: AFM calibration.** We propose to use our calibrated MEMS as an accurate and practical means to calibrate an AFM. Since our device is calibrated for in-plane operation, we use the sidewall of our device as the line of action. By placing our MEMS chip vertically underneath the AFM cantilever stylus, it can be probed with the AFM. The AFM displacement and stiffness can be calibrated by relating the interaction displacement and force measurements of our MEMS sensor against corresponding AFM output readings.

We propose that the AFM cantilever displacement can be calibrated as follows. Given the setup depicted in Figure 2, we have the AFM cantilever press vertically downward upon our calibrated MEMS. This action will result in an initial deflection in the flexures and comb drive of the MEMS, and a corresponding deflection of the cantilever and its beam of light of the AFM.

From this initial state, we notate the reading of the photodiode voltage  $U_{initial}$ , and then apply a voltage  $V$  to the MEMS comb drive so that it will deflect upwards against the AFM cantilever. Upon static equilibrium, a final reading of the photodiode is notated  $U_{final}$ , and the deflection  $\Delta x$  of the comb drive is capacitively measured using (6). The optical level sensitivity (OLS) is measured as

$$\Theta = \left. \frac{\Delta U}{\Delta x} \right|_{\text{calibration}} \quad (13)$$

where  $\Delta x = \Delta x_{AFM}$  in (13) because the AFM base and MEMS substrate are fixed with respect to each other. It is important to note that AFM base or MEMS substrate is not fixed during the initial engagement as the two devices are brought into contact by a piezoelectric stage or other mechanism. For arbitrary  $\Delta U$ , calibrated measurements of AFM cantilever displacements may be determined by

$$\Delta x_{AFM} = \Theta^{-1} \Delta U \quad (14)$$

The uncertainty in AFM displacement or stiffness may be determined by either of the two methods mentioned in Section 2.5.

### 3.2 AFM stiffness

We propose that the AFM cantilever stiffness can be calibrated as follows. Given a measurement of AFM cantilever displacement (14) from an initial photodiode reading of  $U_{initial}$  to a final reading of  $U_{final}$ , the AFM cantilever stiffness can be measured as

$$k_{AFM} = \frac{k \Delta x}{\Delta x_{AFM}} \quad (15)$$

where  $\Delta x$  and  $k$  of the MEMS are measured by (6) and (9). Here  $\Delta x \neq \Delta x_{AFM}$ , unlike in (13), because the AFM base and MEMS substrate are moving with respect to each other during this interaction. In (15), the AFM and MEMS interaction forces are static equilibrium, and are equal and opposite,  $k \Delta x = k_{AFM} \Delta x_{AFM}$ .

## 4 CONCLUSION

In this paper we presented a theoretical study of using our self-calibratable MEMS to calibrate an AFM cantilever displacement and stiffness. We proposed MEMS sensor design and a method of application. Our technique appears to be practical while maintaining accuracy. Measurement uncertainties using our method are clear and easily determined. Measurement accuracy is achieved by eliminating unknowns and implementing accurate definitions of force, displacement, and stiffness.

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