Subthreshold Quantum Ballistic Current and Quantum Threshold Voltage Modeling for Nanoscale FinFET

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ABSTRACT

An analytical modeling framework for quantum ballistic charge transport in the subthreshold regime, and a quantum threshold voltage model for nanoscale double-gate (DG) FinFET are presented. For subthreshold conditions, we assume that the electrostatics of the lightly doped silicon body is dominated by the inter-electrode capacitive coupling between the body electrodes. Hence, the charge is neglected in Poisson’s equation. This decouples the self-consistency between the charge transport, the quantum effects and the body electrostatics. The threshold condition is obtained by self-consistently adjusting the quantum charge potential.

Keywords: double gate FinFET, quantum effects, ballistic transport, threshold voltage, conformal mapping.

1 INTRODUCTION

As MOSFET channel dimensions shrink, effects such as the ballistic component in the drain current and quantum-mechanical confinement increases. A modeling framework capturing these effects is important because: 1) It sets out a framework for modeling of the ultimate MOSFETs. 2) It gives an insight about the ultimate performance limits.

DG structures are among the most promising MOSFET devices in future. A drift-diffusion based quantum mechanical modeling framework for UTB devices operating in the subthreshold and near-threshold regimes has already presented [1]. Here we focus on modeling the quantum ballistic current and the quantum threshold voltage in DG FinFETs. Some of the previous papers have either focused on simulation studies [2,3] or classical thermionic emission currents [4] to model such devices in the ballistic regime. In the present case, we consider ultra-thin devices (UTDs) in the subthreshold regime, where the potential distribution is modeled by solving the 2D Laplace equation using conformal mapping techniques [5,6]. To model quantum effects, we solve the 1D Schrödinger equation in the gate-to-gate (G-G) direction to obtain the eigenfunctions in terms of the parabolic cylindrical functions [7].

The DG device considered here (see Fig. 1) has an insulator thickness $t_{ox} = 1.6$ nm, and an insulator relative dielectric permittivity $\varepsilon_{ox} = 7$. The doping density of the $p$-type silicon body is $10^{15}$cm$^{-3}$. As gate material, we select a near-midgap metal with work function 4.53 eV. Idealized metal contacts with a work function of 4.17 eV (corresponding to that of $n^+$ silicon) are assumed for the source and drain. This ensures equipotential surfaces on all the device contacts. To simplify the modeling, we replace the insulator by an electrostatically equivalent silicon layer of thickness $t'_{ox} = t_{ox} \varepsilon_{si}/\varepsilon_{ox}$, where $\varepsilon_{si}$ is the relative permittivity of silicon. Hence, we define the extended body thickness as $H = W_{fin} + 2t'_{ox}$, where $W_{fin}$ is the fin-width (silicon thickness).

Figure 1: Schematic representation of DG FinFET

2 QUANTUM MODELING

Due to separation of the top electrode by a hard mask, the potential variation along the fin height in DG FinFETs is negligible and thus 2D DG solutions can be directly applied to this 3D structure. The effect of height on quantum confinement does become important for relatively short fin heights ($H_{fin} < 20$nm) [8], but here we consider higher, more practical devices, with sufficient current drivability.

To model the potential in the subthreshold regime, we use conformal mapping techniques to solve the Laplace equation [5]. To obtain a simplified distribution, we assume parabolic potential distribution in the gate-to-gate (G-G) ($x$) direction. This has been shown to be a good approximation in the subthreshold regime [6] and the Laplace potential can be represented as:
\[ \varphi_L(x, y) \approx \varphi_e(y) \left(1 - \frac{4x^2}{H^2}\right) + V_g - V_{FB} \]  

(1)

where \( \varphi_e(y) \) is the difference between the Laplace potential at position \( y \) on the source-drain (S-D) symmetry line and that of the gate-silicon interface of the extended body. \( V_g \) is the gate-source potential, and \( V_{FB} \) is the flat-band voltage of the gate.

Using the above expression for the potential in the Schrödinger equation, we have [1]:

\[ \frac{h^2}{2m^*} \frac{d^2\psi(x)}{dx^2} + \left[q\varphi_e(y) \left(\frac{2x}{H}\right)^2\right] \psi(x) = E\psi(x) \]  

(2)

This can be expressed in terms of the standard equation of parabolic cylinder functions [8] given as:

\[ \frac{d^2\psi(t)}{dt^2} - \left(\frac{1}{4} - t^2 + v\right)\psi(t) = 0 \]  

(3)

where \( t = ax, \alpha = \sqrt{32m^*q\varphi_e(y)/(HH^2)}^{1/4}, v = E/\beta, \) and \( \beta = \frac{1}{2}\sqrt{8q\varphi_e(y)/H^2m^*} \).

This equation has both even and odd solutions. As the potential is symmetric about \( x = 0 \), the wave function must have a definite parity and thus \( \psi(t) \) is given by two different polynomials with even and odd terms:

\[ \psi_{e,o}(t) = A_e(1 + \sum_{n(even) = 2} c_n t^n / n!), A_o(t + \sum_{n(odd) = 3} c_n t^n / n!) \]  

(4)

where \( A_e \) and \( A_o \) are normalization constants and the coefficients \( c_n \) are determined from the recursive relation given in [8]. Depending upon the value of \( t \), these eigenvalues can be determined by truncating the above subpolynomials to some higher term and then finding the numerical solution. For ultra-thin devices operating in subthreshold and near-threshold regimes, the first four terms of the polynomials give sufficient accuracy in the calculation of the lowest subbands. Since higher subbands are not affected by the potential perturbation, they can be better approximated by the eigenvalues of a particle in a box.

For devices operating in the near-threshold regime, the eigenvalues and eigenfunctions approach structure dependent values. In this case, the eigenfunction and eigenvalue corresponding to the first unprimed subband are given as:

\[ \psi_1(x) = \sqrt{\frac{15}{8\pi}} \left(1 - \frac{4x^2}{W_{fin}^2}\right) \]  

(5)

\[ E_i = \frac{4\hbar^2}{m^*_i W_{fin}^2} \]  

(6)

### 3 BALLISTIC CURRENT MODELING

The charge transport is in the \( y \)-direction, and thus the net current, is the difference between the positive flux (filling positive \( k \) states) emitted by the source at the top of the barrier and the negative flux (filling negative \( k \) states) emitted by the drain at the top of the barrier. The net current can thus be evaluated by using Natori’s formalism given as [9]:

\[ I_d = \frac{H_{af}q}{\hbar^2} \left(\frac{kT}{\pi}\right)^{3/2} \sum_{v} \sum_{j} \sum_{z=0}^{m} \psi_{z}^*(v) \ln \left[ 1 + e^{-\frac{E_j(y_m) - E_{FS}}{kT}} \right], \]  

(7)

where \( E_{FS} \) is the Fermi level at the source and \( V_{DS} \) is the applied drain-source voltage. It is assumed that any electron with energy higher than that of the top of the barrier has a transmission probability of 1, and any electron with lesser energy has a transmission probability of 0 (i.e. no tunnelling). The total charge-sheet density at the top of the barrier becomes:

\[ Q_m = \left(\frac{qkT}{2\pi \hbar^2}\right) \sum_{\text{j valleys}} \sum_{j} \sum_{z=0}^{m} \psi_{z}^2(v) \ln \left[ 1 + e^{-\frac{E_j(y_m) - E_{FS} - qV_{DS}}{kT}} \right] \]  

(8)

where \( y_m \) is the position of the top of the barrier, calculated using conformal mapping techniques [6,7].

In general, the total potential inside the silicon body is given as:

\[ \varphi(x, y) \approx \varphi_L(x, y) + \varphi_Q(x, y) \]  

(9)

where \( \varphi_Q \) is the potential due to the charge carriers. In the above-threshold regime, the negative flux from the drain at the top of the barrier decreases when drain voltage is applied. This results in a reduction of the total charge density at this location. Due to this reduction in the charge density, the negative potential \( \varphi_Q \) decreases. In turn, this increases the total potential. (We note that drain induced barrier lowering (DIBL) also gives rise to a slight potential increase, but this effect vanishes in the strong-inversion regime [6]). The net effect is that the positive flux from the source side increases. In the subthreshold regime, however, the potential is fixed by the bias voltages of the contacts and thus we actually see a reduction of charge at the barrier maximum by approximately a factor of two at sufficiently high drain voltages (in reality, we always have a factor of
less than two because of the DIBL effect in the subthreshold regime.

Figure 2 shows the modeled current for a 5 nm thick device, compared with ballistic NEGF (non-equilibrium Green’s function) simulations and constant-mobility DD (drift-diffusion) simulations, both performed using the ATLAS device simulator.

It is of interest to note that in subthreshold the constant-mobility DD mechanism gives rise to similar characteristics as those of the ballistic model, except for a shift in magnitude. For UTBs, the major contribution to the drain current comes from the lowest subband, and thus (7) for the non-degenerate case can be written as:

\[ I_d = H_n q n_{\text{sub}} \frac{2kT}{\pi m^*} \left( 1 - \exp \left( - \frac{V_{ds}}{V_{th}} \right) \right) \]  \hspace{1cm} (10)

where \( n_{\text{sub}} \) is the electron density from the lowest subband at the top of the barrier and \( m^* \) is the transverse effective mass. The drift-diffusion current for the same case can be expressed as [10]:

\[ I_d = H_n m \left( \frac{kT}{\mu n L} \right) \left( 1 - \exp \left( - \frac{V_{ds}}{V_{th}} \right) \right) \]  \hspace{1cm} (11)

where \( \mu_n \) is the electron mobility and the length \( L \) is the distance from the top of the barrier to where charge-sheet density has increased by a factor of ‘e’ (obtained from conformal mapping analysis). Comparing the above two equations, we observe that both equations are essentially the same, except for the effective velocity at the top of the barrier. In the case of ballistic transport, the effective velocity is the thermal velocity determined by the source and the height of the barrier, and is given as:

\[ v_{th} = \frac{1}{2} \sqrt{\frac{2kT}{\pi m^*}} \left( 1 - \exp \left( - \frac{V_{ds}}{V_{th}} \right) \right) \]  \hspace{1cm} (12)

whereas in the case of constant mobility DD, the effective velocity at the top of the barrier is the diffusion velocity and given as:

\[ v_d = \mu_n \left( \frac{V_{th}}{L} \right) \left( 1 - \exp \left( - \frac{V_{ds}}{V_{th}} \right) \right) \]  \hspace{1cm} (13)

4 QUANTUM THRESHOLD VOLTAGE MODELING

The threshold voltage \( V_T \) is defined as the value of gate voltage (for \( V_{ds} = 0 \) V) for which the transition between volume inversion and surface inversion takes place [6].

At \( V_{gs} = V_T \), the electrostatic effects of capacitive coupling and free charges are nearly equal and opposite and the center potential becomes equal to \( V_T - V_{FB} \). Thus, at threshold we have:

\[ \phi_{Qm0} + \phi_{Lm0} \approx V_T - V_{FB} \]  \hspace{1cm} (14)

where \( \phi_{Qm0} \) and \( \phi_{Lm0} \) are the values of the Laplace and charge components at the potential minimum, which will be at the device center (\( y = 0 \)) for \( V_{ds} = 0 \) V. From the analytical solution for the inter-electrode body potential using the conformal mapping technique, we obtain:

\[ \phi_{Lm0} = \left[ 1 - \frac{4}{\pi} \tan^{-1} \left( \sqrt{2} \right) \right] (V_{gs} + V_{fb} - V_{gs} - \frac{1}{2} V_{ds}) \]  \hspace{1cm} (15)

The corresponding threshold voltage based on the classical theory for DG MOSFET can be expressed in the form of a Lambert function [6].

However, for UTB devices, this solution overestimates the threshold voltage. By including the quantum effects, in combination with (14), we can get an expression for quantum threshold voltage. To include the quantum effects, we solve for the quantum charge potential \( \phi_{Qm0} \) by considering the 1D Poisson’s equation along the G-G axis [1]:

\[ \frac{d^2 \phi_Q(x)}{dx^2} = \frac{q}{\varepsilon_s} \sum_{\text{valleys}} \sum_f g_f N_{2D} \ln \left[ 1 + e^{\frac{\phi_F(x) - E_f(x)}{kT}} \right] |\psi_f(x)|^2 \]  \hspace{1cm} (16)

Again, the threshold voltage can be expressed in form of the Lambert function as:
\[ V_{th} = \frac{V_T + V_{FB}}{k_BT/q} \exp \left( \frac{V_{th} - V_T + V_{FB}}{k_BT/q} \right) = \lambda \exp \left( \frac{gV_{th}}{k_BT} \right) \tag{17} \]

where \( \lambda \) is given as:

\[ \lambda = -\frac{15\pi q}{16V_{th}e_W} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{N_{13} \exp \left( -\frac{E_n + E_m/2 + q\phi_n - q\phi_m}{kT} \right)}{kT} \left( \frac{11W_{fin}^2 + 4W_{ox}l_m}{120} \right) \tag{18} \]

Figure 3 shows the modeled threshold voltage compared with the self-consistent Poisson-Schrödinger numerical simulations done using the ATLAS device simulator. The model shows good agreement with the numerical simulations. The small deviation is due to undulations in the G-G potential present at threshold.

5 CONCLUSION

We have developed a compact quantum ballistic current model for nanoscale DG FinFET in the subthreshold regime. It is shown that constant-mobility transport model gives similar characteristics as those of the ballistic transport model. An analytical model for quantum-threshold voltage is also presented. It is shown that quantum confinement enhances the threshold voltage.

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Figure 3: Comparison of modeled and numerically simulated quantum \( V_T \) variation with \( t_{si} \) for a 25 nm long device.