Long-Range Interactions in Quasi-One Dimensional Cylindrical Structures

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ABSTRACT

Casimir forces originating from vacuum fluctuations of the electromagnetic fields [1] are of increasing importance in many scientific and technological areas. The manifestations of these long-range forces at the nanoscale have led to the need of better understanding of their contribution in relation to the stability of different physical systems as well as the operation of various technological components and devices [2]. In this work, we present mathematical methods to calculate the Casimir interaction in various infinitely long cylindrical nanostructures. We will consider a dielectric cylindrical layer with a finite thickness characterized with specific dielectric and magnetic properties and another system of perfectly conducting concentric cylindrical shells. We present analytical expressions and numerical calculations for various cases in terms of the radial dimension, curvature, and material composition of the studied systems. The results from this work can be used to understand long-range interactions in structures such, as carbon nanotubes and nanowires [3,4].

Keywords: Casimir force, nanoscale, mathematical methods, carbon nanotubes.

1 INTRODUCTION

Long-range dispersion forces, such as van der Waals forces (between a pair of unpolarized atoms or molecules), Casimir-Polder forces (between an atom and a macroscopic object), and Casimir forces (between two macroscopic objects), originate from the vacuum fluctuations of the electromagnetic field [5]. They couple electrically neutral objects with no permanent electric and/or magnetic moments and are quantum mechanical in their physical nature. These long-range forces become especially important for systems integrated in nanotechnological devices, where deep understanding of the behavior of various miniaturized components plays a crucial role [2, 6].

Processes, such as friction, adhesion, and ware, are directly related to the Casimir forces and they can be dominant at small scales. For example, Casimir force is found to be prominent in micro- and nanoelectromechanical systems (MEMS and NEMS) [2]. In such tiny devices, the Casimir force can cause mechanical components to stick to one another, resulting in permanent adhesion, an effect called stiction. This effect often results in the malfunction of MEMS and NEMS, causing them to behave erratically.

Furthermore, structures and devices such as multiwall carbon nanotubes consisting of cylindrically wrapped graphene sheets have been shown to be stable due to long-ranged forces [7]. Oscillating carbon nanotubes or buckyballs inside a stationary carbon nanotube have also been demonstrated to be related to such long-ranged forces [8-10].

Therefore, qualitative and quantitative knowledge is needed in order to be able to understand and monitor such long-range forces in various systems. Different geometries, topologies or types of materials can influence the magnitude and sign of the Casimir force [5]. The purpose of this paper is to present a qualitative model of the importance of the cylindrical geometry curvature, number of layers and radial size in the Casimir interaction in cylindrical structures. This is achieved by considering theoretical and mathematical techniques in modeling the Casimir effect.

Of the several theoretical methods that have been developed to calculate the Casimir effect in non-trivial geometries [5], the mode summation method will be of particular interest to us. This mode summation approach involves representing the Casimir energy as a sum of the zero-point energies of the electromagnetic excitations supported by the system. We investigate two specific systems using this method, a dielectric cylindrical layer with finite thickness and perfectly conducting concentric cylindrical shells. Our interest in such systems is motivated by the existence of cylindrical structures, such as double-wall metallic and dielectric carbon nanotubes [3] and carbon multiwall nanotubes made out of metallic shells [11,12].

The rest of the paper is organized as follows. In Section 2 the methodology used is presented. In Sections 3 and 4 the systems used and their corresponding results and discussions are given.

2 MODE SUMMATION METHOD

The mode summation method allows us to express the Casimir energy in a simple and elegant fashion as the sum of the ground state (zero-point) photon energies. One obtains these photon energies from the dispersion relation, $f_{\{p\}}(\chi)=0$ where $\chi^2=\varepsilon\mu\omega^2-k_z^2$ and $\{p\}$ are the complete set of quantum numbers determined by the type of system under consideration.

Here ε and μ are the dielectric and magnetic functions respectively, ω is the frequency and k_z is continuous corresponding to the wave vector along the infinite axial direction of the cylindrical system. For a cylindrical structure, $\{p\} = (n, m, k_z)$ where n is the order of Bessel functions that appear in the dispersion relations, m denotes the number of roots of $f_{\{p\}}(\chi) = 0$.

The Casimir energy can then be expressed by the sum over all modes, as follows:

$$E_C = \frac{\hbar}{2} \sum_{\{p\}} \left(\omega_p - \tilde{\omega}_p \right) \tag{1}$$

The terms ω_p are the eigenfrequencies satisfying $f_{\{p\}}(\chi)$ while $\tilde{\omega}_p$ are the ones corresponding to the reference vacuum with no boundaries present.

3 CASIMIR ENERGY OF A CYLINDRICAL LAYER

3.1 Model and Calculations

The first system that we consider is that of a dielectric cylindrical layer with an inner radius R_1, an outer radius R_2 and an infinite axial direction (see Fig. 1). The dielectric layer has dielectric and magnetic functions ε and μ respectively, and it is placed in an infinite medium of dielectric and magnetic functions ε_m and μ_m respectively.

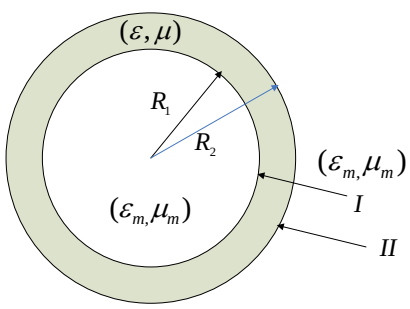

Figure 1: Cylindrical layer of finite thickness.

For this model we define the Casimir energy in the following form [3]:

$$E_C(s) = \frac{\hbar}{2} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \sum_{n,m} \left[\omega_{n,m}^{-s}(k_z) - \tilde{\omega}_{n,m}^{-s}(k_z) \right] \tag{2}$$

The sums over n and m in Eq. (2) are both divergent. The divergence over m is removed by using the Residue's theorem while that over n is regularized using the Riemann ζ-function procedure. The parameter s, specified as a complex number, enables us to employ the Riemann ζ-function regularization procedure and the exact Casimir energy is retrieved by setting $s \to -1$. In order to facilitate the calculations for the Casimir energy we impose the condition $\varepsilon\mu = \varepsilon_m \mu_m = c^{-2}$. In other words, the speed of light c is taken to be constant across each interface.

For this model, the dispersion relation can be separated into the pure magnetic (TE) and pure electric (TM) modes, $f_{\{p\}}(\chi) = f_{\{p\}}^{TE} f_{\{p\}}^{TM}$. The Casimir energy then becomes of the form [3]:

$$E_C(s) = \frac{\hbar}{4\sqrt{\pi}\,\Gamma\left(\frac{s}{2}\right)\Gamma\left(\frac{3-s}{2}\right)} c^{-s} \times \sum_{n=-\infty}^{\infty} \int_0^{\infty} dy\, y^{1-s} \frac{d}{dy} \ln \frac{f_n^{TE}(iR_1 y, iR_2 y)\, f_n^{TM}(iR_1 y, iR_2 y)}{f_n^{TE}(i\infty)\, f_n^{TM}(i\infty)} \tag{3}$$

Here we have used the relation $\chi^2 = \varepsilon\mu\omega^2 - k_z^2$ and made the substitution $y = \mathrm{Im}\,\chi$. The dispersion relations $f_n^{TE,TM}$ provide us with our electromagnetic modes where $f_n^{TE,TM}(\infty)$ are the dispersion relations when there is no boundaries in the system and $\Gamma(s)$ is the gamma function.

3.2 Results and Discussion

The Casimir energy for the dielectric cylindrical layer, given by Eq. (3) is solved using analytical and numerical techniques [3] and various limiting cases in terms of the radial dimension, curvature, and material composition of the cylindrical layer are considered. Also, by introducing the parameter $\xi = \frac{\varepsilon - \varepsilon_m}{\varepsilon + \varepsilon_m}$, it allows us to make calculations for practically important limiting cases such as a dielectric-diamagnetic cylindrical layer as well as two concentric perfectly conducting thin shells.

Fig. (2a) indicates that the energy is negative, and in the limit $\alpha \to 1$, where $\alpha = R_2 / R_1$, the energy behaves as $|E_C| \sim 1/(\alpha - 1)^3$. Fig. (2a) and Fig (2b) show that in the limit $R_2 \to \infty$, the system approaches the limit of a single perfectly conducting cylindrical shell [13] and $E_C \to 0$. The Casimir energy per unit area of a dielectric-diamagnetic plate [14] in the limit of $R_1, R_2 \to \infty$ when $d = R_2 - R_1 = const$ is also recovered.

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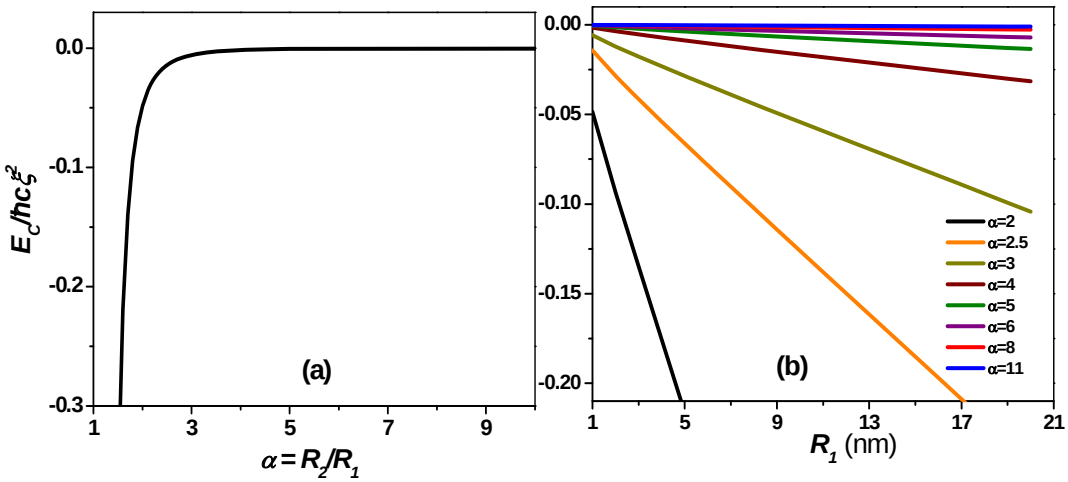

Figure 2: Dimensionless Casimir energy per unit length for the cylindrical dielectric layer as a function of **(a)** – the ratio of the outer and inner radii R_2/R_1, **(b)** – the inner radius R_1.

4 CASIMIR ENERGY OF PERFECTLY CONDUCTING CYLINDRICAL SHELLS

4.1 Model and Calculations

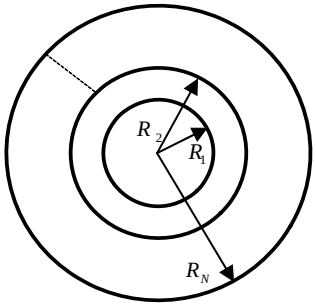

Figure 3: Infinitely long perfectly conducting and concentric cylindrical shells.

The second system considered is that of N perfectly conducting, infinitely long, concentric cylindrical shells immersed in a medium. The radii of the shells are R_i where $i=1,2,\ldots N$. (see Fig. 3).

In order to calculate the Casimir energy for the system of perfectly conducting cylindrical shells, one needs to remove the divergences present. This is achieved by taking the difference between the energy of the system of concentric shells and the energy of the individual isolated cylindrical shells. In this way the Casimir energy is expressed in a more transparent way and the remaining divergences are cancelled out [4]. After doing the appropriate modifications, the Casimir energy becomes of the following form:

$$\tilde{E}_C = -\frac{\hbar c}{8\pi} \times \sum_{n=-\infty}^{\infty}\int_0^{\infty} dy y^2 \frac{d}{dy}\ln\frac{f_n^{TE} f_n^{TM}\left[f_{n,i}^{TE}(i\infty) f_{n,i}^{TM}(i\infty)\right]^N}{\prod_{i=1}^{N}\left[f_{n,i}^{TE}(iR_i y) f_{n,i}^{TM}(iR_i y)\right] f_n^{TE}(i\infty) f_n^{TM}(i\infty)} \quad (4)$$

The dispersion relations here are given as $f_n^{TE,TM}(iR_1 y, iR_2 y, \ldots, iR_N y)$ for N shells with $y = \mathrm{Im}\,\chi$. $f_{n,i}^{TE,TM}(iR_i y)$ are the dispersion relations for a single cylindrical shell with radius R_i and $f_{n,i}^{TE,TM}(i\infty)$ are the dispersion relations with no boundaries present [4].

4.2 Results and Discussion

To illustrate the behavior of the Casimir energy for multiple shells, we will consider the case $N=3$ shells. First case we consider is when the radii of the shells are varied in such a way as to keep the distance between them constant (see Fig. 4(a)). We find that in the limit of $R_1, R_2, R_3 \to \infty$ and $d_1 = d_2 = const$ where $d_1 = R_2 - R_1$ and $d_2 = R_3 - R_2$, the Casimir energy per unit area behaves as $E_C \approx -\hbar c\pi^2\left(1/720 d_1^{\,3} + 1/720 d_2^{\,3}\right)$ confirming a result obtained in Ref. [15].

Fig. 4(b) illustrates the case when the inner radius is kept constant, while the outer two are varied. As R_2 increases, the Casimir energy becomes practically

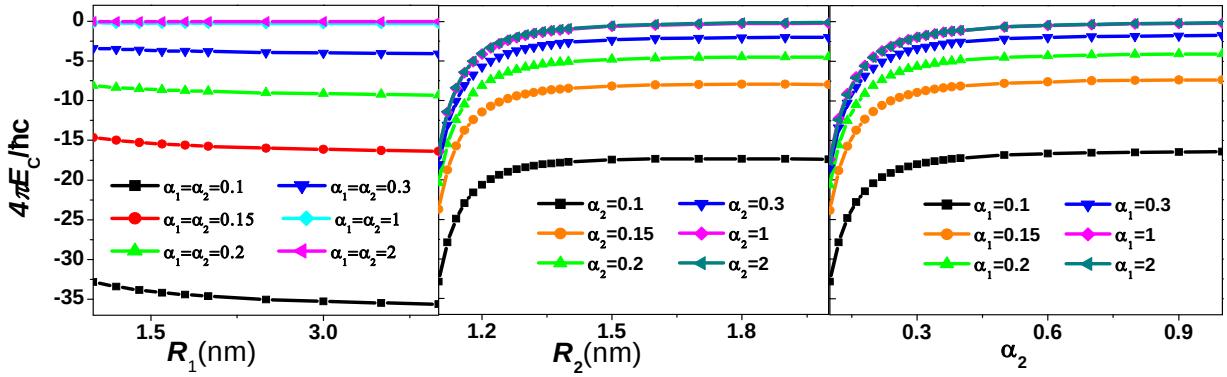

Figure 4: The dimensionless Casimir energy for the case of *N=3* shells: **(a)** as a function of the inner radius R_1; **(b)** as a function of the radius of the second shell R_2; and **(c)** as a function of separation between the two outer shells. Here $\alpha_1 = R_2 / R_1$ and $\alpha_2 = R_3 / R_2$.

a constant with our system behaving more like a single cylindrical shell and two perfectly conducting plates. The final case considered consisted of keeping the two inner radii constant while the outer one, R_3, is varied (see Fig. 4(c)). For the $R_3 \to \infty$ limit, the energy approaches that of two perfectly conducting cylindrical shells infinitely separated from a conducting parallel plate [4].

5 CONCLUSION

In this paper, we have investigated the zero-point energy for two different systems, the first one being a cylindrical layer with a finite thickness and the other one a system of N perfectly conducting, infinitely long cylindrical shells by making use of the mode summation method. We were able to successfully remove the divergences for both cases and obtained physically finite results for various interesting limits. For instance, we recovered the well-known Casimir formula for the energy per unit area of two parallel perfectly conducting plates [1] separated by a distance $d = R_2 - R_1 = const$ in the limit of $R_1, R_2 \to \infty$, when we considered the model of cylindrical layer. For our second model, we analyzed various limits in the case of three shells, and found that our result agrees with Ref. [15] in the limit of $R_1, R_2, R_3 \to \infty$ and $d_1 = d_2 = const$ where $d_1 = R_2 - R_1$ and $d_2 = R_3 - R_2$, in which case our system corresponds to three parallel plates.

In regards to practical applications, the case of perfectly conducting concentric cylinders might be of particular interest as a qualitative model of the Casimir interactions in a multi-wall carbon nanotube system. More thorough and realistic analysis is necessary to describe the Casimir interaction in multi-wall carbon nanotubes, by taking into account realistic electromagnetic properties.

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