Analog Nanoelectronic Computation: I. Simulation Of Dynamical Systems

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ABSTRACT

We describe here a new strategy for computation and computer design optimized for simulation of complex (chaotic) dynamical systems. This strategy has the potential for performance many orders of magnitude higher than conventional digital computers. It combines three emerging technologies, collectively referred to as *nanologic*: (1) analog nanoelectronics; (2) adaptive analog arrays; and (3) topologic. Analog nanoelectronics provides the opportunity to develop circuits using fewer (but more complex) devices, fully parallel computation, lower power, higher speed, and simpler architectures. Adaptive analog arrays provides for prototyping, adaptive precision, and self-optimization. Topologic means that the simulating circuit has the same global topology as the system being simulated; it provides a simple design path to the most efficient analog architecture. In this paper we describe nanologic, and show examples of its application to simulation of chaotic dynamical systems.

Keywords: dynamics, analog computing, nanoelectronics, nanologic, topologic

1. INTRODUCTION

One of the most extensive uses of computers is *simulation*. Unfortunately, there is a large class of such computations that cannot be done with conventional digital computers. These problems are part of a larger group of computational problems referred to as *intractable.*

Among the important classes of simulation problems that are often intractable are dynamical systems exhibiting nonlinear, complex, and chaotic behavior [1,2,3]. The goal in studying such systems generally is not to describe the exact behavior of a particular system in time, but rather to understand the behavior of a set of similar systems. This is essentially a topological concept. Indeed, the current trend in complex dynamics is to classify behavior using *topological analysis* [4,5].

In this paper, we present a new approach for computation and computer design for simulating complex dynamical systems, exploiting the good match of analog nanoelectronics to topological dynamics. We will effect this using a concept we refer to as *topologic*, which is one of three components of the strategy called *nanologic*.

2. CIRCUIT ANALOGUES OF TOPOLOGICAL DYNAMICAL SYSTEMS

Chaotic dynamical systems are commonly described by a topological structure called a branched manifold (BM), which is an idealization and dimensional reduction of the attractor. The branched manifold is 2D ribbon aggregation embedded in a 3D space. The system point moves cyclically on this ribbon. FIG. 1(a) shows a typical branched manifold, taken from the work of Gilmore [4].

Our first step in this work is to idealize the topological structure of the BM to a geometrical structure we will call a model branched manifold (MBM), shown in Fig. 1(b). The MBM is topologically identical to the BM.

Next, we note that any MBM can be assembled from a small number of canonical pieces, shown in Fig. 1(c). These include pieces for extension, scaling, shifting, bending, twisting, rotating, splitting one branch into two, and merging two branches into one. The pieces have a flow direction, corresponding to the direction of flow of time.

The pieces can be assembled into arbitrary complete MBMs according to simple rules: the pieces must join in their common plane, there must be no overlaps, the edges must match, the angles of all bends, turns, and twists must be 90º, the manifold must be closed, and the flow must be coherent. This is indicated in Fig. 1(d).

Locally, each piece can be represented by 3 variables: the independent dynamical variable x along a transverse cut called the Poincare section; the density of orbits ρ across the section; and the direction of time t. These will be associated with the 3 laboratory axes 1,2,3, as shown in Fig. 1(e).

Since each piece has 3 variables, we can associate a 3-wire circuit with each piece, such that the circuit behaves as an electronic analogue of the piece. For instance, in Fig. 1(f) we show the split piece and its analogue circuit. Line 1 of the circuit (the analogue of the independent variable) is sensed, and an analog switch opens one of the two output 3-wire circuits. The current flowing into this piece is thereby switched into the proper outgoing circuit, depending on the value of the independent variable, which is how the attractor, and the BM, work. We have devised a set of simple circuit analogues for all the canonical pieces of the MBM. We call these *circuit manifold pieces*.

Just as the MBM can be assembled by connecting the pieces together, we can assemble a complete circuit analogue by connecting the circuit manifold pieces. Fig. 1(g) shows the assembly of a split piece and a merge piece, and its circuit analogue. Any of the circuit pieces can be attached to any other, building up a complete circuit that has the same global topology as the MBM. Thus, Fig. 1(h) shows the MBM and Fig. 1(i) shows the analogue circuit. We call such circuits *circuit manifolds* (CM).

Although the full MBM has 3 variables, and therefore the CM has 3 wires, if the CM has no knots we can use a 1-wire circuit that performs repeated transformations of the independent variable. Fig. 1(j) shows a fragment of a 1-wire CM. This circuit is labeled to show the parts that correspondence to the split, twist, scale, and merge parts of the MBM.

An example of a complete 1-wire CM is shown in Fig. 1(k), an 18-module ring. The node voltages on the ring correspond to the values of a period-18 unstable orbit of a chaotic dynamical system. As emphasized by Gilmore, such unstable orbits serve to classify the major modes of behavior of a chaotic dynamical system.

Fig. 1(l) shows how 1-wire modules can be assembled to carry out operations that are the analogues of iteration. Here, the successive modules perform successive transformations that are the analogues of iteration steps.

Computation is done with these circuits by fixing some circuit values (parameters), varying some values (independent variables), and measuring the remaining values (dependent variables). The relation between the dependent variables and the independent variables constitutes the desired simulation of the physical system.

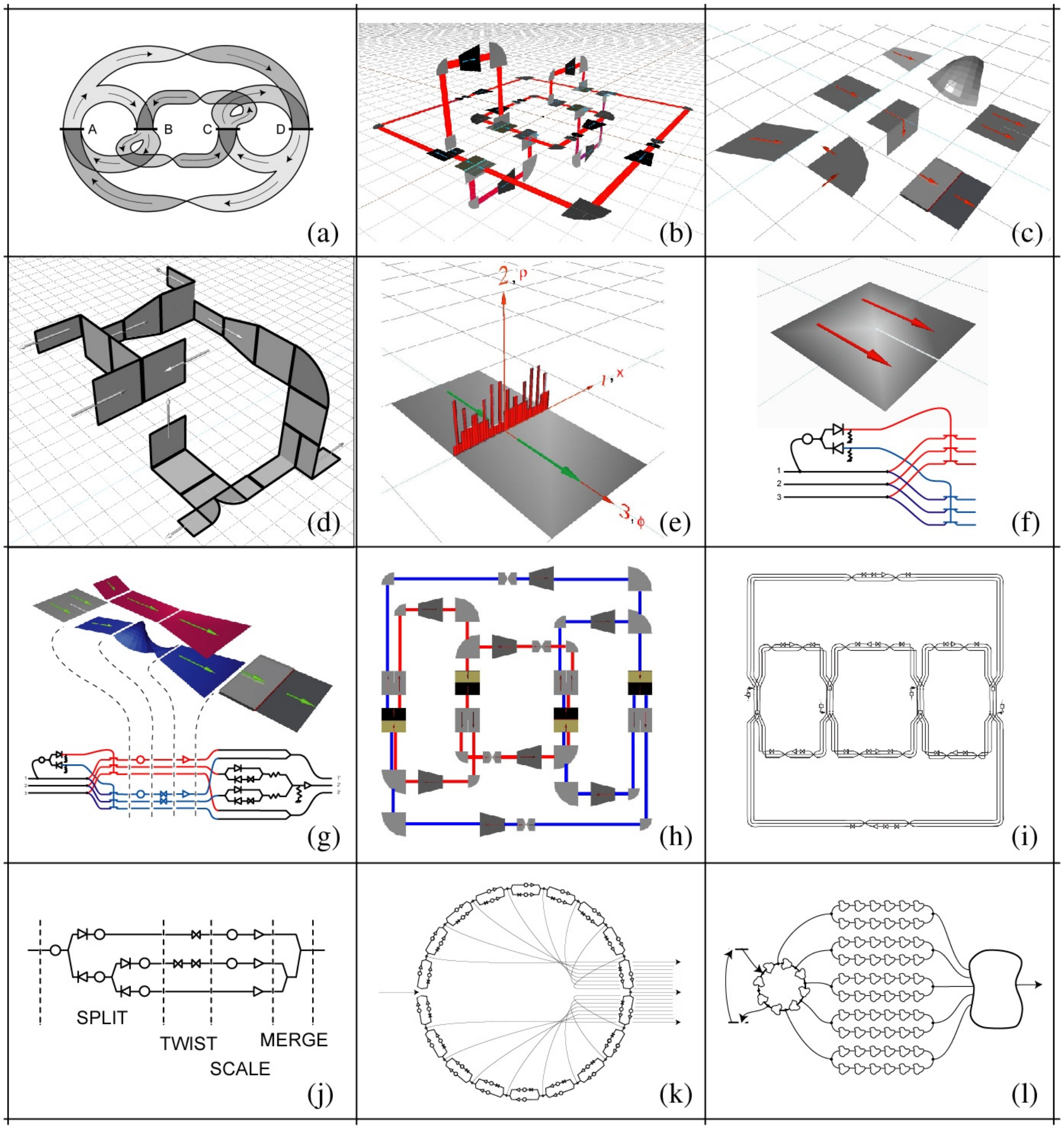

Figure 1 – Summary of the major concepts comprising analog simulation of topological dynamics.

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3. ANALOG CIRCUITS

The present strategy is intrinsically an analog scheme. This arises from several considerations:

(1) Compatibility of analog data with human-important problems, especially involving chaotic systems found in Nature;

(2) Greater speed due to processing of the problem in full parallelism using the entire machine;

(3) Compatibility of analog circuits with nano-electronic devices, which have complex behavior that is difficult to force into simple discrete or linear behavior.

We expect that these circuits to be fabricated as VLSI chips containing many analog functional blocks. Hence we envision the chips as being field-programmable analog arrays (FPAA). There is a currently vigorous research and development activity in the field of analog array processors [6,7]. Experience by Hasler [8,9] suggests the feasibility of an array of 10^4 configurable analog blocks on a single chip

4. NANOELECTRONICS

The third component of the present strategy (after topologic and analog arrays) is *nanoelectronics*. The introduction of nanoelectronic devices in place of the simple linear devices shown in Fig. 1 presents the opportunity to develop circuits with very high behavioral complexity while maintaining low device count. For instance, stacks of resonant tunneling diodes (RTD) can be used to generate nonlinear and discontinuous maps that represent mathematical catastrophes, which are used to describe many types of physical systems that exhibit sudden changes, such as eddies in turbulent fluids, deformation of structures, and rapidly changing social systems. The advantage of using higher complexity devices in computers is well-understood.

Devices are readily available [10] that can be considered hybrid continuous/discrete, or analog/digital. Using such devices in a processor will endow that processor with the ability to store and process hybrid analog/digital data, which is the province of the present strategy. Nanoelectronics also offers advantages in lower power, higher speed, fewer devices, and simpler architectures.

Both individual nanoelectronic devices and VLSI analog electronics have been developed to engineering subjects. We note that despite the availability of a large variety of nanoelectronic devices [11], technology for fabricating nanoelectronic analog VLSI circuits is not widely available. We emphasize that the present strategy in no way depends on the availably of such technology (it can be fabricated using any electronic technology). Stability of such devices will be a critical issue [12].

5. IMPLEMENTATION

For analog circuits, *programming* implies the process of physically assembling the circuits and setting parameter values. This is essentially an *aggregation* process. The present strategy, based on modules assembled by connecting them in topological arrays, is intrinsically an aggregation procedure. However, in order to keep the computer from growing without bound, we will have to have some process for replacing a complex assembly of devices with an equivalent (and presumably simpler) circuit. This process is referred to as *evaluation*. Repeated cycles of aggregation/evaluation constitute *computation*.

The present strategy blurs the distinction between *data* and *control*, hence blurs the distinction between data and programming. It is simply not meaningful to separate data and programming, either logically or physically. Problem solving should be thought of as a single process of *specification* and *solution*: the solution emerges simultaneously with the specification of the problem. In this sense, even the aggregation/evaluation paradigm is inadequate to describe the operation of the analog circuit manifolds described in this invention. To a great extent, programming *per se* is inherently meaningless in the present strategy.

It is natural to ask about the performance of the circuit manifolds described herein as computers, and whether they could compete favorably with conventional computers. While it is perhaps dangerous to compare the mature digital technology with the incipient nanoelectronic analog array circuit manifold technology described here, we can offer a rough basis for such a comparison:

(1) <u>VLSI technology</u>: Nanoelectronics offers advantages of reduced device size and power, and increased device density, speed, and complexity; these factors should provide an advantage to nanoelectronic circuit manifolds of $10\text{-}10^3$.

(2) <u>Logic</u>: Analog arrays can have throughput $10^2\text{-}10^5$ times digital arrays, as shown by Twigg and Hasler [9] and others.

(3) <u>Architecture</u>: Assembling the circuit manifolds as topological analogues of the system attractor should bring advantages in throughput of $10^2\text{-}10^4$.

Table 1 brings these advantages together. On the bottom line, we make a rough estimate of what we might expect in combining them. The smallest figure (10^3) refers to a period of research, technology demonstration, and verification. The intermediate figure (10^6) refers to a period of product development. The highest figure (10^9) refers to projected optimized products.

Clearly, the overall efficiency of these circuits will depend on the extent to which nanoelectronics can be included.

TABLE 1

	Conventional digital	Nanologic hybrid	Advantage NL/digital.
Technology	Microelectronics	Nanoelectronics	$10\text{-}10^3$
Logic	Digital arrays	Analog arrays	$10^2\text{-}10^5$
Architecture	Bit storage	Topologic	$10^2\text{-}10^4$
		TOTAL	$10^3\text{-}10^6\text{-}10^9$

6. DISCUSSION

Most human-important problems are qualitative: we neither have complete and precise input, nor do we want complete and precise output of instances of solutions. Rather, we need qualitative information about the system, its general dependence on controls, and discovery of unexpected features. Problems in the following areas are often of this kind: artistic expression, climate and weather, control, engineering, image processing, pattern recognition, language, medicine, politics, and psychology.

As an example, consider the challenge of planning for global climate change. This system has many aspects that would make it a good candidate for the kind of approach we describe here. What we seek is relatively coarse simulations that generate scenarios, together with indicators of their dependence on controls (e.g., petroleum exhaustion, introduction of nuclear energy, deforestation, land use changes, etc.), rather than a detailed high-precision description of a single instance.

Appropriate applications of this approach will have the following characteristics:

(1) The problem is important;
(2) Input data is qualitative, incomplete, ambiguous;
(3) Physical models do not exist;
(4) The system behavior is complex;
(5) The behavior has both local and global character;
(6) It involves numerous dynamic processes;
(7) The system may exhibit discontinuities;
(8) The dynamics may be chaotic;
(9) Conventional computers can't solve the problem;
(10) The behavior has structure;
(11) We do not need detailed precise data as output;
(12) We are interested in controlling the system;
(13) We don't care about the details.

It may be asked how we can specify circuits and problems in this seemingly vague, nonspecific domain. The answer is inherent in the topological foundation of this approach: we are not demanding numerical agreement of a simulation with a real physical system, but the qualitative behavior of a set of systems connected by control parameters. Thus, we can be rather cavalier in the details—we can miss the behavior quantitatively by a lot—but we look for qualitative aspects of the behavior, in the hope and expectation that the simulation will give us some insight into the behavior of the system and how to control it. We need not be concerned with whether the fragments agree in detail with a real physical system. Although this may sound hopelessly sloppy, it is not—it is in fact the central motive for attacking intractable simulations, namely to find out (roughly!) "what's happening" [13].

Chaotic dynamical systems is exemplary of the large class of problems that cannot be solved with conventional digital computers but can be successfully attacked with the present approach. This approach derives its exceptional advantages from the casting of the computational algorithm into an electronic analog circuit that is a physical analogue of the system being simulated. The two systems are, by design, topological equivalents. Implementing these circuits with nanoelectronics will provide significant increase in performance. The challenges of analog nanoelectronics are well-met by the projected payoff of large-scale analog simulation.

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