

# High-stability numerical algorithm for the simulation of deformable electrostatic MEMS devices

X. Rottenberg<sup>1,2</sup>, B. Nauwelaers<sup>2</sup> W. De Raedt<sup>1</sup> and D. Elata<sup>3</sup>

<sup>1</sup> IMEC v.z.w., Division MCP, Kapeldreef 75, B3001 Leuven, Belgium  
xrottenb@imec.be

<sup>2</sup> K.U.Leuven, ESAT, Kasteelpark Arenberg 10, B3001 Leuven, Belgium

<sup>3</sup> Technion, Faculty of Mechanical Engineering, Haifa 32000, Israel

## ABSTRACT

This paper presents a novel high-stability electrically-driven algorithm for the simulation of the electro-mechanical actuation of electrostatic MEMS devices. The stability of this algorithm improves on that of voltage- and charge-drive algorithms. Key in our algorithm are the use of a local charge density as driver for an adapted relaxation algorithm and the adequate selection of the bias node in the mesh. The high stability of this algorithm allows probing the electromechanical equilibrium locus way beyond the V- and Q-drive pull-in instabilities. The new algorithm allows investigating the effect of dielectric charging in deformable electrostatic MEMS devices and especially the narrowing of their equilibrium locus due to dielectric charging non-uniformities. We implement this algorithm in 2D for clamped-clamped beams of rectangular cross-section and take into account, among other things, distributed dielectric thickness, permittivity, rest air gap, actuation electrode and linearly distributed dielectric roughness.

**Keywords:** dielectric charging, electrostatic actuation, numerical algorithm, high stability

## 1 INTRODUCTION

Electrostatic actuators are commonly used MEMS building blocks for various applications. In the case of RF-MEMS devices, these actuators are used for example to realize resonators, tunable capacitors and capacitive switches. The general actuation characteristics of typical devices, e.g. parallel-plate or comb-drive actuators, are well known and described in analytical form, e.g. stability or unstability of the voltage-drive actuation of parallel-plate actuators. Designers have however to turn to numerical techniques to assess the actuation characteristics of complex non-standard devices or of standard devices perturbed by parasitic phenomena, e.g. dielectric charging.

In a previous work [2], Rottenberg et al. extended the analytical model for the electrostatic actuation of MEMS devices in the presence of parasitic uniform surface charging of the dielectric [3] in order to account for non-uniform distributions of charges in the bulk of the dielectric and of air gaps at rest in the device. They showed in particular that the spatial covariance of charge and rest air

gap distributions breaks the symmetry of the actuation characteristics and that the spatial variance of the charge distribution can produce a narrowing and finally a closure of the pull-out (-in) window that can result in the stiction (self-actuation) of the device. Strikingly, these phenomena can even occur for a total zero trapped charge.

The analytical model developed in [2] assumes rigid plates. Under this assumption, only the mean, variance and covariance of the charge and rest air gap distributions have an impact on the actuation of the device. In actual devices however, the mobile electrodes deform during the actuation. This introduces a non-uniform distribution of the air gaps in the structure, non-present in rest position, and modifies the impact of the trapped charges on the actuation characteristics. This modification was illustrated in [5] for the clamped-clamped beam shown in Figure 1. However, the poor stability of both voltage- and charge-drive numerical algorithms implemented limited the demonstration of the impact of conjunct distributed charging and progressive deformation.

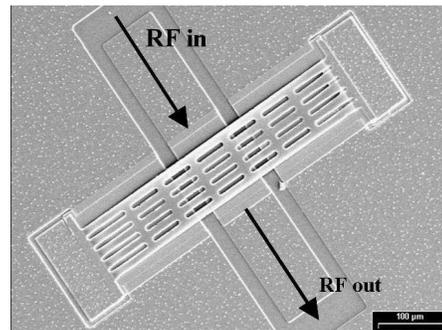


Figure 1 SEM of a typical RF-MEMS shunt capacitive switch in thin-film technology

This paper presents a novel high-stability algorithm for the simulation of the electromechanical actuation of electrostatic MEMS devices. Its stability improves on that of voltage- and charge-drive algorithms. It further offers a purely electrical simulation alternative to previous work [1] based on displacement control of the electromechanical response to drive the simulation. Key in our algorithm are the use of a local charge density as driver for an adapted relaxation algorithm and the adequate selection of the bias node in the mesh. The high stability of this algorithm

allows probing the electromechanical equilibrium locus way beyond the voltage-drive and charge-drive pull-in instabilities. The new algorithm facilitates investigation of the effect of dielectric charging in deformable electrostatic MEMS devices and especially the narrowing of their equilibrium locus due to dielectric charging non-uniformities [2]. We implement this algorithm in 2D for clamped-clamped beams of rectangular cross-section and take into account, among other things, distributed dielectric thickness, permittivity, rest air gap, actuation electrode and linearly distributed dielectric roughness.

## 2 ALGORITHM DESCRIPTION

Figure 1 shows a typical RF-MEMS capacitive shunt switch, consisting of a 1/500/100 $\mu\text{m}$  thick/long/wide Al bridge under 50MPa tensile stress. The bridge is suspended 2 $\mu\text{m}$  above a 100 $\mu\text{m}$  wide actuation electrode coated with a 200nm thick high-k ( $\epsilon_r=25$ ) dielectric. It is modelled as the perfect clamped-clamped beam of Figure 2 and discretized along its length as depicted in Figure 3 and detailed in [4].

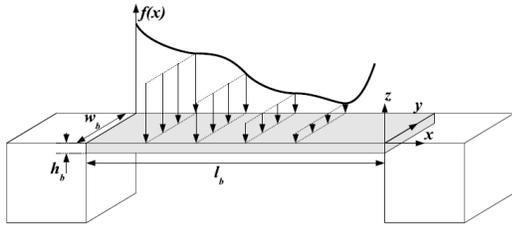


Figure 2 Shunt capacitive switch modelled as an ideal clamped-clamped uniform beam with distributed electrostatic load.

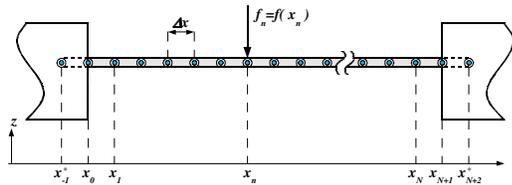


Figure 3 Discretization scheme for the simulation of an ideal clamped-clamped uniform beam.

For such a deformable device, both V- and Q-drive are unstable. Indeed, even under fixed  $Q_{\text{bias}}$ , charges can reorganize on the bridge and flow in an avalanche towards the valley defined by the deformed device. A more stable actuation scheme is thus needed in order to probe the actuation-profile way through  $V_{\text{PI}}^{+/-}$  and  $Q_{\text{PI}}^{+/-}$ , respective limits of the stable V- and Q-drives.

A simple numerical way to prevent the destabilizing Q-drive charge redistribution is to fix the local charge on a selected node  $n$ , i.e.  $q(x_n)=q_n$ , instead of fixing the total charge on the device, i.e.  $Q_{\text{bias}}$ . While the V-drive and Q-drive algorithms imposed a global electrical variable, the novel  $q_{\text{I}}$ -drive algorithm imposes a local electrical variable.

It is as a result more binding and more stable. Remark that this algorithm is in a way more subtle than that proposed in [1] as it does not impose a local geometrical effort, i.e. the displacement of a node, but a local electromechanical effort. Imposing the bias  $q_n$  corresponds indeed to imposing the force applied to the node  $n$ .

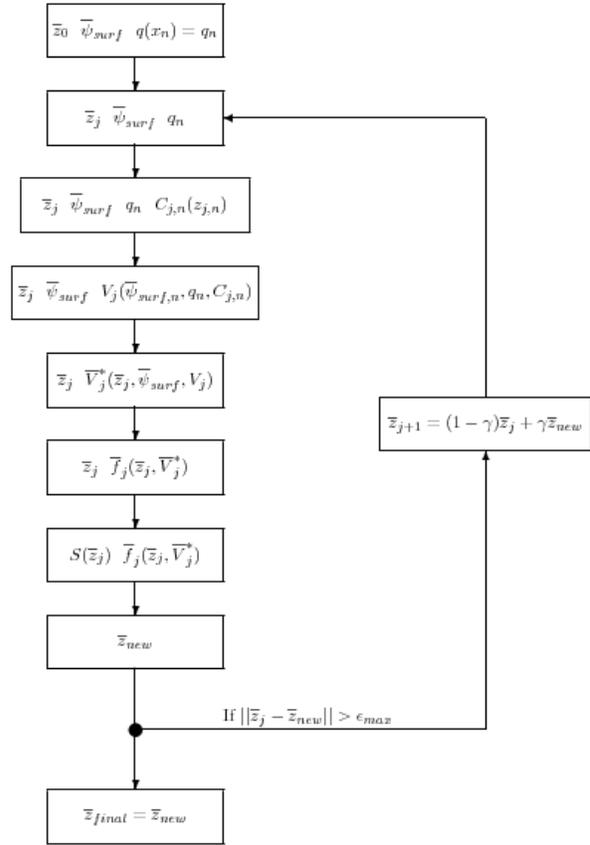


Figure 4 Numerical algorithm for the local-charge actuation of deformable devices with distributed surface charge.

The novel iterative relaxation scheme is detailed in Figure 4. It is a slightly modified version of the V-drive algorithm. From the deformation at a given iteration step, i.e.  $\bar{z}_j$ , the capacitance  $C_{j,n}$  associated with the bias-node  $n$  is computed. From this capacitance, the algorithm computes the voltage  $V_j$  needed to accommodate the forced charge located on the node  $n$ , i.e.  $q(x_n)=q_n$ , in association with the equivalent surface charge distribution located under the same node, i.e.  $\psi_{\text{surf},n}$ . With this voltage and the other parameters, the iteration is carried on following a modified V-drive scheme based on [2] and [4]. The distributed effective voltage  $V^*$  and resulting electrostatic force are first computed. The displacement  $\bar{z}_{\text{new}}$  balancing this distributed force is then evaluated. Should the algorithm have converged, the simulation stops. Otherwise, a step is performed from  $\bar{z}_j$  in the direction of  $\bar{z}_{\text{new}}$  and a

novel iteration starts. At each iteration, the voltage on the beam is thus recomputed to maintain the local charge on the bias node.

A simple physical insight brings a solution to the numerical stabilization of the simulation. Remark nevertheless that imposing the charge on a portion of a metal armature is highly non-physical. The proposed stabilization technique remains thus a numerical trick in opposition to the Q-drive that can be physically implemented with its limitation [5][6].

### 3 SIMULATION RESULTS

Figure 5 shows the result of the simulation of the device from Figure 1 using the local-charge algorithm in absence of trapped charge in the dielectric.

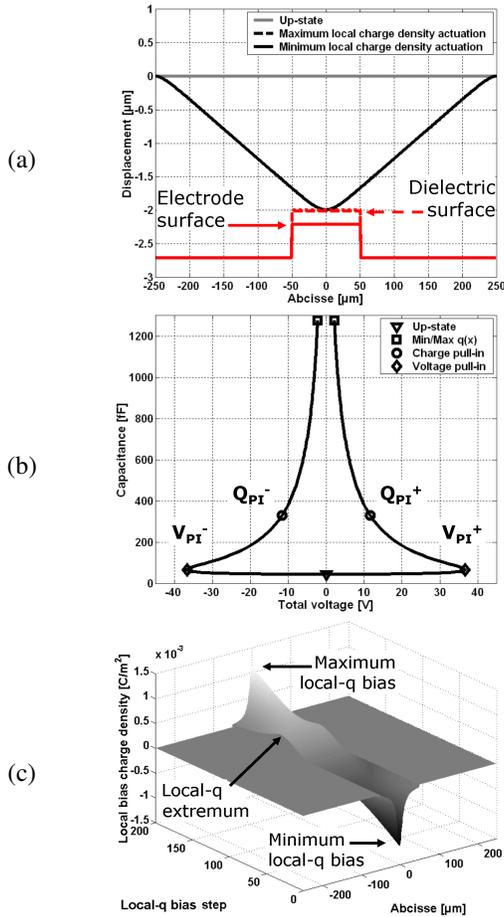


Figure 5 Local-charge drive simulation of a clamped-clamped beam in case of ideal uncharged dielectric layer; (a) beam deformations at maximum opening and at extreme stable positions for the local-q drive algorithm, (b) C-V profile stable for the local-q actuation, (c) charge density on the bridge during the local-q actuation.

Figure 5(a) shows the deflections at rest and at contact with the dielectric layer, demonstrating the stability of our algorithm. Figure 5(b) depicts the symmetric C-V profile up

to contact (diamond and round markers show respectively voltage and charge pull-in's). The charge map Figure 5(c) shows that the centre node is a proper bias node as its local-q monotonically increases. In contrast, the local-q above the electrode-edges exhibits two extrema, similar to the Q-drive instabilities. In this case, working with off-centre bias nodes may partially stabilize the simulation. A general strategy for bias node selection consists in choosing the node with the highest rate of change of its local charge density.

In presence of centered linearly distributed trapped charges in the dielectric, the V-drive presented in Figure 6(a) partially depicts the expected narrowed and asymmetric C-V due to the covariance of deflection and charge distributions [2]. At maximal opening, the beam is not flat. The maximal deflection is noticeably different for  $V_{PI}^+ > 0$  and  $V_{PI}^- < 0$ .

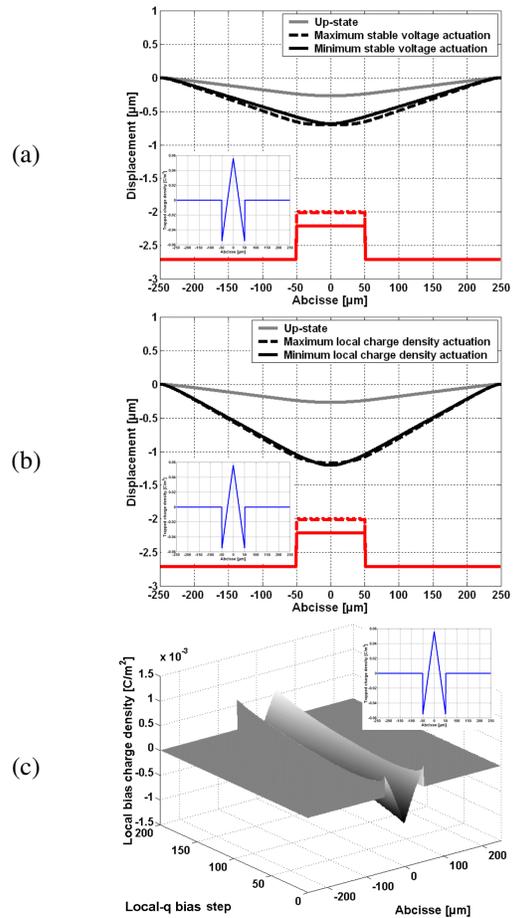


Figure 6 Simulation of a clamped-clamped beam in case of centred linear symmetric equivalent surface charge distribution trapped in the dielectric layer with zero mean; beam deformations at maximum opening and at extreme stable positions for (a) the voltage drive algorithm and (b) the local-q drive algorithm, (c) charge density on the bridge during the local-q actuation.

In comparison, the local-q-drive, Figure 6(b), tracks the equilibrium much further. Positive and negative branches of the locus converge. So do the beam shapes. The charge map Figure 6(c) strongly differs from Figure 5(c). The local-q of each node exhibits an extremum. The bias node had therefore to evolve during the actuation from the centre to the edges respectively at low and high local-q.

Figure 7 presents a comparable but more realistic case of a centered, net positive, cosine charge distribution in the dielectric. The portion of equilibrium locus made accessible by the local-q simulation shows that, as expected, the C-V profile is clearly shifted upwards in voltage, narrowed and asymmetric. Despite this voltage shift, the charge map remains comparable to that in Figure 6(c).

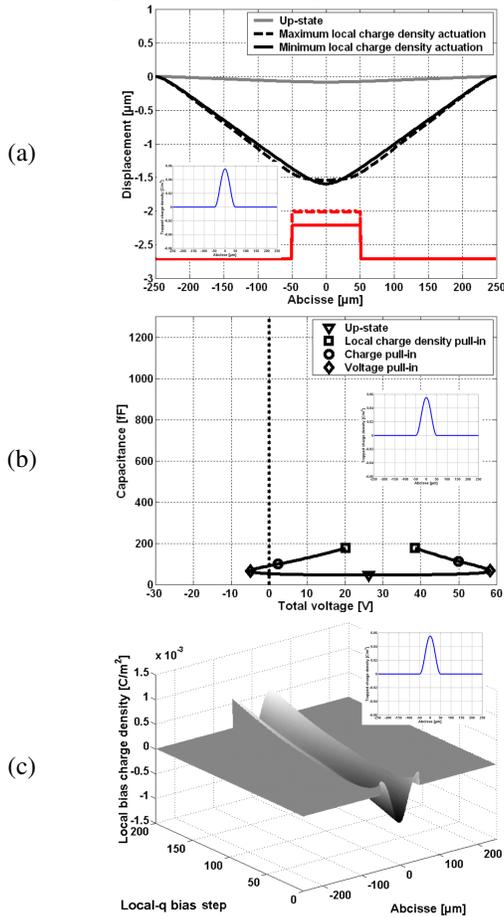


Figure 7 Local-charge drive simulation of a clamped-clamped beam in case of centred positive cosine equivalent surface charge distribution trapped in the dielectric layer; (a) beam deformations at maximum opening and at extreme stable positions for the local-q drive algorithm, (b) C-V profile stable for the local-q actuation, (c) charge density on the bridge during the local-q actuation.

## 4 CONCLUSIONS

We presented a novel high-stability algorithm for the simulation of the electromechanical actuation of electrostatic MEMS devices. Its stability improves on that of voltage- and charge-drive algorithms. It further offers a purely electrical simulation alternative to previous works based on displacement control of the electromechanical response to drive the simulation. Key in our algorithm are the use of a local charge density as driver for an adapted relaxation algorithm and the adequate selection of the bias node in the mesh. The high stability of this algorithm allows probing the electromechanical equilibrium locus way beyond the voltage-drive and charge-drive pull-in instabilities. The new algorithm facilitates investigation of the effect of dielectric charging in deformable electrostatic MEMS devices and especially the narrowing of their equilibrium locus due to dielectric charging non-uniformities. We implement this algorithm in 2D for clamped-clamped beams of rectangular cross-section and take into account, among other things, distributed dielectric thickness, permittivity, rest air gap, actuation electrode and linearly distributed dielectric roughness.

## 5 ACKNOWLEDGEMENTS

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