Dielectrophoretic ratchets for linear and rotary nanomotors

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ABSTRACT

Dielectrophoresis (DEP) is a phemomenon of induced particle motion in non-uniform electric fields. In this paper, we will describe how the phenomenon can be used to design electrokinetic ratchet devices to provide both linear and rotary motion. Such devices exploit Brownian motion through the imposition of an asymmetrical potential energy field, to provide motion. Since Brownian motion is a prerequisite, such devices are most applicable to the transport of nanoparticles. In this paper, we present the results of modelling of such devices capable of linear translation, and of particular novelty, rotary motion and show the torque of the rotary motor is of the order of that generated by a bacterial motor.

Keywords: nanomotor, dielectrophoresis, ratchet, simulation

1 INTRODUCTION

Thermal (or Brownian) ratchets are devices that employ asymmetrical potential energy fields for the rectification of Brownian motion [1-3]. Pulsed (or flashing) ratchets are a sub-class of ratchet structures [4] that operate by periodically imposing an asymmetric potential energy profile across a volume containing particles of sufficiently small size to allow for significant Brownian motion, as shown schematically in figure 1. When this potential energy field is applied, the particles collect at the points of lowest potential energy. After the particles have collected, the potential energy field is released, allowing the particles to diffuse away by Brownian motion. After a period the field is reapplied, forcing most particles to return to the same energy minimum. However, since the potential energy profile is asymmetrical, particles diffusing in one direction have a greater chance of diffusing out of the local potential energy well and into the area of effect of the adjacent well. Those particles which migrate across this barrier will collect in the next potential energy minimum along after the field is reapplied, as indicated by the arrows in the figure. It can be seen that more particles move to the right than to the left, and there is therefore a net drift of particles to the right.

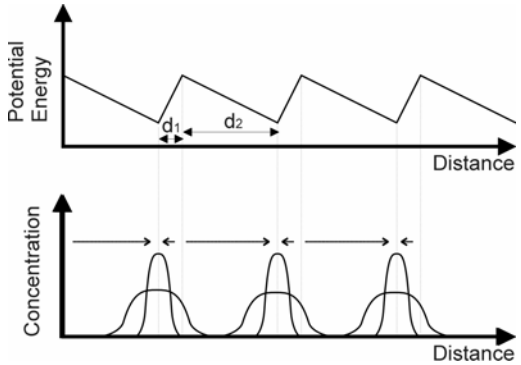

Figure 1. A schematic showing how the repeated application of an asymmetric potential energy profile can rectify Brownian diffusion. When the field is applied, the concentration of particles is localised at the potential energy minima; once released, they diffuse away. Those diffusing a distance greater than d_1 to the right will be collected at the next potential minimum to the right, causing a net drift rightwards.

One method of realising an asymmetric potential energy profile is through the use of an electric field generated by asymmetrical electrode structures of appropriate geometry. If an electric field varies in an asymmetric but periodic manner along an axis, it can be used to attract particles to regions of high electric field by dielectrophoresis [5]. Dielectrophoresis is the induced motion of polarisable particles in non-uniform fields, which generates a force proportional to the gradient of the electric field squared. The application of dielectrophoresis for ratcheting was demonstrated to manipulate colloidal particles in dielectrophoretic ratchet structures. Particles responding to dielectrophoretic force are attracted up an electric field gradient (or may, under appropriate conditions, be repelled down it), so that the minimum potential energy is achieved at the point of highest electric field strength. Since the force is related to the electric field gradient, the greater this gradient is, the stronger the force on the particles, and the more effective the ratcheting mechanism. In this paper we describe how numerical modelling can be used to engineer dielectrophoretic ratchet devices capable of both linear for the propulsion of nanoparticles in electrode arrays.

Furthermore, we demonstrate through numerical simulation that similar principles can be applied to the development of *rotary* motors based on dielectrophoretic ratchets capable of producing similar torque outputs to bacterial rotary motors.

2 DIELECTROPHORESIS

When a dielectric particle is suspended in a spatially non-uniform electric field, the interaction of the applied field and induced dipole generates a force on the particle [6-8]. This force, termed dielectrophoresis, has been widely used as a means of manipulating particles on the micron scale. The dielectrophoretic force, F_{DEP}, acting on a homogeneous, isotropic dielectric sphere, is given by:

$$F_{DEP} = 2\pi r^3 \varepsilon_m Re\left[K(\omega)\right]\nabla E^2 \qquad (1)$$

where r is the particle radius, ε_m is the permittivity of the suspending medium, ∇ is the del vector operator, E is the local *rms* electric field and $Re\left[K(\omega)\right]$ the real part of the Clausius-Mossotti factor, given by:

$$K(\omega) = \frac{\varepsilon_p^* - \varepsilon_m^*}{\varepsilon_p^* + 2\varepsilon_m^*} \qquad (2)$$

where $\varepsilon_m{}^*$ and $\varepsilon_p{}^*$ are the complex permittivities of the medium and particle respectively, and $\varepsilon^* = \varepsilon - \frac{j\sigma}{\omega}$ with σ the conductivity, ε the permittivity and ω the angular frequency. The frequency-dependence of $Re\left[K(\omega)\right]$ indicates that the force on the particle varies with the frequency. The magnitude of $Re\left[K(\omega)\right]$ also varies depending on whether the particle is more or less polarisable than the medium. If positive, then particles move to regions of highest field strength (positive dielectrophoresis); the converse is negative dielectrophoresis where particles are repelled from these regions. Since a key attribute of div- and curl-free fields is that local field minima (though not local maxima) are permitted, electrode geometries may be devised which have enclosed potential energy minima at isolated positions near the geometry. Such local minima, bounded in all directions parallel to the plane in which the planar electrodes have been fabricated, allow the fabrication of particle "traps".

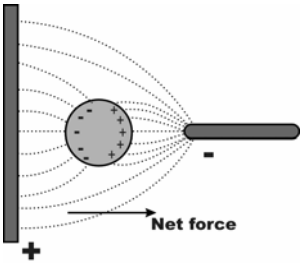

Figure 2. Dielectrophoresis occurs due to the interaction of a polarisable particle and a non-uniform electric field, and acts towards or away from the field gradient.

3. MODEL

In order to study the effectiveness of dielectrophoretic ratchets and their dependence on the electric field intensity across the electrode structure, electric-field ratchets were simulated using the Maxell 2D (Ansoft Inc., Pittsburgh USA) software suite. Simulations were performed to a final energy error of 0.1%, corresponding to about 5000 elements. The simulated electrodes were energised with ±5V. For the **linear** ratchet, the electrode array simulates was as shown schematically in figure 3a. The locations and values of the maxima and minima of the electric field strength were determined along the centre line between the electrodes. From this the following parameters were determined: a dimensionless potential energy asymmetry parameter (which can take values between 0 and 1, respectively representing symmetry and complete asymmetry), as given by the expression

$$\Lambda = \frac{d_2 - d_1}{d_2 + d_1}, \qquad (3)$$

where d_1 and d_2 are as shown in figure 1; the difference in electric field strength between maximum and minimum; and the "forward" and "backward" dielectrophoretic forces (with "forward" defined as the direction in which particle motion is biased) were also examined. For the **rotary** ratchet structure, the electric field was determined as generated by four stator electrodes is as shown in figure 3b. The rotary Brownian motor consists of two or more stator electrodes and a rotor. The stator electrodes are arranged around a central axis, and contain offset cutouts which, when placed together, form a circle. The electrodes are energized with sinusoidal potentials applied such that opposing voltages appear on adjacent electrodes. A dielectric medium such as water occupies the inter-electrode gap, which also contains the rotor.

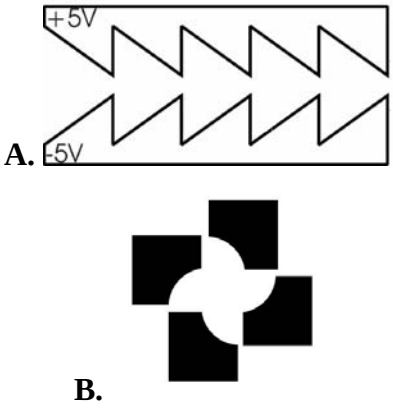

Figure 3. Geometries of linear (A, top) and rotary (B, bottom) ratchet electrodes.

4. RESULTS

4.1. Linear ratchets

Taking the asymmetry parameter first, it was found that the asymmetry of the system increases as the depth of the sawtooth is decreased, for all of the inter-electrode distances examined. There is also a logarithmic dependence on the distance between opposing electrode tips; the greatest value of asymmetry (0.7) was found where the electrode tips are 2.5μm apart, and the maximum depth of the sawtooth is 0.5μm. Analysis of the data for variation of all three parameters showed that the asymmetry can be fitted (RMS error of 9.3%) to the empirical expression

$$\Lambda = \left(\frac{a}{Xb + a} \right) e^{-\left(\frac{Yc}{a} \right)}, \qquad (4)$$

where a is the length of the period of one saw tooth, b is the gap between opposing electrode tips and c is the depth of the sawtooth. X and Y are dimensionless constants having values of approximately 5(±0.5) and 1.7(±0.1).

This result indicates that the maximum asymmetry can be found where the sawtooth is at the smallest variation from two parallel, straight electrodes seems counterintuitive; the case of two sawtooth electrodes with a sawtooth depth of 0μm is that of two parallel, straight electrodes with no variation in electric field strength. However, these two statements can be resolved by examining the difference between maximum and minimum electric field strength along the centre line. As the depth of the sawtooth is reduced, both the maximum and minimum electric field strength are increased by the same amount; examination of the difference between these values (which governs the potential energy profile) remains approximately the same (varying by no more than 10%) for all sawtooth depths for a given inter-electrode gap. This remains the case until the depth of the sawtooth is approximately 10-15μm, at which point the electric field maximum is almost equal to that expected in the parallel-plate case. As the depth of the sawtooth is reduced beyond that point, the minimum value of potential energy continues to rise as the sawtooth depth is reduced. Note is that this threshold is only weakly dependent on the inter-electrode gap, with the diminution of difference in electric field strength starting at smaller sawtooth depths for smaller inter-electrode gaps.

From these results, we find that the shape of the potential energy is not directly related to the geometry of the electrodes responsible for generating the potential energy field, but is in fact a complex function different aspects of that electrode geometry. Furthermore, the greatest asymmetry in the potential energy field occurs for very small deviations from the case of two parallel electrodes, though this does correspond to small changes in electric field strength along the inter-electrode gap. Where the magnitude of the force is taken into consideration, there exists what might be regarded as an optimum configuration where both the magnitude of the force and the asymmetry parameter are high, which occurs at ratchet depths of approximately 0.2-0.25x the length of the sawtooth period.

4.2. Rotary ratchets

Simulations of the structures shown in figure 3b indicate that the regions of highest electric field occur at the electrode corners, whilst the region of lowest electric field forms a line slightly offset from this first line; in angular terms, there is an asymmetry that reflect the one seen in the potential energy pattern in the linear ratchet case. A nonspherical rotor will align along the axis between the electrode tops when the field is applied; when the field is then removed, the rotor will rotate under the action of rotational Brownian motion in a similar manner to the linear motion of particles in a linear ratchet device.

For example, consider a rotor between the two vertical electrodes, with a starting position that is "vertical" in figure 3b. when the field not applied, the rotor rotates under Brownian motion. If the rotor has rotated by an angle greater than *a* but less than *b* in a clockwise direction, then reapplication of the field will cause the rotor to advance 90° clockwise to be aligned along the horizontal axis. So long as the rotor does not rotate anticlockwise by an angle greater than *b*, then on the reapplication of the field, the rotor with either remain at the same location or advances 90° clockwise.

We can determine the effectiveness of the device in rectifying rotary Brownian motor by using the asymmetry parameter Λ. Simulation of the electrode array indicates that for the geometry shown, a=28° and b=62°, giving a value of Λ=0.37. In order to maximize the effectiveness of the motor, the activation of the pulsed potential must be timed for the most likely advance on reapplication of the field. From the work of Einstein [9] on Brownian motion, we can determine the root mean square (RMS) angle

through which a spherical particle will rotate under Brownian motion, which we can define as the modulus value for which there is a 50% chance of exceeding or not. Since this RMS can either be clockwise or anticlockwise, the probability has an upper limit of 0.5.

In order to maximize the chance of rotating, the mean angle of rotation should be at the center of the band bounded by angles *a* and *b*; that is at 45°. The likelihood of a transition is governed by the width of that band (and hence the chance of a particle having moved into it).

Although there is no analytical expression for the RMS distance moved by ellipsoids due to Brownian motion, we can make an approximation by equating the orientational and translational energies. It is possible to show that a prolate ellipsoid with major and minor axes of 1μm and 0.5μm respectively, suspended in water at 17°C, will rotate an average distance of 45° in 0.054 seconds, with a standard deviation of 66.6°. Analysis of the dielectrophoretic torque exerted on the particle is difficult; since the force is axisymmetrical, no "conventional" dielectrophoretic force (due to field gradients along the rotor) will be exerted. However, the presence of higher-order terms such as electric quadrupoles means that the ends of the rotor can be treated as independent dipoles. If the rotor is treated as two independent particles attracted to the high-field points from the 45° position, we estimate the torque exerted on the rotor means that the return to alignment should take somewhat less than 0.1ms for an applied RMS alignment potential of 10V and exerting a torque of approximately 1.7×10^{-15} Nm during that period, equivalent to that exerted by a flagellar motor found on some types of motile bacteria [10].

3 CONCLUSION

One of the advantages of nanomanipulation is the ability to take advantage of phenomena not found at larger scale lengths, such as Brownian motion and dielectrophoretic force. The ratchet structures described here employ both phenomena to manipulate, sort and rotate particles on the anoscale, with a plethora of applications for the growing nanotechnology industry.

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