Nanotube in a periodic potential: A conveyer belt for electrons

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ABSTRACT

We consider a carbon nanotube gated slightly away from the half-filled π-band, in the presence of an externally applied periodic potential. We show that the electron states can become locked by the potential when the electron density (counted from half-filling) corresponds to a rational number of electrons per potential period. At such densities, excitation gaps open up in the spectrum. When gapped, the electron system is protected from small perturbations caused by disorder, temperature and other effects. This robustness can be utilized to realize a quantized charge pump, in which a slowly moving periodic potential induces quantized current. For the gapped states with a fractional number of electrons per period, the adiabatic current will correspond to pumping on average a fraction of an electron per cycle (equivalently, to pumping at a fraction of the base frequency) as a result of electron interactions.

Keywords: carbon nanotube, charge pump, quantized current

1 INTRODUCTION

Since their discovery [1], carbon nanotubes (NTs) remain in focus of both basic and applied research [2]. Besides their important technological potential [3], [4], nanotubes are a testing ground for novel physical phenomena involving strong electron interactions [5]–[9]. Experimentally, effects of electron-electron interactions in nanotubes have been observed in the Coulomb blockade peaks in transport [10], [11], in the power law temperature and bias dependence of the tunneling conductance [12]–[14], in the power law dependence of the angle-integrated photoemission spectra [15], and, recently, in the phenomena involving the low-energy SU(4)-flavor physics [16], [17].

The aim of this presentation is to describe a setup [18]–[20] in which the coupling of an external periodic potential to the nanotube's electronic system is proposed (i) as a probe of electron interactions by means of the commensurability effects; and (ii) as a vehicle to realize the adiabatic charge transport.

2 OVERVIEW OF THE RESULTS

We focus on the electron properties of single-wall NTs in a periodic potential whose period $\lambda_{\rm ext}$ is much greater than the NT radius a, $\lambda_{\rm ext} \gg a$. Such a potential can be realized using optical methods, by gating, or by an acoustic field. In all of these cases, the realistic period $\lambda_{\rm ext}$ is of the order 0.1–1 μm.

Near half-filling, the electrons in a NT are conventionally described in terms of the Dirac fermions of the four polarizations [2] (herein called "flavors"). Below we identify incompressible electron states, characterized by excitation gaps, that can arise when the average NT electron number density $\bar\rho$ (counted from half-filling) is commensurate with the period $\lambda_{\rm ext}$ of the external potential:

$$\bar\rho = \frac{m_{\rm tot}}{\lambda_{\rm ext}}\,, \quad m_{\rm tot} = 4m\,. \tag{1}$$

In Eq. (1), m is the number of the NT Dirac fermions of each of the four flavors per potential period.

At what density values m can the spectral gaps open?

The single-particle treatment of Ref. [18] maps the problem onto that of the Bloch electron, resulting in the spectrum of minibands separated by minigaps as a consequence of the Bragg diffraction on the external potential. Filling up an integer number m of minibands corresponds to adding m electrons of each flavor per "unit cell" (the period $\lambda_{\rm ext}$). Thus, minigaps open up when the density (1) is *integer*: $m = 0, \pm1, \pm2, \ldots$ [18]. In other words, the wave nature of the Bloch electron leads to commensurate states with integer density.

In Refs. [19], [20] it was shown that electron-electron interactions dramatically change the spectrum, adding incompressible states at *rational* densities $m = p/q$, in which commensurability is induced by interactions.

In a fractional-m state, the NT electron system is locked by the external potential into a $q\lambda_{\rm ext}$-periodic commensurate configuration (such as the ones schematically represented in Fig. 1). Naturally, the states with the lower denominator q are more pronounced. Realistically, due to finite NT length and temperature, only a few states with small enough q can be detected. However, these fractional-m states are important as the corresponding minigaps are interaction-induced (and vanish in the noninteracting limit). Measurement of such

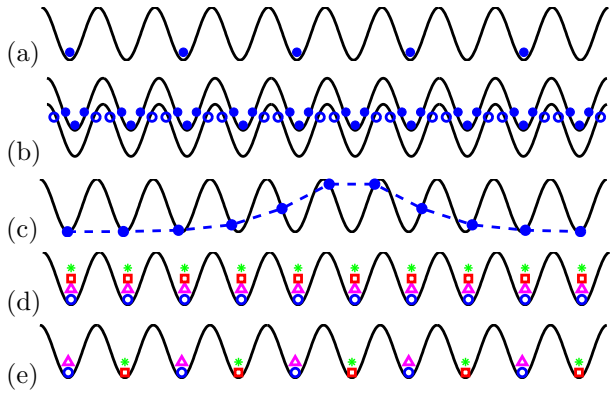

Figure 1: Incompressible electron states (schematic). Semiclassically, the states are characterized by the numbers (n_e, n_h) of the electrons and holes in the potential minima and maxima correspondingly, with the total density $4m = n_e - n_h$. (a) The $(1/2, 0)$ state. (b) The $(3, 2)$ state with $4m = 1$. Here large potential amplitude breaks the Dirac "vacuum" by placing holes (open circles) into the potential maxima. (c) Example of an excitation over the $(1, 0)$ state. Finite energy cost of the excitation gives the spectral gap. (d) The $m = 1$ incompressible state allowed by the Bloch theory of noninteracting electrons. Four kinds of labels mark the four electron polarizations. (e) The interaction-induced $m = 1/2$ state. (Figure taken from Ref. [20].)

minigaps can provide a direct probe of interactions between electrons.

From a practical standpoint, measurement of the gaps can identify the incompressible states characterized by quantum coherence on a macroscopic scale. A challenging experimental proposal [18] is to realize the Thouless pump [21] by taking advantage of the semimetallic NT dispersion. In such a setup, a *quantized current* is predicted to arise whenever the chemical potential is inside the minigap created by the adiabatically slowly moving potential wave. The approach of Refs. [19], [20] suggests that, due to electron interactions, the commensurability (1) will result in additional current plateaus, corresponding to pumping on average of a fraction of unit charge per cycle. Equivalently, electron interactions could allow one to realize a novel effect of adiabatic pumping of charge at the fraction of the base frequency of the potential modulation.

From a theoretical viewpoint, the excitation spectrum is linked to the general theory of commensurate-incommensurate transitions [22]–[26]. In this approach, an excitation over the commensurate state (an incommensuration) is represented by a *phase soliton* [illustrated in Fig. 1(c)], whose energy gives the corresponding spectral gap.

In Refs. [19] and [20] the phase soliton method has been generalized onto the case of strongly interacting massive Dirac fermions of multiple polarizations. According to our treatment, the external periodic potential can lock electrons into incompressible states at commensurate densities. As a semiclassical illustration of this phenomenon, in Fig. 1(d,e) the electrons of the different flavors are marked by the four different symbols.

In general, both Bragg diffraction and the Coulomb interaction contribute to the excitation gaps. In order to take into account both factors we employed the bosonization technique [27], that maps the problem of the massive interacting Dirac fermions onto the nonlinear bosonic system. In this language, an electron is represented by a composite sine-Gordon soliton [9] of the four interacting bosonic modes. The size of this soliton ℓ semiclassically represents the "size of an electron", i.e., roughly, the spatial extent of its wavefunction (note that, strictly speaking, the system is in the many-body state).

The diffraction is a dominant factor to form the incompressible states when the system is controlled by the Luttinger-liquid fixed point [27]. In this case, the electrons are extended objects, $\ell \gg \bar{\rho}^{-1}$, and the effects of the fermionic exchange between them are important.

On the other hand, the Coulomb interaction plays a major role when the electron system is in the Wigner-crystal regime [9]. In this case one can think of the system as a set of single-electron wavefunctions of an extent $\ell < \bar{\rho}^{-1}$, localized on the "sites" (that would correspond to the point-like electrons in the classical limit) due to the strong Coulomb field of their neighbors.

The external potential, by bringing the additional length scale $\lambda_{\rm ext}$ to the problem, naturally distinguishes between the two sides of the Luttinger-liquid — Wigner-crystal crossover. Technically, this distinction occurs on the level of a saddle point of the nonlinear bosonic action considered in detail in Refs. [19] and [20]. Below we outline the main results of our investigation.

2.1 Gaps in the Semiclassical Limit

In a recent Letter [19] we showed that, in the semiclassical limit, when electrons are nearly pointlike ($\ell \ll \lambda_{\rm ext}$), the long-range Coulomb interaction leads to the sequence of incompressible states (the so-called devil's staircase) representing the pinned Wigner crystal. Gaps open at rational densities $m_{\rm tot}$, and the Coulomb interaction sets the gap energy scale. An example of such a state with $m_{\rm tot} = \frac{1}{2}$ is shown schematically in Fig. 1(a). Remarkably, the Dirac character of the electrons in a nanotube brings about a set of incompressible states in which the "Dirac vacuum" is broken when the potential amplitude exceeds the gap at half-filling. In this case [20], physically different incompressible states can correspond to the same total density (1) with

$$4m = m_{\rm tot} = n_e - n_h \,. \qquad (2)$$

To further characterize these states one specifies [20] the pair of numbers (n_e, n_h) of electrons and holes in the potential minima and maxima correspondingly. An example of the $(3, 2)$ state with $m_{\rm tot} = 1$ is schematically shown in Fig. 1(b).

2.2 Gaps Due to Bragg Diffraction

In the opposite limit of the weakly coupled electrons ($\ell \gg \lambda_{\rm ext}$), the spectral gaps open due to the Bragg diffraction [18], [20]. In this case, the electron wavefunctions are delocalized over many periods, and the excitation gaps occupy a small part of the spectrum. The fermionic exchange is important, and adds to the cost of an excitation in which an extra electron of a particular flavor is added to the commensurate configuration. Specifically, in Ref. [20] we explicitly demonstrated how the lowest-denominator fractional state $m = \frac{1}{2}$ appears when the electron interactions are turned on. For this state we have found the effective action that describes excitations over the commensurate configuration, and obtained the charge and the SU(4)-flavor excitation gaps.

3 QUANTIZED CURRENT

When gapped, the electron system is protected from small perturbations caused by disorder, temperature and other effects. This observation is instrumental to realizing the adiabatic charge pump, long envisioned by Thouless [21]. In the Thouless setup, applied to the electrons in a nanotube [18]–[20], the external periodic potential, moving adiabatically slowly with a frequency f, will generate *quantized current* (Fig. 2)

$$j = 4mef\,. \tag{3}$$

Semiclassically, the charge is transported by a moving potential wave in a conveyer-belt fashion. In Ref. [18] the electric field of a surface acoustic wave (SAW) propagating in the underlying piezoelectric substrate was proposed for the role of such a conveyer belt.

Operation of the charge pump requires adiabaticity [28], [29]

$$k_B T,\ hf \ll \Delta_m\,. \tag{4}$$

The typical minigap values Δ_m are estimated [18]–[20] in the meV range, and the adiabaticity condition (4) is realistic. The feasibility of the Thouless pump in the NT-SAW setup of Ref. [18] is further corroborated by recent pumping experiments involving SAWs. In particular, in the pumping of electrons between the two 2D electron gases through a pinched point contact [30] the achieved quality of current quantization is close to metrological [29], [31]. Recently the SAW-assisted pumping has been demonstrated through the laterally defined

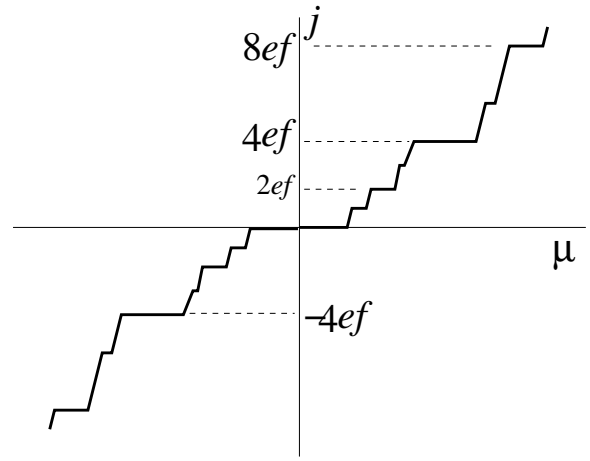

Figure 2: Mapping incompressible electron states in the adiabatic charge pump setup (schematic). Shown is the current j measured as a function of the gate volgate μ, with $\mu = 0$ corresponding to the half-filled nanotube. Plateaus correspond to the non-dissipative (quantized) electric current induced by the adiabatically slowly moving periodic potential. In addition to the integer-m plateaus, the ones with fractional m appear due to electron-electron interactions. The device pumps electrons for $\mu > 0$ and holes for $\mu < 0$. (Figure taken from Ref. [19].)

quantum dot [32], as well as through the semiconducting nanotube whose working length L matched the SAW period, $L = \lambda_{\rm ext}$ [33], [34].

A practical realization of the proposed pumping setup can become a first implementation of the Thouless transport. Besides being instrumental to study electron interactions in a nanotube, it could realize the "conveyer belt" for electrons with a possibility of pumping current quantized in fractions of the unit charge per cycle (corresponding to the fractional-m states of Refs. [19] and [20]). In other words, such a setup would exemplify a charge pump operating at a fraction of the base frequency. The control on the charge transfer potentially achievable in this setup can be a crucial ingredient in areas as diverse as engineering of nanoscale devices, quantum information processing, and metrology.

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