

Correlation of Experimental and Numerical Results on Electrostatically Actuated Micro-Beams

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ABSTRACT

The aim of this paper is to validate numerical simulations of electromechanical coupling in micro-structures using some experimental results. The micro-structures studied here consist in a micro-bridge and two cantilever micro-beams. Multi-physics models of micro-electro-mechanical systems (MEMS) based on the finite element method (FEM) are used to model the strongly coupled electro-mechanical interactions and to perform static analyses taking into account large displacements. In all the cases treated here the numerical results are in very good agreement with the experimental results.

Keywords: multi-physics model, electro-mechanical coupling, finite element method, experimental validation, non-linear systems

1 INTRODUCTION

Classical methods used to simulate coupling between electric and mechanical fields are commonly based on staggered procedures, which consist in computing quasi-static configurations using two separate models [1]. In this modelling research, a fully coupled electro-mechanical finite element formulation is proposed, which allows static equilibrium positions to be computed in a non-staggered way, and which provides fully consistent tangent stiffness matrices. When a voltage is applied between the membrane and the electrodes, electrostatic forces appear which force the membrane to bend. When the applied voltage increases, the electrostatic force becomes dominant and for some limit value, the plates stick together. The corresponding critical voltage is called the pull-in voltage. It is one of the most important design parameters in this type of micro-systems. Using the formulation proposed in this paper, the algorithm is able to pass over the pull-in voltage and to enter in the unstable area. The fully coupled methodology provides more reliable results than staggered methods.

2 FINITE ELEMENT FORMULATION

A consistent way of deriving a finite element discretisation for the coupled electro-mechanical problem con-

sists in applying the variational principle on the total energy of the coupled problem. It includes the electric and mechanical energies. The expression of energy density results from thermodynamic considerations [2]. Usually Gibb's energy density is used:

$$G = \frac{1}{2} \mathbf{S}^T \mathbf{T} - \frac{1}{2} \mathbf{D}^T \mathbf{E} \quad (1)$$

where \mathbf{T} is the stress tensor, \mathbf{S} the strain tensor, \mathbf{D} the electric displacement tensor and \mathbf{E} the electric field.

The internal forces may be obtained using the virtual work principle (see [3]). The internal energy of the coupled problem on a volume V is:

$$W_{int} = \frac{1}{2} \int_V \mathbf{S}^T \mathbf{T} dV - \frac{1}{2} \int_{V(\mathbf{u})} \mathbf{D}^T \mathbf{E} dV = W_m - W_e \quad (2)$$

where W_m is the mechanical energy and W_e the electric one.

The variation of the total energy with respect to the displacement \mathbf{u} and to the electric potential ϕ yields the mechanical internal forces \mathbf{f}_m and the electric equilibrium equation f_e , respectively:

$$\begin{cases} \mathbf{f}_m \cdot \delta \mathbf{u} = \delta_u W_{int} = \delta_u W_m - \delta_u W_e \\ f_e \delta \phi = \delta_\phi W_{int} = \delta_\phi W_m - \delta_\phi W_e \end{cases} \quad (3)$$

These equations provide the equilibrium equations with the unknowns \mathbf{u} and ϕ . In (3), $\delta_u W_m$ and $\delta_\phi W_e$ can be treated as in the standard variational calculus for uncoupled electrostatics and mechanics. Further the mechanical energy is independent from the voltage: $\delta_\phi W_m = 0$. The variation of the electric energy due to the displacement \mathbf{u} is the contribution of the electrostatic forces. After some developments [3], we obtain

$$\delta_u W_e = \frac{1}{2} \int_V \mathbf{D}^T \mathbf{F} \text{grad} \delta \mathbf{u} dV \quad (4)$$

where \mathbf{F} is a matrix function of the space derivatives of ϕ . This term represents the electrostatic forces on the structure. From (3) and (4), a fully coupled finite element formulation can be built following classical discretisation procedures.

The variation of the mechanical and electrostatic forces with respect to small potential and displacement perturbation is an important characteristic of the coupled

system since it allows linear vibrations to be evaluated around equilibrium positions. The tangent stiffness matrix around a position (\mathbf{u}_0, ϕ_0) may be obtained by linearisation of the internal forces

$$\begin{pmatrix} \mathbf{K}_{uu}(\phi) & \mathbf{K}_{u\phi}(\phi) \\ \mathbf{K}_{\phi u}(\phi) & \mathbf{K}_{\phi\phi}(\phi) \end{pmatrix} \begin{pmatrix} \Delta \mathbf{U} \\ \Delta \Phi \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{f}_m \\ \Delta \mathbf{Q} \end{pmatrix} \quad (5)$$

The total coupled matrix is symmetric. The matrix $\mathbf{K}_{\phi\phi}$ is the same as the stiffness matrices of the purely electric problem and need not be further discussed. The other terms need to be derived from the total energy [3]. These terms depend on the electric field and the coupled problem is thus nonlinear.

Finite elements based on this monolithic formulation were developed and implemented in a finite element program called *oofelie* powered by Open Engineering sa.

3 CONFRONTATION TO EXPERIMENTS

The numerical simulations are now validated through the comparison with experimental results. In the framework of the ARC project ("Action de Recherche Concertée"), MEMS were fabricated. The used technology is called PolyMUMPS which stands for "Polysilicon Multi-User MEMS Processes". The design and testing of these devices were performed by researchers of IEF ("Institut d'Electronique Fondamentale"), which is a CNRS research centre in Paris. The fabrication was subcontracted in a company called MEMSCAP. Different designs and structures were fabricated, but only the electrostatic actuated beams namely one electrostatic micro-bridge made in the first Polysilicon layer called Poly1, and two cantilever beams, one made in the first Polysilicon layer (Poly1) and one in the second one (Poly2) are considered here. The material characteristics and the dimensions of the micro-beams are given in table 1. L is the length of the beam, t the thickness and d the gap. L_{elec} is the length of the lower electrode deposited on the substrate. A picture of the studied micro-bridges is presented in figure 1.

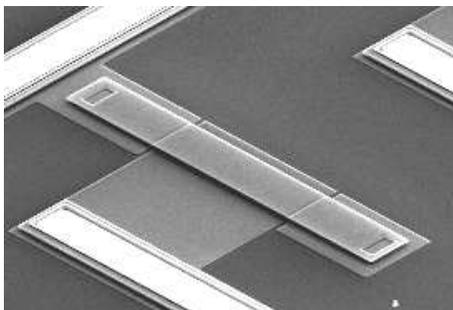


Figure 1: The studied micro-bridge.

Polysilicon Properties			
E	158± 10 GPa	ν	0.22±0.01
Poly 1 Micro-Bridge			
L	200 μm	t	2.19 μm
L_{elec}	100 μm	d	1.87 μm
Poly 1 Cantilever Micro-Beam			
L	175 μm	t	2.5 μm
L_{elec}	168 μm	d	1.87 μm
Poly 2 Cantilever Micro-Beam			
L	175 μm	t	1.4 μm
L_{elec}	168 μm	d	2.6 μm

Table 1: Dimensions and characteristics of the studied micro-devices.

3.1 Micro-Bridge

The first type of structure considered here is a micro-bridge. Prestress is present in the layer due to the fabrication process. A buckling effect may be observed, and a displacement of about 164 μm is measured at the beam centre.

3.1.1 Buckling Simulation

The first step to model these micro-devices is to estimate the prestress induced in the mechanical structure by the fabrication process. To determine exactly the stress distribution on the thickness of the beam the entire process should be simulated. In this study, only a mean prestress will be added to the structure. The solution of the problem is found using the Riks Crisfield algorithm. A 30 MPa prestress is necessary to achieve the 164 nm initial displacement at the centre of the beam. In figure 2 the deformation of the beam due to the prestress is shown.

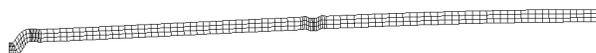


Figure 2: Initial deformation of the structure due to the prestress (10 \times).

3.1.2 Pull-in Curve

Starting from the buckled configuration a voltage is applied between the electrode and the beam. The electro-mechanical problem is solved using a Riks-Crisfield algorithm. The exact shape of the anchor was not accurately measured. The different shapes presented in figure 3 were tested. For each ones the mean prestress is adapted to obtain the observed initial deformation. The displacements of the beam centre when the voltage increases are plotted in figure 3 for each anchor shapes. The geometry of the anchor has thus a certain effect on

the pull-in curve. In figure 3 the numerical results represented by dots, diamonds and squares overestimate the experimental data represented by circles.

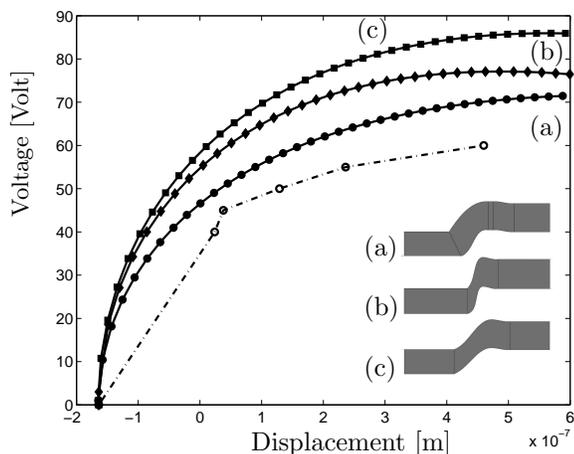


Figure 3: Pull-in curve for different shapes for the anchor of micro-bridge.

3.2 Poly1 Cantilever Micro-Beam

The second device treated here is a cantilever beam made in the layer Poly1. The initial configuration of the beam is relatively flat.

3.2.1 Pull-in Curve

The static electro-mechanical problem is solved using a Riks-Crisfield algorithm. The displacement of the extremity of the beam when the voltage increases is plotted in figure 4. The numerical results are plotted in black dots, and the experimental results are in circles. The numerical results overestimate the experimental data.

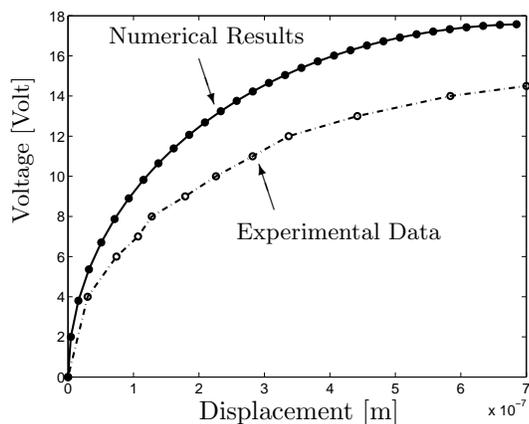


Figure 4: Pull-in curve for the cantilever beam.

3.3 Poly2 Cantilever Micro-Beam

Now the same beam is realised in the Polysilicon layer Poly2. The beam is thinner than the Poly1 ones, and the gap between the electrodes is larger.

3.3.1 Prestress

The initial shape of the beam is deformed due to a gradient of prestress inside the structure. In cantilever beam a mean stress on the thickness as shown in figure 5 is relaxed when the beam is released. To deform the beam as the measured initial configuration a gradient of prestress such as in figure 6 has to be added in the mechanical element.

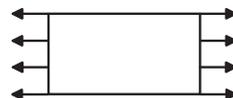


Figure 5: Constant prestress.



Figure 6: Gradient of prestress.

Using a Riks-Crisfield algorithm the beam is deformed as shown in figure 7. The deformation is magnified by 30. The maximum prestress applied to the surface is about 2 MPa to achieve the measured deformation of the beam.



Figure 7: Initial deformation of the structure due to the prestresses (30 \times).

3.3.2 Pull-in Curve

Starting from the prestressed beam the Riks-Crisfield algorithm is used to compute the deformation due to the electrostatic force. The displacement of the extremity of the beam when the voltage increases is plotted in figure 8. The black dots represent the numerical results and the circles the experimental data. The prestress bends the beam down so that the voltage needed to deform it is lower.

3.4 Model Updating

In all of these examples the numerical results overestimate the experimental data. A lot of reasons may cause this difference namely the shape of the anchor, the dispersion on Young's modulus and other external parameters such as the ambient temperature. For a better agreement between numerical predictions and experimental results Young's modulus of the polysilicon is updated. The Poly1 cantilever beam is taken as reference beam, because no prestress influences its pull-in curve. To fit the experimental data of the Poly1 beam

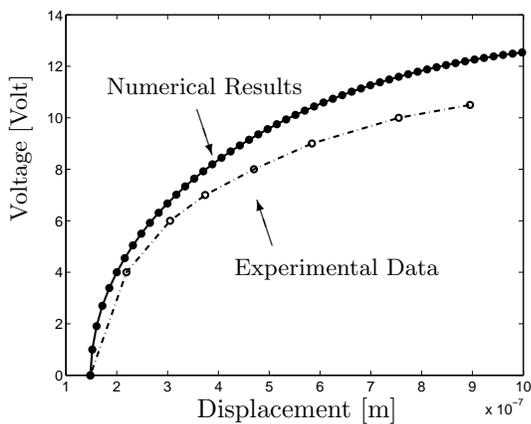


Figure 8: Displacement of the extremity of the beam due to the electrostatic forces.

the Young's modulus is reduced to 112GPa as shown in figure 9.

The update value of Young's modulus is then applied to the Poly2 cantilever beam problem. The gradient of prestress to obtain the measured displacement is reduced to 1.4 MPa on the surface. The numerical results fit very well the experimental data as shown in figure 9. By using the same Young's modulus for the micro-bridge the mean prestress is reduced to 22 MPa. The numerical results (line) and the experimental data (circle) are compared for the micro-bridge in figure 10. The numerical results are in good agreement with experimental data.

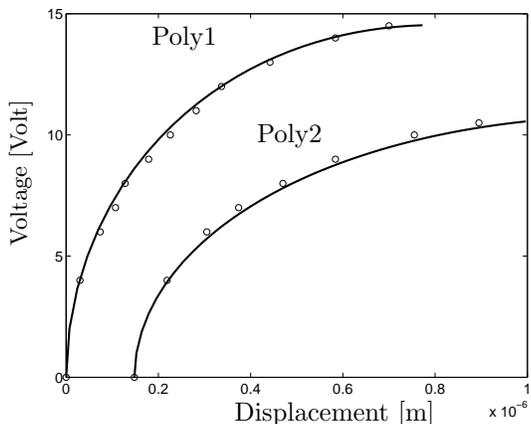


Figure 9: Comparison of numerical results and experimental data for the cantilever beams.

4 CONCLUSIONS

In this paper the numerical computations using the strong coupled electro-mechanical formulation are compared with experimental data. Two different types of devices are studied: a micro-bridge and two cantilever micro-beams. The comparison between the numerical

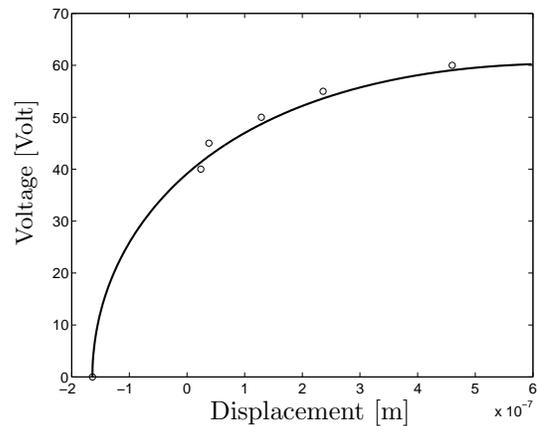


Figure 10: Comparison between the numerical (line) and experimental (circle) results for the micro-bridge.

results and the experimental results calls for further fine tuning of the model. First the pull-in curve depends on the shape of the anchor. No precise measurements of the anchor were provided. Secondly two different types of prestress are present in the structures: Poly1 has a constant prestress on its thickness visible in the micro-bridge and released in the cantilever beam. In Poly2 layer a gradient of prestress is present, which deforms the cantilever beam. In each model the pull-in curve is overestimated. The model is updated to obtain a better agreement between numerical results and experimental data. Young's modulus is then reduced to fit the Poly1 cantilever beam curve. With this updated value the pull-in curves of the micro-bridge and the Poly2 cantilever beam are in good agreement with experimental data. However this model updating may be not unique. Different solutions updating the anchor stiffness and the beam stiffness may achieve to the same results. To perform a better model updating more experimental measurements are needed such as the natural frequencies of the structure.

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