Computationally Efficient Dynamic Modeling of MEMS

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ABSTRACT

Traditional modeling work in MEMS includes simplified PDE/ODE formulation, based on physical principles, and Finite Element Analysis. More recently, reduced order modeling techniques using Krylov subspace decomposition have been proposed in the context of nodal analysis. This modeling technique makes it possible to predict the dynamic behavior of more complex MEMS, but the computational engine is still a traditional cubic order solver.

In this paper we apply a new modeling approach for complex MEMS based on a linear O(n+m) (n- number of bodies, m – number of constraints) solver for rigid multibody dynamics. As direct applications, we present simulation and experimental results of models for thermally driven MEMS actuators, compared against established simulation tools, namely FEA (Intellisuite), AUTOLEV, and SUGAR 3.0.

Keywords: MEMS modeling and simulation, compact modeling, O(N) simulation

1. Introduction

During the last 15 years modeling and simulation tools for Micro-Electro-Mechanical Systems (MEMS) have evolved from simplified PDE/ODE formulations based on physical principles, to MEMS specific layout, process and FEA tools (Intellisuite, CoventorWare). The algorithm complexity associated with these tools in dynamic simulation is cubic in the number of mesh elements, $O(N^3)$, with additional overhead and numerical stability problems associated with the treatment of intermittent constraints. More recently, reduced order modeling techniques using Krylov subspace decomposition have been proposed in the context of nodal analysis (SUGAR) [7,8]. This modeling technique makes it possible to predict the dynamic behavior of more complex MEMS, but the computational engine is still a traditional $O(n^3)$ solver (that is, the number of mathematical operations need to perform the simulation at each integration time step increases as a cubic function of the number of beam elements n). Compared to FEA, however, the MEMS system is described by n large beam elements, with n<<N.

In this paper we apply a different modeling approach for complex MEMS for use with a fully nonlinear $O(n+m)$ (where n is the number of generalized coordinates and m the independent algebraic loop constraints) Recursive Coordinate Reduction (RCR) solution method for rigid body dynamics [1,5]. Such computationally efficient multibody simulation methods have been receiving increasing attention since the first functional $O(n)$ algorithm was developed by Armstrong in 1979 [2]. Recent advances in this area have extended linear order solutions to include kinematic loops, link flexibility, and intermittent constraints which are needed to characterize the operation of very complex MEMS devices. In this paper, we illustrate the benefits of an RCR formulation for MEMS actuated via thermal flexure actuator banks.

Our approach involves approximating the continua (the flexible MEMS parts) via a series of rigid bodies which are interconnected by stiff springs. Once the rigid body model is obtained, the RCR formulation is applied to the approximate system to realize a simulation. Nodal analysis uses an analogous approach, decomposing the flexible components into a series of interconnected beam elements, but the manner of solving the resulting set of differential equations is conventional (i.e. mass-matrix inversion by direct methods).

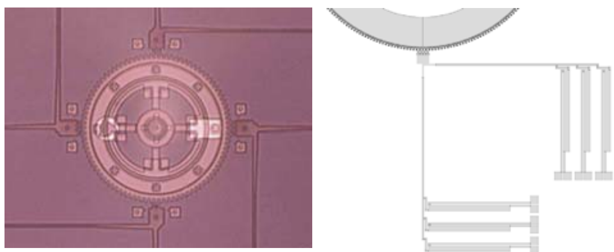
We pay particular attention to two issues that need addressing specifically in the context of MEMS, namely:

- How to best approximate the actual structural and dynamical characteristics of the flexible MEMS device using a rigid body/flexible link model.
- How to organize the computation of dynamical equations of motion so that they can be solved in linear time at each iteration.

The resulting simulation method represents a significant improvement in performance relative to what is possible for MEMS simulation using established methods. The performance is not only reflected in a linear execution time, but also in the numerical stability of the algorithm.

2. Motivation for an O(n+m) simulation tool for complex MEMS

Figure 1 shows an example of a MUMPS rotary stage actuated by orthogonal banks of bimorphs [6]. If we consider a single quarter stage arm of the rotor, namely an XY stage, we could approximate it as a series of rigid bodies and flexible joints, or as a set of interconnected beam elements. As an example, Figure 2 shows a quarter stage approximate model that includes n=123 generalized coordinates. This model is an interconnection of a simpler single thermal bimorph models, each containing 12 generalized coordinates. The number of additional constraint equations is also large, three being associated with each kinematic loop present in the system (m~½n). The manner in which the discretization is obtained is in itself a non-trivial task [3,4], and will be presented in the next section of this paper. To this multibody model, electro-thermal and damping effects may be added through applied forces or, in the case of thermal expansion, prescribed displacements.



(a) (b)

Fig 1: Actual MUMPS rotary stage (a), using 4 orthogonal thermal bimorph banks (b).

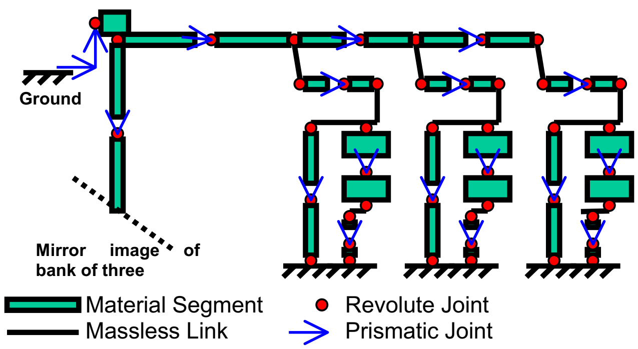


Fig 2: Rigid body/flexible link discretization model for XY stage

After approximating the flexible system by a set of interconnected rigid bodies, the new efficient linear order constrained multi-rigid-body solution scheme of Recursive Coordinate Reduction (RCR) may be applied to the model.

The dynamic behavior of the XY stage can be simulated using tools of different computational complexity such as Finite Elements, Nodal Analysis, and Rigid Body Dynamical solvers, as shown in Table 1. It is apparent that for situations where n and/or m are large, a cubic computational cost is required for each temporal integration step. A truly linear O(n+m) formulation becomes extremely attractive for complex systems.

Methods	Pro(s)	Con(s)
F.E.M. (Abaqus, Ansys, Intellisuite, CoventorWare)	Available now High fidelity Multi-role	Very computationally expensive Poor Contact Element performance
$O(n^3)$ (ADAMS, DADS, AUTOLEV, SUGAR)	Available now Moderate cost	Still computationally expensive for complex structures. Do not handle unilateral constraints
$O(n+nm^2+m^3)$ $O(m+n)$ (RCR)	Lowest cost. Most unilateral constraints Minimum Contact Model	Not generally used Not all unilateral constraints

Table 1: Advantages and disadvantages of various simulation tools available for dynamical simulation

3. Rigid Body – Flexible Link Model

The model discretization shown in Figure 2 assumes that a set of stiffnesses and masses can be found, and therefore, the bending moment at each of the interconnection point is directly proportional to the angular displacement between joints. In fact, this is only true for constant cross section beams of equal lengths, and therefore we make use of a more general joint spring formulation in which the bending moment at the i-th joint is expressed solely in terms of its displacement and the displacement of its immediate neighbors.

In order to approximate the MEMS XY stage with a rigid body/flexible link model it was necessary to address the following questions:

Q1: *If a flexible beam is split into N rigid bodies, what is the best position of the split points such that the first N fundamental modes match the FEA, SUGAR (nodal analysis), and/or the analytical model?*

If N=2, for a beam with length L and cross-sectional moment I. Theoretically,

$$K = E\frac{I}{L} \text{ and } I = fW^2T$$

where T is the beam thickness, W is the beam width, E is Young's Modulus, and f is a correction factor. The correction factor is theoretically equal to 0.5, but from SUGAR and AUTOLEV simulations it was best fit to 0.43.

Q2: *Given the layout of a MEMS device, how does one obtain the values of inertias, stiffnesses, and position of the links in the rigid body/flexible link model?*

Most MEMS device layouts are described in standard formats, such as .gds file formats. A particular format, the Caltech Interchangeable Format (.cif) has been used in the past to describe a MEMS device geometry composed of beams of constant width. It has also been used in conjunction with SUGAR by translating it into a SUGAR-specific net list. We use the same method to describe the geometry of a MEMS device.

Q3: *What kind of interconnects are necessary to describe the discretized approximate model for a MEMS device composed of beams?*

We have identified and modeled four types of beam interconnects necessary to describe the quarter stage shown in Figure 2, namely:

- Rotary flexible joint between beams with same cross-section (type 1).
- Rotary flexible joint between beams with different cross-section (type 2).
- Rotary flexible T-joint (type 3).
- Prismatic flexible joint (type 4).

Q4: *How are backbending, and other nonlinear thermomechanical or electrostatic effects included in the model?* Backbending effects (i.e. thermally induced plastic deformations), thermal expansion effects (i.e. the basis for thermal actuation, proportional with the electrical input power), or electrostatic deflections (i.e. comb drive forces or attraction between plates) can be modeled as externally imposed reduced order nonlinear "gap" functions. For example, a thin MEMS beam expanding by thermo-electrics behaves similarly to a gap function of a type 4 joint proportional to the square of the voltage applied through the link. Moreover, a backbent bimorph bank shown in Figure 3 is equivalent to a negative spring constant shortening the beam.

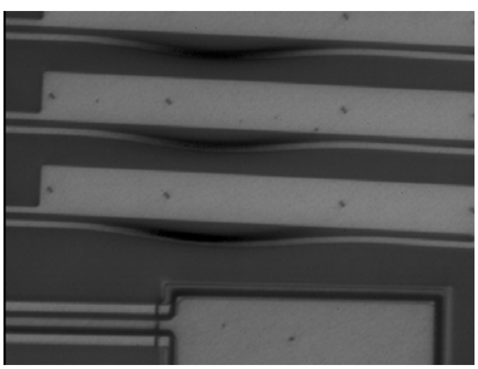


Fig 3: Beam shortening due to plastic backbending deformation of bimorph hot arms.

Q5*: Using the continua to rigid body rigid body/flexible link conversion rules outlined below, how close are the structural modes of the resulting structure to those computed using FEA?*

A three-bimorph bank structure was transformed to a rigid body/flexible link model. Table 2 shows modal analysis results comparing SUGAR, FEA, and AUTOLEV, showing a reasonable match for the first seven flexible modes.

Mode	Intellisuite	AUTOLEV	SUGAR3.0
#	Hz	Hz	Hz
1	83200.30	87004.09	86506.01
2	405413.00	316201.84	N/A
3	405926.00	316424.22	N/A
4	421218.00	330743.72	N/A
5	613638.00	653664.79	649154.89
6	626903.00	664621.89	662160.46
7	773568.00	846293.33	829454.79
8	1130000.00	2002156.90	1879437.60
9	1140000.00	2119784.30	2084951.20
10	1160000.00	2543336.50	3073013.30

Table 2: Comparison of modal analysis results using different simulation tools.

4. Implementation of RCR

The recursive coordinate reduction was applied to the rigid body/flexible link model of the quarter stage using code written in MATLAB, and compared to results obtained in AUTOLEV, SUGAR 3.0 as well as FEA. The RCR method is already the topic of several journal articles and it is to these articles [1,5] that the reader is referred for the equations and detailed derivations.

However during the verification of the numerical exactness of the RCR implementation within MATLAB it was observed that AUTOLEV introduces numerical instabilities that prohibit accurate computation of anything more complex than a single bimorph. This instability is attributed to the symbolic nature of AUTOLEV that implements a constraint solution via Gaussian Elimination without regard to the numerical values of the matrix elements (it does not pivot). The instability is observed as a divergence from the RCR solution and an inability to temporally integrate the equations of motion with an error controlled variable step integrator.

As a result, we compared RCR against dynamic simulations obtained using FEA analysis (namely Intellisuite) as well as Sugar 3.0. Dynamic simulations were performed with the quarter stage model released from rest after bending to a horizontally applied 100μN force.

The X time-dependent coordinate of the complete quarter stage is shown in Figure 4. Figure 5 further illustrates the accuracy of the approximation by showing the planar trajectories of the simulations plotted on 1:1 scaled axis.

Also shown with the trajectories are the results obtained from SUGAR 3.0. These SUGAR curves have been obtained by running a simulation with the constant mass and stiffness matrix that are generated for modal analysis. This linear simulation is not representative of the intended dynamic capabilities of SUGAR, however such functionality is not currently present in that package, possibly due to numerical integration instability for stiff systems. It is interesting to note that for our system the mass matrix inversion of the SUGAR matrices generates a condition number warning in MATLAB.

By observing that the character of the FEA result is maintained, the results provide solid evidence that both the RCR implementation is correct and that the rigid body modeling technique is a meaningful approximation.

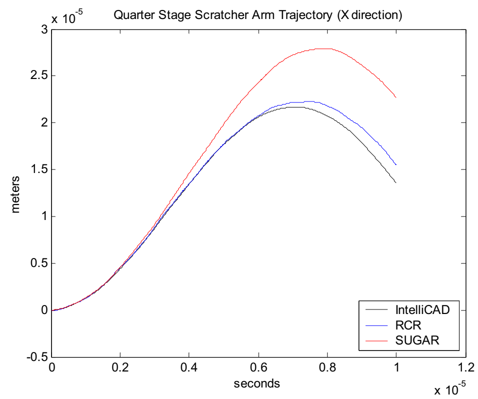


Fig 4: Comparison between FEA, RCR and Sugar 3.0, showing XY stage displacement vs. Time.

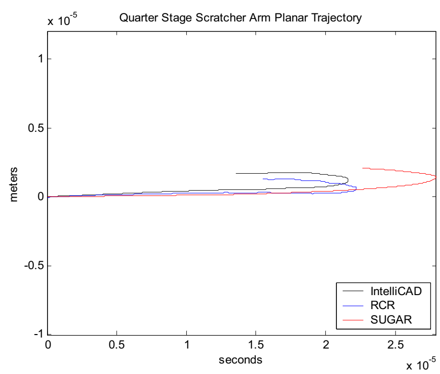


Fig 5: Comparison between FEA, RCR and Sugar 3.0, showing XY stage tip 2D trajectory.

5. Conclusion and Future Work

In this paper we presented a new approach for modeling and dynamic simulation of MEMS devices based on the linear complexity Recursive Coordinate Reduction (RCR) algorithm. The algorithm has been tentatively used in the past for simulating rigid multibody dynamics with a large number constraints and degrees of freedom.

Applying this algorithm to MEMS requires the conversion of a MEMS layout consisting of interconnected beams into a rigid body and stiff spring model. Currently, the conversion from a .cif layout is done through a SUGAR .net file using four types of joints. In the current implementation the RCR is used as an add-on MATLAB toolbox, and can be used in parallel with SUGAR. Further work is necessary in automating the MEMS layout conversion, model conversion (i.e. parameter calculations), and the general application of the RCR.

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