Relaxation-based Circuit Simulation for Large-scale Circuits with Lossy Transmission Lines[1]

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ABSTRACT

This paper integrates a time-domain transmission line calculation method and relaxation-based circuit simulation algorithm, and investigates the transient response simulation of large-scale circuits with transmission lines. Experimental results have been provided to justify the performance of proposed methods.

Keywords: transmission line, circuit simulation, Waveform Relaxation, large-scale circuit.

INTRODUCTION

Due to the advance of modern integrated circuit manufacturing technique, the device size has been ever decreasing and circuit speed has been ever increasing. Hence the circuit interconnect delay plays an increasingly important role. Recently, many techniques, e.g. [1], have been proposed to simulate the transmission line effects of interconnects. But most of them don't consider the situation of dealing with large-scale circuits. In this paper, we propose an efficient and accurate method that is based on relaxation method [2] to perform the transient responses analysis of large-scale circuits containing transmission lines.

Our approach is to use Waveform Relaxation (WR) [2] as the basic algorithm, in which the transmission lines are treated as usual time-domain circuit elements. To do this, a time-domain calculating method [1] is used to "transform" the transmission lines into two-pin equivalent elements whose values are updated routinely. Some techniques have been added onto this WR-based method to pursue more performance, such as the *separated-time-step scheme* (STS) and method to vary *inner time step* (time steps used by transmission line calculation method, while the usual time steps are called *outer time step* here).

Some profits can be earned by using this scheme. First, the transmission lines usually use very small inner time steps, which will limit the magnitude of outer time step. By partitioning transmission lines into subcircuits, the small time steps can be "enclosed" in subcircuits containing them. Second, the induced multi-rate problem [2] can be solved by WR algorithm very well. Finally, relaxation-based algorithms have lower time complexities, so they usually show better efficiency in dealing with large-scale circuits. These will be justified by our experiments.

The following section briefly describes the used transmission line calculation method, WR algorithm, STS, and method to vary inner time steps. Then a section expressing our experimental results follows. Finally, we give a conclusion remark section.

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USED METHODS

Transmission Line Calculation

The used transmission line calculation method is based on the Method of Characteristic [1][3]. Consider a coupled uniform transmission line whose resistance, inductance, capacitance, and conductance per unit length are R, L, C, and G respectively. The electrical behaviors (voltages v, and currents i) are described by the following Telegraph's equation:

$$\frac{\partial}{\partial x} v(x,t) = -L\frac{\partial}{\partial t} i(x,t) - Ri(x,t) \qquad (1)$$

$$\frac{\partial}{\partial x} i(x,t) = -C\frac{\partial}{\partial t} v(x,t) - Gv(x,t) \qquad (2)$$

, in which x denotes distance, t denotes time. These equations are partially decoupled (on the L and C sense only) by:

$$v(x,t) = Xu(x,t), \quad i(x,t) = (X^T)^{-1} j(x,t) \qquad (3)$$

to get:

$$\frac{\partial}{\partial x} u(x,t) = -\hat{L}\frac{\partial}{\partial t} j(x,t) - \hat{R}j(x,t) \qquad (4)$$

$$\frac{\partial}{\partial x} j(x,t) = -\hat{C}\frac{\partial}{\partial t} u(x,t) - \hat{G}u(x,t) \qquad (5)$$

, where X is the eigenvector matrix of LC, $\hat{L} = X^{-1}L(X^T)^{-1}$, $\hat{C} = X^T CX$, $\hat{R} = X^{-1}R(X^T)^{-1}$, and $\hat{G} = X^T GX$. Note that both $\hat{L}$ and $\hat{C}$ are diagonal matrices. Consider the kth equation of both (4) and (5):

$$\frac{\partial u_k(x,t)}{\partial x} + \hat{L}_k \frac{\partial j_k(x,t)}{\partial t} = -\sum_{l=1}^{N}[\hat{R}_{k,l}\, j_l(x,t)] \qquad (6)$$

$$\frac{\partial j_k(x,t)}{\partial x} + \hat{C}_k \frac{\partial u_k(x,t)}{\partial t} = -\sum_{l=1}^{N}[\hat{G}_{k,l} u_l(x,t)] \qquad (7)$$

,where $1 \le k \le N$, and N is the number of lines of this transmission line. Now we will apply the Method of Characteristic. We use the equations:

$$\frac{dx}{dt} = g_k \text{ and } \frac{dx}{dt} = -g_k \text{ (where } g_k = (\hat{L}_k \hat{C}_k)^{-0.5}\text{)}$$

to define the characteristic lines in the x-t plane, which are called the kth characteristic a and b lines respectively. Considering the differentiating along the kth characteristic a line:

$$\frac{d^a}{dt}[u_k(x,t) + Z_k\, j_k(x,t)] = -\sum_{l=1}^{N}[g_k \hat{R}_{k,l}\, j_l(x,t)] - \sum_{l=1}^{N}\frac{\hat{G}_{k,l} u_l(x,t)}{\hat{C}_k} \qquad (8)$$

, in which $Z_k = (L_k / C_k)^{0.5}$ is the *characteristic impedance* of the kth transmission mode. Similarly, along the kth characteristic b line, we have:

$$\frac{d^b}{dt}[u_k(x,t) - Z_k j_k(x,t)] = -\sum_{t=1}^{N}[g_k \widehat{R}_{k,l} j_l(x,t)] - \sum_{l=1}^{N}\frac{\widehat{G}_{k,l} u_l(x,t)}{\widehat{C}_k} \qquad (9)$$

Now, solve (9) by Forward Euler (FE) integration method:

$$u_k(t_n) - Z_k j_k(t_n) = u_k(t_{n-1}) - Z_k j_k(t_{n-1}) + \Delta t_i \sum_{l=1}^{N}[g_k \widehat{R}_{k,l} j_l(t_{n-1})] - \Delta t_i \sum_{l=1}^{N}\frac{\widehat{G}_{k,t} u_l(t_{n-1})}{\widehat{C}_k} = V_{0,k} \qquad (10)$$

Notice that both x-axis and t-axis have been divided into segments here (Δt_i is the segment length of t-axis). Equation (10) is expended into matrix form to represent N lines:

$$u(t_n) - Zj(t_n) = V_0 \qquad (11)$$

Then, by substituting (3) into (11) we get:

$$i(t_n) = G_0 v(t_n) + I_0 \qquad (12)$$

, where $G_0 = (X^T)^{-1} Z^{-1} X^{-1}$, and $I_0 = -(X^T)^{-1} Z^{-1} V_0$. Equation (12) represents the $x=0$ end terminal equivalent circuit of this transmission line. The $x = D$ (D is the length of this transmission line) end terminal equivalent circuit can be obtained by processing (8) similarly. The result is:

$$u_k(t_n) + Z_k j_k(t_n) = u_k(t_{n-1}) + Z_k j_k(t_{n-1}) + \Delta t_i \sum_{l=1}^{N}[g_k \widehat{R}_{k,l} j_l(t_{n-1})] + \Delta t_i \sum_{l=1}^{N}\frac{\widehat{G}_{k,t} u_l(t_{n-1})}{\widehat{C}_k} = V_{D,k} \qquad (13)$$

$$-i(t_n) = G_D v(t_n) + I_D \qquad (14)$$

, in which $G_D = G_0$, $I_D = -(X^T)^{-1} Z^{-1} V_D$. Equations (12) and (14) represent the time-domain equivalent circuits of a transmission line, Figure 1.

WR-based Algorithm

In equation (10) and (13), the inner time step, $\Delta t_i = t_n - t_{n-1}$, is used, which should be small enough to certify numerical stability and solving accuracy. If Δt_{iok} is a fine value for Δt_i, the x-axis of the x-t plane would be divided into at lease $M = \lceil D/(\Delta t_{iok} g_{\max}) \rceil$ segments. We call the u/j values on the conjunctions of these sections *inside-state*(st_i), and the values of I_0, I_D, G_0, and G_D *outside-state*(st_o) in this paper. They are combined to represent the states of a transmission line. Follows is the WR-based circuit simulation algorithm:

Algorithm 1(WR-based Circuit Simulation):

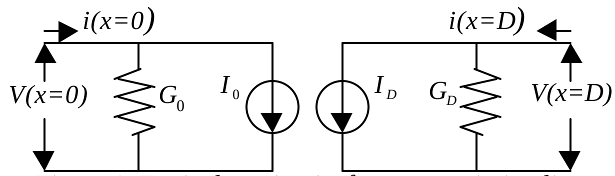


Figure 1: Equivalent circuits for a transmission line.

Partition the analyzed circuits into N subcircuits;
Perform DC analysis to get initial values;
Schedule subcircuits into appropriate order: S_i, $1 \le i \le N$;
for(k=1, $converge$=0; ! $converge$; k++) {
    for (i=1; i <= N; i ++) {
        Reset transmission lines' states to initials;
        if(S_i contains transmission line)
            /* separated time step scheme */
S1:   **if**(! *STS_condition*) turn STS off;
            else turn STS on;
        for (*time*=0, *time* <= maximum simulation time;) {
            if(S_i contains transmission line T) {
A: *tsim* = *time* + Δt_i;
B: st_o(*tsim*) = F(st_i(*time*));
            }
            else {
                Pick an appropriate time step *stp* for S_i;
C: *tsim* = *time* + *stp*;
            }
S2:   Solve S_i by using direct approach;
            if(S_i contains transmission line T)
D: st_i(*tsim*) = G(st_i(*time*), uj_b(*tsim*));
            time = *tsim*;
        }
        Save waveforms of S_i into waveform tables;
    }
    if(all waveforms converge) *converge* = 1;
}

Notice that only the outer time steps of subcircuits containing transmission line(s) have to be synchronized with the denser inner time steps (line A), and those of other subcircuits (line C) are not affected. The function F() (line B) exploits equation (10), (11) and so on to calculate the current outside-state. Function G() (line D) exploits both equation (10) and (13), then uses old inside-state and uj_b(*tsim*) (*u/j* values on the two boundaries of the transmission line) to calculate the current inside-states.

STS and Method to Vary Inner Time Steps

Separated time step scheme (STS) separates inner and outer time steps, making inner and outer time steps go on their own rate. Figure 2 shows how STS solves the "outer" time point at *time*+*stp*, where the transmission line's states at *time_tr* and outer time point at *time* have been calculated well initially. STS works as follows:

Algorithm 2(Separated Time Step Scheme):

tsim_tr = *time_tr* + Δt_i;
while(*tsim_tr* <= *time*+*step*) {
    st_i(*tsim_tr*) = G(st_i(*tsim_tr*- Δt_i), uj_b(*time_tr*));
    if(*tsim_tr*+ Δt_i > *time*+*stp*)
        st_o(*tsim_tr*) = F(st_i(*tsim_tr*- Δt_i));
    tsim_tr += Δt_i;
}
Solve S_i by using direct approach;

Once STS is turned on, Algorithm 2 would replace the three lines, starting from line S2, of Algorithm 1. It is obvious that STS saves the calculations on “redundant” outer time steps. But STS derives approximation solution rather than exact solutions, it can only be used in the first few WR relaxation. The condition *STS_condition* (such as “not all fan-in waveforms have at least 70% converged portions”) is used in line S1 of Algorithm 1 to turn off STS in later few WR relaxations.

When STS works, the outer time step and inner time step would be separated. We now consider the variable inner time steps. Since outer time steps have been chosen to consider accuracy already, we can make the magnitude of inner time steps be not smaller than a fixed portion of outer time step, identical to allow a maximum number of inner time steps be calculated in an outer time step. We’ve defined a constant *skip_STS_factor* for this maximum limit. Once the number of inner time steps in an outer time step exceeds *skip_STS_factor*, only *skip_STS_factor* inner time points would be distributed uniformly in this outer time step. Saving the calculating at inner time points is accomplished by by-passed (or latency) [2] technique.

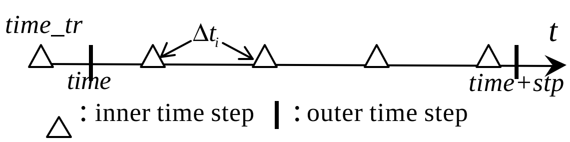


Figure 2: Operation environment of STS.

EXPERIMENTAL RESULTS

The methods proposed have been implemented in the program MOSTIME [4]. Figure 3 to 6 demonstrate the accuracy of this implementation by showing simulations results of a one-subcircuit analog circuit and a synchronous counter. The electrical parameters of transmission lines have been shown in figures. Both Line#1 and Line#2 are of .01 meter long and are divided into 10 uniform sections, i.e. M is 10 (note that these parameters for transmission lines are still valid in following examples). In HSPICE, the transmission lines are emulated by cascading 25 stages of lump elements. We can see that very good waveform matches in both cases.

In Table 1 the one-bit ALU circuit Alu, shown in Figure 7, is cascaded to form circuits of different sizes, and then be simulated. We can see the time complexity is nearly linear. In the “WR+STS” case, the STS (and method to vary inner time steps) is used, in which the *STS_condition* is “not all fan-in waveforms have at least 80% converged portions” and the value of *skip_STS_factor* is 10 (both which are valid in following examples). We can see that STS doesn’t increase the number of WR relaxations, moreover it reduces the simulation time dramatically. The waveforms obtained by both methods are completely the same, which is due to that MOSTIME uses same codes to undertake the last few WR relaxations, and check the convergence by a same criterion (.05v absolute tolerance).

Table 2 summarizes the situations of simulating some circuits by using various algorithms (Direct method of MOSTIME doesn’t include the sparse matrix solver), where

Table 1: Simulation time for ALU

# Stages		1	2	3	4	8
# Nodes		52	104	156	208	416
#Transistors		102	204	306	408	816
# Subcircuits		29	58	87	116	232
# Transmission Lines		1	2	3	4	8
Used CPU Time (#relaxations)	WR	26(4)	52(5)	80(4)	121(4)	232(4)
	WR+STS	15(4)	33(5)	46(4)	66(4)	132(4)

Cpu time is in Pentium II-233 Seconds

Inner# is the totally number of inner time steps. Notice the Direct method works relatively badly when the circuit is big and contains fewer transmission lines (such as Alu and 10-stage Inv5), which is because that many “redundant” outer time points are caused by the denser inner time points. This observation certifies the importance of “bounding” transmission lines into subcircuits. Then we note that STS works better when circuits have higher ratio of nodes in subcircuits embedding transmission lines (such as Invs2, 10-stage Invs2), since STS mainly saves the number of calculations on this kind of subcircuits. The bigger ratio of relaxations turning on STS, the more saving can be got (such as Alu), too. Generally speaking, WR-based method causes much more inner time points, which is due to the iteration behavior of relaxation-based algorithm.

CONCLUSION

In this paper, a time-domain method for transmission line calculation is used, so what proposed can be easily implemented. Moreover, by using WR, both the large-scale circuit problem and the “serious” multi-rate problem are solved well. The proposed method shows good efficiency in deriving accurate transient response waveforms of large-scale circuits containing transmission lines, and it can be used in other circuit simulation algorithm (such as STWR [5]) capable of solving the multi-rate problem. We think this work could have obvious contributions to circuit design community.

REFERENCE

[1.] Jun-Fa. Mao, Ernest S. Kuh, “Fast simulation and sensitivity analysis of lossy transmission lines by the method of characteristics,” *IEEE Tran. on Computer-aided Design*, Vol. 44, No. 5, pp. 391-401, May 1997.

[2.] A. R. Newton, and A. L. Sangiovanni-Vincentelli, “Relaxation-based electrical simulation,” *IEEE trans. Computer-Aided Design*, vol. CAD-3, pp. 308-331, Oct. 1984.

[3.] Chun-Jung Chen, Wu-Shiung. Feng, “Relaxation-based transient sensitivity computations for MOSFET

Table 2: Performance of algorithms.

Circuits	Various Algorithms							
	Direct		WR			WR+STS		
	Time	Inner#	Time	Relax#	Inner#	Time	Relax#	Inner#
Invs2	3	8909	6	2	17818	4	3	17915
Invs5	5	8909	9	3	26727	8	4	26471
10-stage Invs2	168	89090	82	4	311815	65	5	256584
10-stage Invs5	1233	89090	145	7	552358	143	9	550418
Alu	1319	17818	26	4	71272	15	4	36741
Sct	151	14967	143	34	493911	137	34	464827

Cpu time is in Pentium II-233 Seconds

circuits," *IEEE Trans. on Computer-aided Design*, Vol. 14, No. 2, pp. 173-185, Feb. 1995.

[4.] F. H. Branin. Jr., "Transient analysis of lossless transmission lines," *Proceeding of the IEEE*, pp. 2912-2913, 1967.

[5.] Chun-Jung. Chen, and W. S. Feng, "Transient sensitivity computations of MOSFET circuits using Iterated Timing Analysis and Selective-tracing Waveform Relaxation," *Proceeding of 31st Design Automation Conference*, pp. 581-585, June 1994.

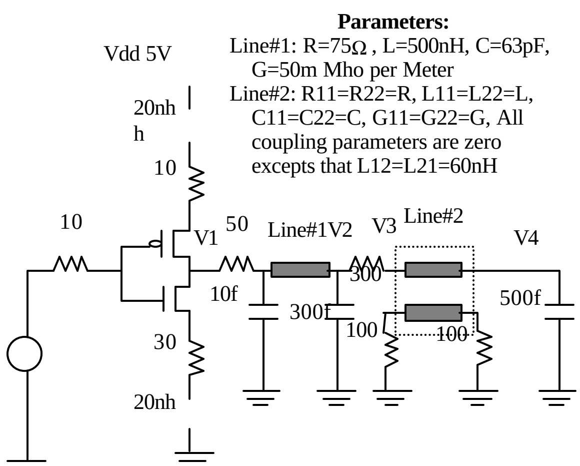


Figure 3: Schematic of an analog circuit.

Figure 6: Waveforms comparison for Figure 5.

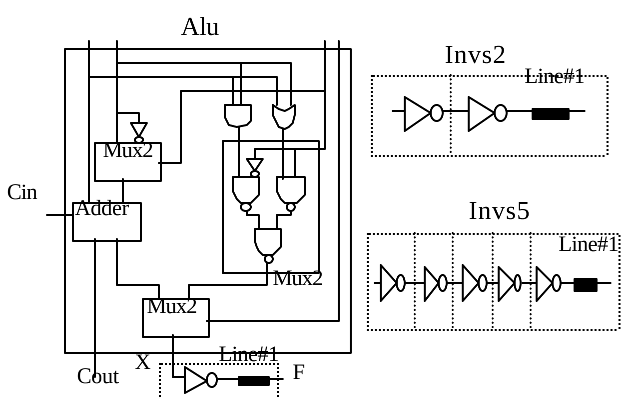
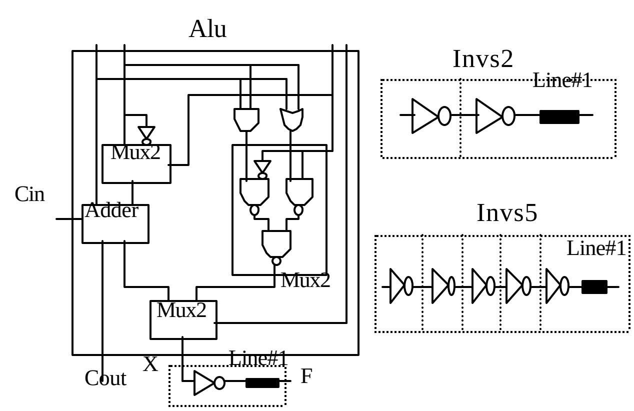


Figure 7: Schematics of the three testing circuits, in which each dashed-line rectangle (and logical gates in Alu) forms a subcircuit.

Figure 4: Waveform comparison for Figure 3.



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