System Level Simulation of a Digital Accelerometer

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ABSTRACT

This paper describes the derivation of a detailed model of a micromachined accelerometer and its implementation in a commercial SPICE package. The sensing element is modeled at a system level, the control and interface electronics at a component level. The SPICE simulation allows very realistic insight into the system behavior, however, the simulation time is considerable long.

Keywords: Micromachining, accelerometer, sigma-delta-modulation, simulation

INTRODUCTION

Micromachined inertial sensors have an enormous growth potential for automotive, medical, inertial navigation and guidance applications. However, the development to market time for MEMS in general is still considerably long which is one of the most restricting reasons for commercial exploitation. One important reason for this is the lack of suitable simulation tools which for MEMS are still under development [1], it is therefore a promising approach to use standard electronic circuit simulation packages which can be used to model micromachined components at system level and the interface electronics at component level.

In this work an accelerometer, including the capacitive, bulk-micromachined sensing element and the control electronics, was simulated using a commercial SPICE package. The sensing element is incorporated in a force-feedback loop which yields an increase in bandwidth, dynamic range and linearity. The control system strategy relies upon an oversampling conversion technique or sigma-delta modulation which yields an inherently digital sensor and has superior system stability compared to analogue, closed loop devices [2]. The control strategy embeds the sensing element in a sigma delta modulator loop and uses electrostatic forces to counter-balance the inertial force acting on the proof mass caused by acceleration. Under normal operating conditions this maintains the mass close to the center position between the electrodes, consequently, nonlinear effects, which dominate for larger mass deflections introduced by the damping, the conversion from a voltage to an electrostatic force and the suspension system, are linearized to first order.

MATHEMATICAL MODEL

Before using SPICE to simulate at component level it was important to derive a detailed mathematical model of the accelerometer. This model included:

- the micromachined sensing element considering the effects of squeeze film damping, offset due to manufacturing tolerances and conversion from the mechanical to the electrical domain,
- the sigma-delta modulator control structure,
- the compensator,
- the reset mechanism employing electrostatic force feedback,
- conversion from the feedback voltage to the electrostatic forces including the effects of the current position of the mass.

The sensing element was modeled as a standard mass-spring-damper system. The damping mechanism is determined by squeeze film damping, for this particular sensing element and for the bandwidth of the accelerometer the squeeze number is small, hence the damping coefficient, b, can be found to be [3, 4]:

$$b(x) = \mu A^2 \left(\frac{1}{(d_1 - x)^3} + \frac{1}{(d_2 + x)^3}\right) \quad (1)$$

where μ is the viscosity of air, A the area of the electrodes, x the deflection of the mass and d_1 and d_2 the gaps from the mass to the top and

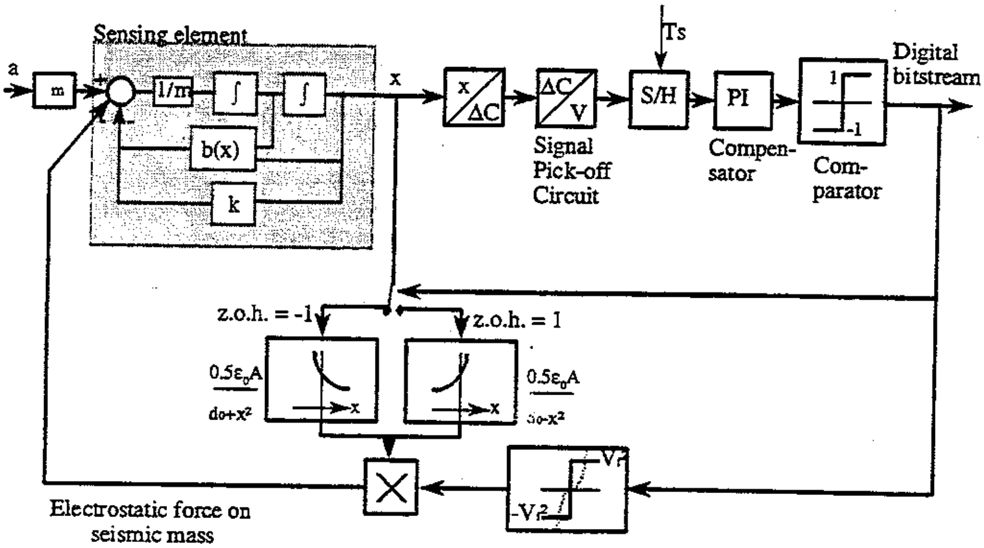

Figure 1: Mathematical model of the digital accelerometer

bottom electrodes respectively. Ideally, the gaps to either side should be equal ($d_1=d_2=d_0$), however, due to manufacturing tolerances up to 20% mismatch was observed.

If small deflections of the seismic mass are assumed – which for closed loop operation is a valid assumption – eq. (1) reduces to [5]:

$$b = 0.42\mu W^3 L / d_0^2 \qquad (2)$$

where W and L are the length and width of the seismic mass and d_0 the nominal gap.

However, the more general expression of eq. (1) was used for the model in order to simulate conditions in which the seismic mass is deflected considerably from its rest position; such a condition can be can caused by a shock in acceleration.

For the conversion from the position to the differential change in capacitance, ΔC, the following equation was used:

$$\Delta C = \varepsilon_0 A\left(\frac{1}{d_1 - x} - \frac{1}{d_0 + x}\right) \qquad (3)$$

where ε_0 is the dielectric constant of vacuum.

The conversion from the differential change in capacitance to a voltage is realized by a switched capacitor circuit which applies a voltage step to the top electrode and a voltage step of the same magnitude but of opposite polarity to the bottom electrode. This can be modeled by a charge amplifier producing an output voltage V_{out} of:

$$V_{out} = V_{Step}\frac{\Delta C}{C_i} \qquad (4)$$

where V_{step} is the voltage step and C_i the integrating capacitor of the charge amplifier. The output voltage is then stored in a sample and hold circuit and passed on to a PI compensator. The comparator, whose output states correspond to the mass being either above or below the center position, can be modeled as an ideal relay and determines to which electrode a voltage pulse is applied. The feedback force acting on the seismic mass during one operating cycle is given by:

$$F_{fb} = 0.5\varepsilon_0 AC\frac{V_f^2}{d_{1/2} + Cx^2} \qquad (5)$$

where C is the state of the comparator and is ±1. Since the system is oversampled, the movement of the seismic mass during one operating cycle is negligible ($x \ll d_{1/2}$) and to first order the feedback force can be assumed as constant.

The mathematical model derived above is suitable for implementation in a standard simulation package such as MATLAB/SIMULINK, however, to include effects from nonideal electronic components SPICE is a much more realistic simulation tool.

SPICE SIMULATION MODEL

The SPICE simulation model can be divided into three parts: 1) the sensing element simulated with the Analogue Behavioural Modeling Library, 2) the digital electronic section and 3) the analog electronics section. The three parts of the models are shown in fig. 2a-c.

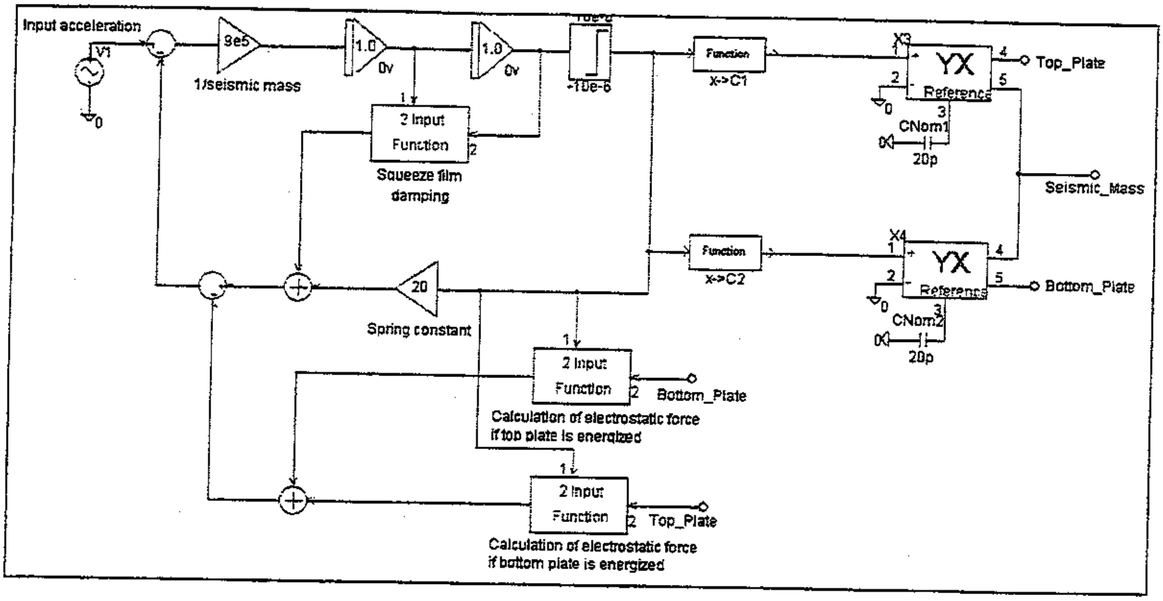

Figure 2a: SPICE model of the micromachined sensing element

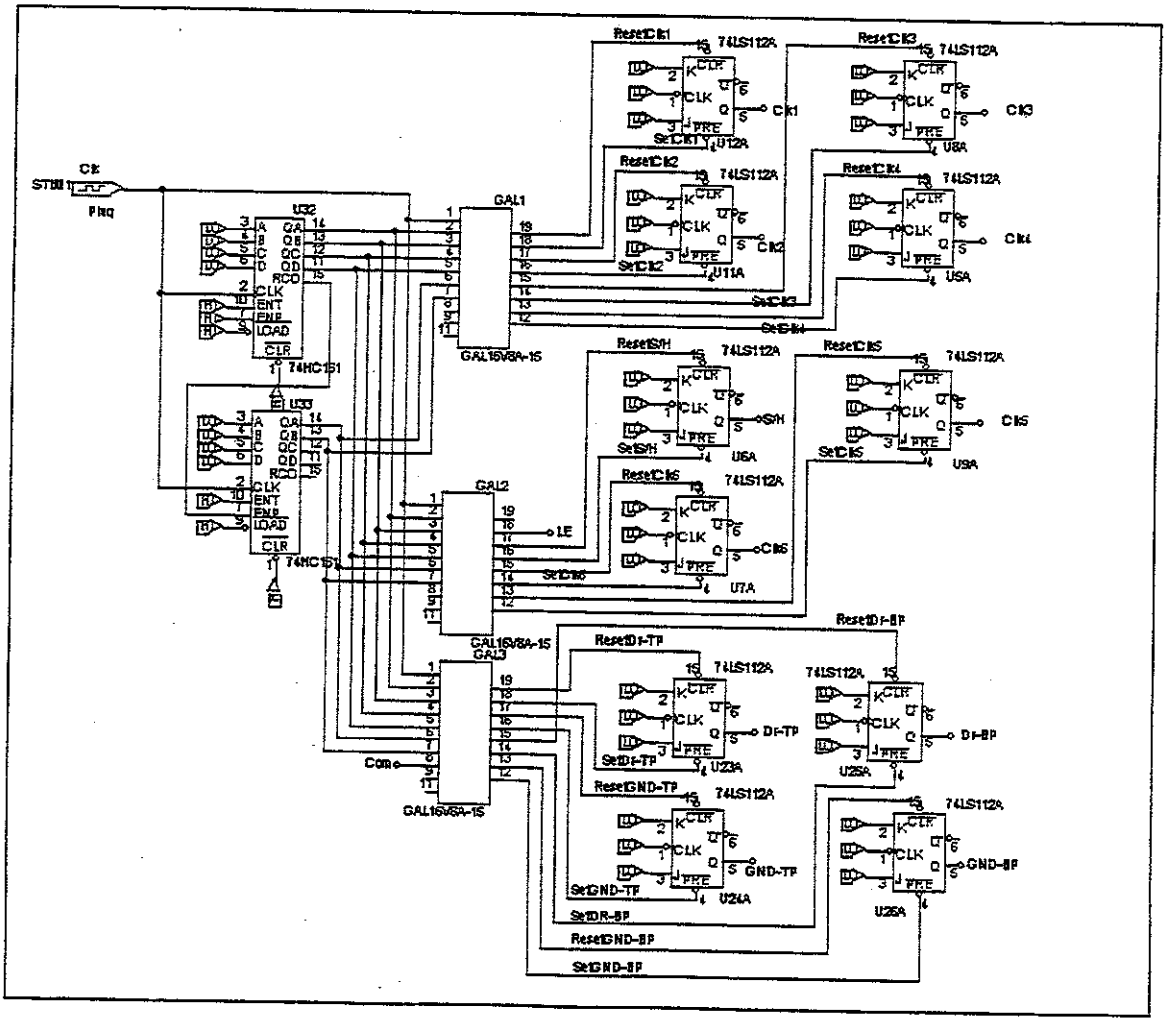

Figure 2b: SPICE model of the digital part of circuit

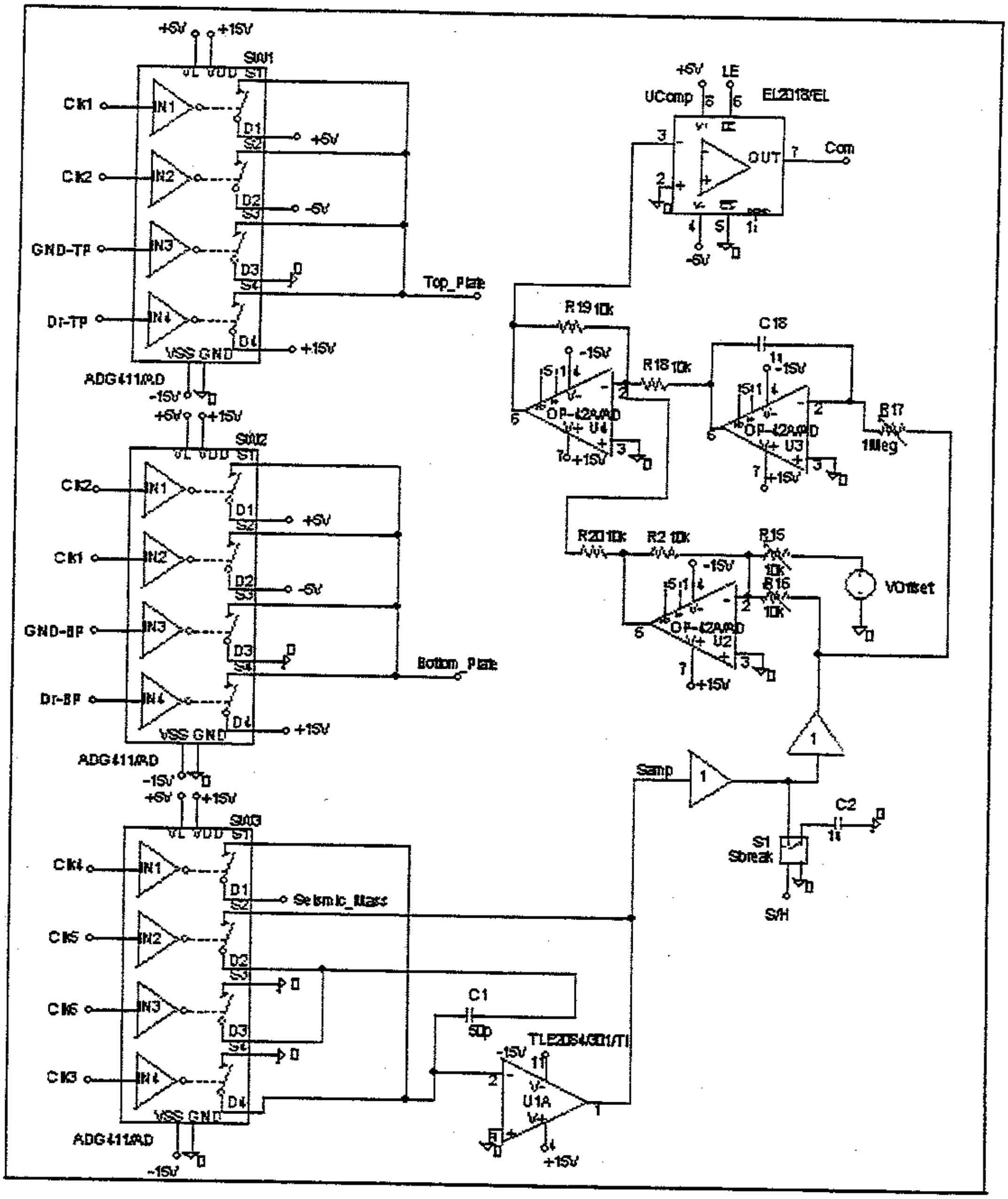

Figure 2c: SPICE model of the analog part of the circuit

The sensing element is an exact implementation of the mathematical model derived above. The function blocks required are implemented as voltage dependent voltage sources and are readily available in SPICE.

Conversion from the position of the mass to the momentary values of the two capacitors of the sensing element accomplished by a function block and a variable admittance.

The digital part of the circuit consists of two 8-bit counters from which three Gate Array Logic ICs (GALs) are used to derive logic control signals. The same JEDEC files required to program the GALs in the hardware realization can be used in the SPICE simulation. The signals of the GALs control several JK-flipflops which, in turn, control the analog switches in the analog section circuit used to create the different waveforms applied to the three terminals of the sensing element. These waveforms produce the voltage steps required for position sensing and apply the feedback voltage to pull the mass back to the center position between the electrodes. Between the sensing and the feedback phase the

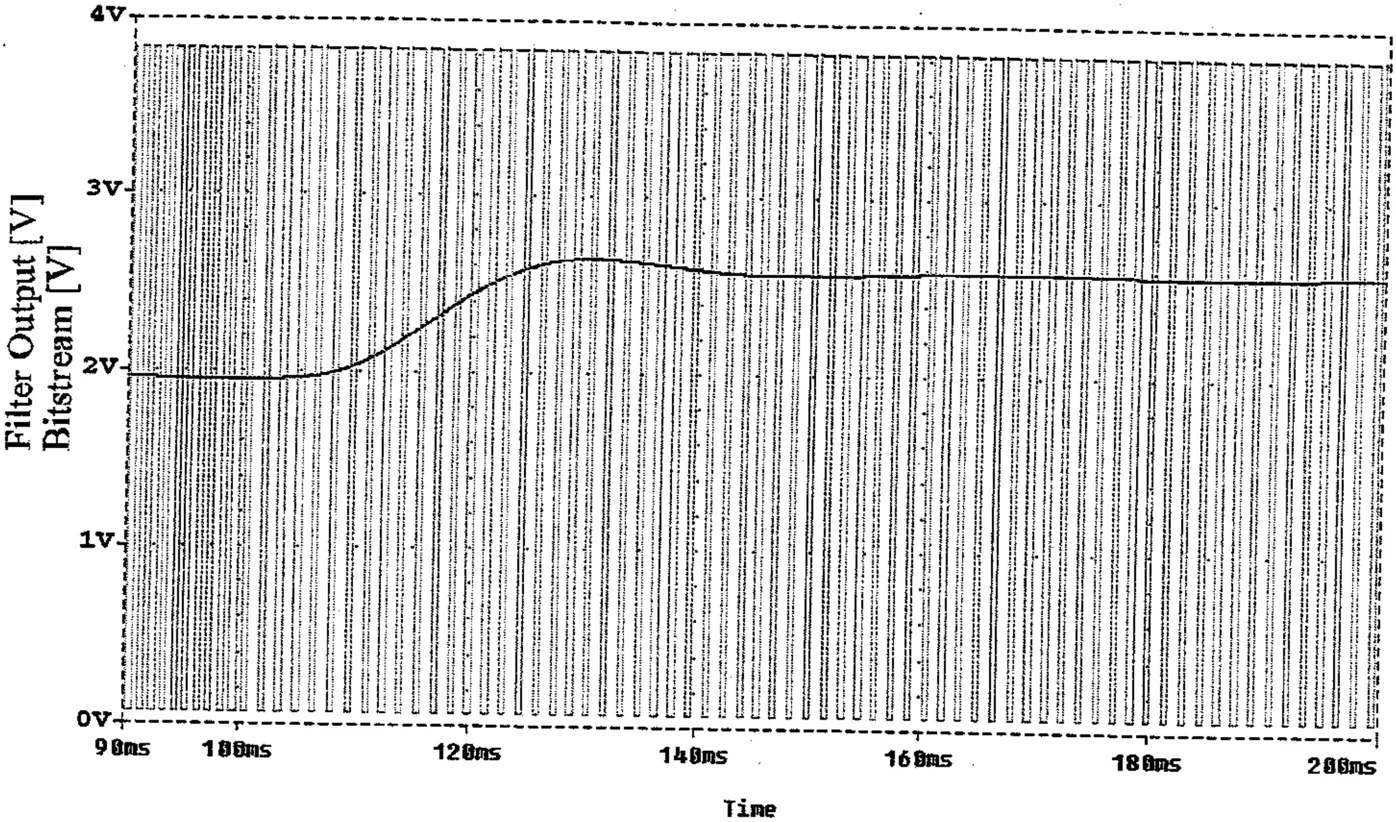

Figure 3: Response of the system to a 1g step in acceleration applied at 120 ms. The rapidly changing signal is the output bitstream, the other signal shows the low-pass filter output.

charge is removed from all capacitors by grounding all terminals.

In the analog part the charge amplifier produces an output voltage according to eq. (4) which is stored in a sample and hold (S/H) circuit. In the SPICE version used no model of a standard S/H IC was available, hence an idealized S/H was created consisting of a switch, a capacitor and unity gain buffers at the input and output of the device The signal from the S/H is passed on to various operational amplifiers which realize the PI and an offset control which can be used to compensate for offsets introduced by the signal pick-off and for those produced by the seismic mass due to manufacturing tolerances. The output signal of the comparator is then used as an input signal to the GALs and, in turn, determines which electrode is energized in the next feedback cycle.

A more detailed description of the circuit would go beyond of the scope of this paper and can be found in [2].

It should be noted that the SPICE options which determine the accuracy (Vtol, Reltol) had to be increased by a factor of 100, otherwise the output from the sample and hold stage introduced considerable random variations.

SIMULATION RESULTS

The simulation model was used to predict the overall system performance, the effect of different types of compensation (P and PI), verification of the theoretically predicted modes of oscillation in the unforced condition and the effects due to manufacturing tolerances.

A typical simulation result is shown in Fig.3 for which a step in acceleration of 1g magnitude was assumed. The output consists of a serial, pulse density modulated bitstream which is usually subjected to a low-pass filter (which is modeled in SPICE as well). The simulation results agreed very well with measurement results on the actual hardware prototypes [2].

The model allows to simulate the response of the digital accelerometer to different input acceleration waveforms. In addition, it permits to determine the main sensor parameters such as sensitivity, dynamic range, linearity and bandwidth and to investigate the effects of various parameter variation on the system performance.

A severe disadvantage of simulating at such a detailed level is the simulation time. To obtain a 200 ms long transient response a simulation run on a Pentium 233 took approximately six hours. Consequently, it is virtually impossible to obtain a frequency response, since many transient simulations would have to be run with different input magnitudes.

CONCLUSIONS

The model of a digital, micromachined accelerometer derived in this work proved to be a valuable tool to predict and evaluate the system performance before implementing the sensor in hardware. SPICE is suitable to implement models of non-electrical components which are described at a behavioral level, consequently, the entire microsensor system can be simulated comprising the sensing element and the interface electronics. The simulation can help to develop alternative interface electronics and control strategies [6]. The approach relies on readily available simulation tools and is relatively easily applicable to other micromachined sensors (e.g. gyroscopes [7]) provided that a lumped parameter mathematical model for the micromachined part is available.

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