An efficient numerical algorithm for extracting *Pull-In Hyper-Surfaces* of electrostatic actuators with *Multiple Uncoupled Electrodes*

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ABSTRACT

In this work the stability of electrostatic actuators with *multiple uncoupled voltage sources* applied to separate isolated excitation electrodes is analyzed. These electrodes are used to drive a *continuous deformable element.* A *novel computational strategy* and an *efficient numerical scheme* for simulating such problems are presented. The theory of the stability of electrostatic actuation and specifically the notion of the Pull-In state are generalized to such systems. In this respect, for an actuator with *K*-uncoupled electrodes and *N*-degrees of freedom, the Pull-In is shown to be a *K-1* dimensional hyper-surface in a *N*+*K* dimensional state-space of the actuator. The novel strategy enables the extraction of the Pull-In hyper-surface by scanning the voltage space along voltage hyper-rays. Along these rays a *DIPIE* scheme is used to rapidly extract the Pull-In parameters, resulting in an efficient multi-*DIPIE* scheme. To illustrate the strategy a clamped-clamped beam actuator with multiple electrodes is analyzed.

Keywords: Pull-In Hypersurface, Multiple uncoupled voltage sources, *DIPIE*, Electrostatic actuation

1 INTRODUCTION

Electrostatic actuators are widely used in *MEMS* devices [1]. These actuators are often characterized by an inherent instability above a certain applied voltage. The onset of instability is referred to as the Pull-In state. Some devices utilize this inherent Pull-In instability for digital switching. In other devices a wide dynamical range is required and the Pull-In phenomenon should be avoided.

Most electrostatic actuators utilize a *single voltage source* applied between two electrodes. Each of these electrodes may have an arbitrary shape and spatial distribution (Fig.1a) [2].

The electro-mechanical response of *MEMS* devices that utilize electrostatic actuation can be enriched by using *multiple uncoupled voltage sources* applied to separate isolated electrodes (Fig.1c). The multiple voltage sources may be used to drive an actuator with few (e.g., one [3] or two [4]) degrees-of-freedom (DOF), or to drive a continuously deformable element (e.g., beam or membrane [5,6]).

The stability problem of electrostatic actuators with a single source was widely addressed and various methods and algorithms for extracting the Pull-In parameters have been suggested [1-3]. The Pull-In instability emanates from the fact that above a critical voltage the actuator has *no equilibrium state* (Fig.1b).

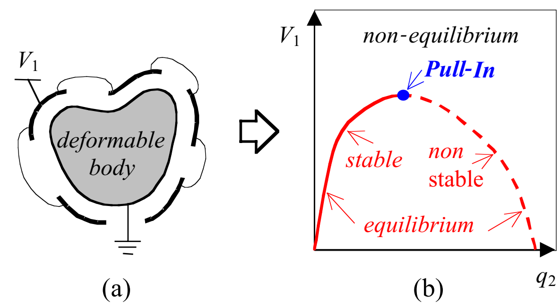


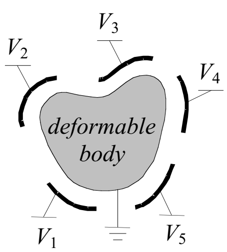
Figure 1 – Electrostatic actuator with a single distributed electrode. (a) schematic description, (b) the stability domain.

The stability problem of electrostatic actuators with an *arbitrary number of sources* and an *arbitrary number of degrees-of-freedom* was not fully addressed.

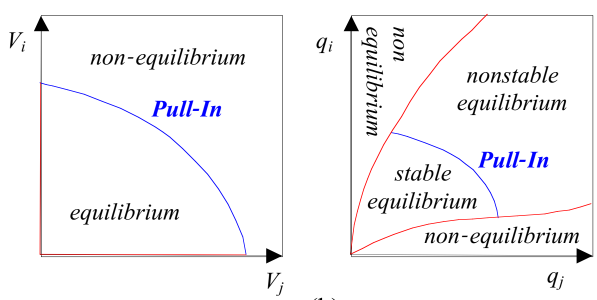
In this work the stability problem of actuators with multiple uncoupled voltage sources is analyzed. The dimensionality of the state-space of the problem is discussed and the notion of the Pull-In state is generalized for these systems. Various strategies for extracting the Pull-In states of the system are discussed, and a novel strategy that assures complete extraction is presented. The novel strategy together with the rapid *DIPIE* algorithm [9] enables efficient Pull-In extraction within a short run time.

2 MULTIPLE VOLTAGE SOURCE ACTUATION

A general electrostatic actuator constructed from a conducting deformable body and a finite number of *K* uncoupled electrodes (Fig. 2a) is considered. It is assumed that the deformation can be represented by *N* degrees of freedom q_i (i=1,...,N). Each of these electrodes forms a free space capacitor C_j (j=1,...,K) with the body. Uncoupled energy sources apply K voltages V_j (j=1,...,K) across each of these capacitors. This results in K generalized electrostatic forces that are applied on the body. In response the body deforms and mechanical restoring forces develop. Equilibrium states are those for which K-electrostatic forces are balanced by the mechanical restoring forces. These equilibrium states may be either stable or unstable. Above certain combinations of the applied voltages, no equilibrium states for the system exist. These combinations are equivalent to the Pull-In voltage of the single voltage actuation (Fig. 1b), all these combinations constitute a Pull-In hyper-surface the dimensionality of which is discussed in the following.



(a)



(b)

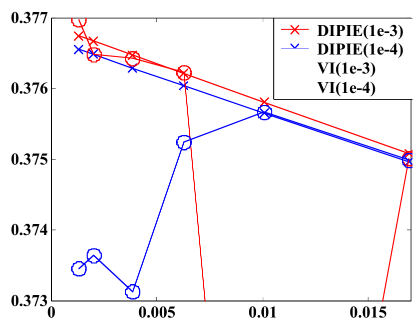
Figure 2 – Electrostatic actuator with multiple uncoupled voltage sources (a) schematic description, (b) cross-sections of the stability domain.

In an electrostatic actuator with K uncoupled electrodes and N degrees of freedom, the equivalent of the 1-*D* line of equilibrium states in the 2-*D* voltage-displacement domain (Fig.1b), is a K-dimensional *hyper-volume* of equilibrium states in a (K+N)-dimensional space. Also, the equivalent of the pull-in 0-*D* point in the single electrode problem is a *K-1* dimensional *hyper-surface* within the K-dimensional *hyper-volume* of equilibrium states.

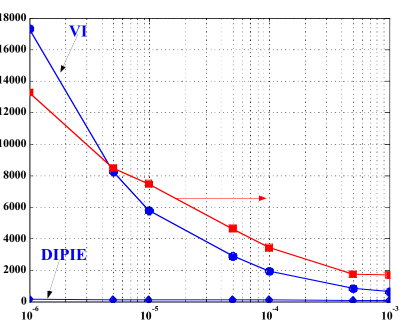
In order to visualize the stability domains of the problem, Fig. 2b illustrates representative 2-*D* cross-sections of the (K+N)-dimensional domain. In the 2-D voltages domain the Pull-In hyper-surface separates the equilibrium and non-equilibrium domains of the actuator. The equilibrium domain folds the stable and unstable states in these projections. In the 2-D space of representative displacements the Pull-In hyper-surface separates the stable equilibrium and unstable equilibrium domains of the actuator.

3 PULL-IN HYPER SURFACE EXTRACTION

Several strategies may be employed to map the Pull-In Hyper-surface of the considered electrostatic actuator with K uncoupled electrodes. A straightforward strategy is to scan the voltage domain by fixing all but one voltage and to execute a *voltage-iteration* (*VI*) numerical scheme [3-8] with the single variable voltage. In the *VI* scheme the N upper bounds of each of the voltages are a-priory unknown. Moreover, the authors have recently shown that the *VI* scheme has poor convergence characteristics in terms of consistency and run-time (Fig.3) [9].



(a)



(b)

Figure 3 - Comparison of the convergence characteristics in terms of (a) consistency and (b) run-time between the VI-scheme and the DIPIE-scheme.

Another critical issue that makes this strategy invalid in several actuators is that the variable voltage scan may originate in a non-equilibrium domain. Thus the solver cannot identify whether an equilibrium state exists for the fixed set of voltages.

To illustrate this point a 2D projection of the stability domain of the DMD™ [3] on the applied voltages space is shown in Fig. 4. First consider the fixed V_2 scan-lines 1 and

2, which are lower than a critical value of V_2. It is clearly seen that the scan originate from an equilibrium state and thus the scan of V_1 can easily meet the Pull-In hyper-surface. Second, consider the scan lines 3 and 4, which are above this V_2 critical value. These scan lines originate from a non-equilibrium state, thus preventing the identification of the Pull-In hyper surface for these V_2 values.

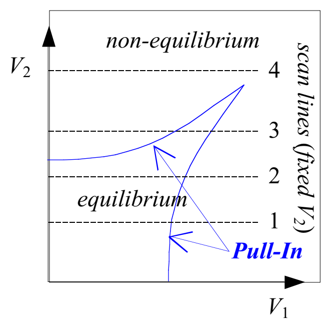


Figure 4 - Typical concave Pull-In hyper-surface of a DMD™ exhibiting the fixed voltages strategy scan lines

The present work suggests a novel strategy, namely scanning the voltage domain along *hyper-rays* of fixed voltage ratios (i.e., $V_i=\alpha_i V_1$ i=2..N). All these hyper-rays originate from the unloaded state (V_i=0 i=1..N) unlike the above *VI* strategy, thus solving the above mentioned problem.

Moreover, this novel strategy can be implemented using a *DIPIE* scheme [9] (recently presented). By using the *DIPIE* scheme, the remaining variable voltage (V_1) is eliminated from the calculation resulting in a Voltage-Free calculation. Moreover, the *DIPIE* scheme has excellent convergence properties in terms of consistency and run-time (>100 time faster than the *VI* scheme) (Fig.3).

4 CLAMPED-CLAMPED BEAM EXAMPLE

To illustrate the scheme, the equilibrium states of the continuous clamped-clamped beam actuator with two asymmetric, uncoupled electrodes (Fig.5) are calculated.

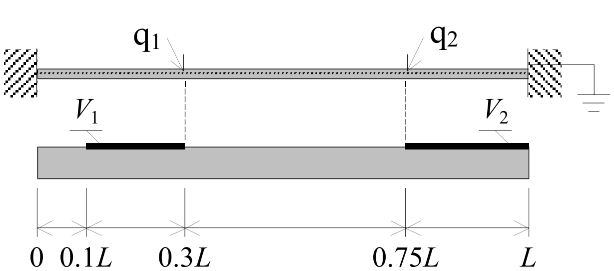


Figure 5 - A schematic view of a clamped-clamped beam actuator with two asymmetric, uncoupled electrodes.

Figure 6a describes the solution in the applied voltages domain. In this domain the voltage hyper-rays are straight lines along which the *DIPIE* algorithm was employed to obtain the point of intersection with the Pull-In hyper-surface.

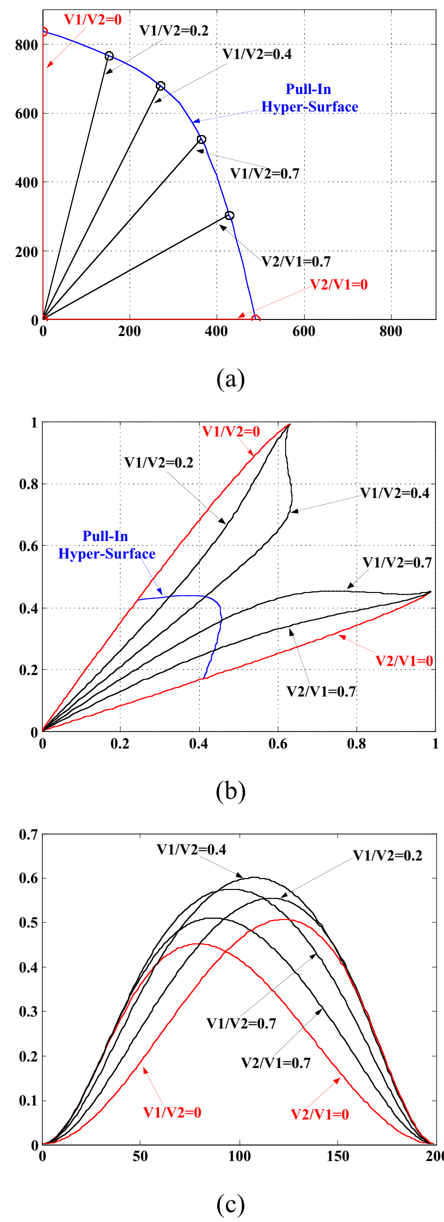


Figure 6 – Stability analysis of a clamped-clamped beam actuator with two uncoupled electrodes, (a) voltages-domain and (b) displacements-domain exhibiting Pull-In hyper-surface and selected equi-voltage-ratio hyper-rays, (d) selected Pull-In deflection mode.

Figure 6b describe the solution in the representative-displacements domain. In this domain, the hyper-rays are curved lines that originate from the unloaded state. The hyper-rays pass from the stable equilibrium domain to the non-stable equilibrium domain crossing the Pull-In hyper-surface between the two domains. In Fig. 6a the stable and non-stable regions along the hyper-rays appear on top each other. The beam *Pull-In deformations* for selected Pull-In states appear in Fig.6c.

5 SUMMARY

In this work a novel strategy for extracting the Pull-In hyper-surface of electrostatic actuator driven by multiple, uncoupled voltage sources, was presented. The novel strategy enables the mapping of the stability domain of actuators for which currently used strategies fail. Moreover, the novel strategy enables the utilization of the rapid and efficient *DIPIE* scheme, enabling the complete extraction of the Pull-In hyper-surface within a very short run time. These features were demonstrated on the widely addressed clamped-clamped beam actuator. Using the novel strategy the Pull-In hyper-surface was mapped with in a few minutes whereas current voltage iterations based algorithms require the same run time to extract a single point on the hyper-surface.

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