

# Phase-lead Compensator to Improve the Transient Performance of Comb Actuators

Yijian Chen, Yashesh Shroff and William G. Oldham  
EECS Department, University of California, Berkeley, CA 94720

## ABSTRACT

The transient performance of electrostatic actuators is important in high-frequency switching operation of MEMS devices [1]. Optimization of the transient behavior of a simple actuator without introduction of controller network or modification of actuator shape has been studied recently [2, 3]. However, it is desirable to further improve the transient performance by electrical or mechanical means.

For this purpose, we present a simple electrical phase-lead compensator and a novel shape design of the comb drive (a mechanical phase-lead compensator). Both of them will increase the modulation force impulse to the actuator at the initial stage with little influence on the behavior at the later (settling) stage. Numerical simulation is carried out to show that the rise time can be reduced.

**Keywords:** Controller, phase-lead compensator, comb drive, overshoot, rise time, settling time, transient optimization.

## 1 INTRODUCTION

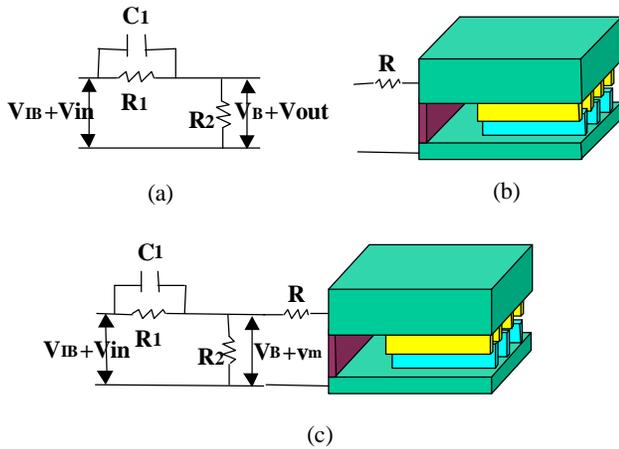


Fig. 1. A comb drive and a phase-lead compensator.

Different forms of controllers, for example, PID controllers, phase-lead and phase-lag controllers, are widely used in large-scale industrial processes [4]. However, there are fewer applications of controllers in the MEMS area, mainly due to the difficulty of introducing complex networks in MEMS devices. In this paper, we investigate two easy-to-fabricate controllers to improve the transient performance of the comb drive. A typical electrical phase-lead compensator and a comb drive are

shown in Fig. 1 where (a) is the phase-lead network, (b) is the comb drive with a resistor to introduce electrical damping and (c) is their combination. A bias voltage ( $V_B$ ) of 1.414 V and a modulation voltage ( $v_m$ ) of 0.586 V are chosen in our simulation for low power operation. In Fig. 2, two shapes of comb drive are compared. In the next section, it will be shown that shape design of (b) is mechanically equivalent to a phase-lead compensator.

Extensive transient simulations are made to examine the effect of electrical phase-lead compensator (Fig. 1(c)) and new shape design (Fig. 2(b)). We adjust the parameters [3] to change the rise time while keeping the tolerable overshoot 10% above the equilibrium value. By this way, we are able to get the optimal response with the minimum rise time and a fixed overshoot (10%).

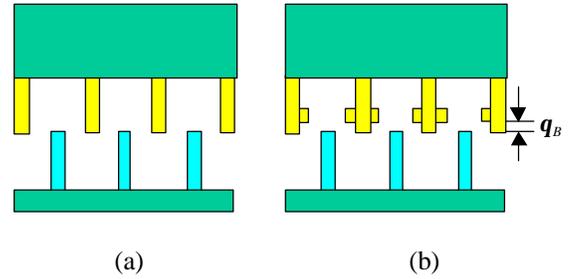


Fig. 2. Comparison of the shape of two comb drives with no voltage applied (not to scale, front view). A tooth is added in (b) to increase the capacitance derivative,  $\frac{\partial C}{\partial q}$ , where  $q$  is the angular displacement.

Both structures are biased by a bias voltage  $V_B$ . In both Structures, we consider a bias displacement of  $q_B = 0.5^\circ$ , and a modulation displacement of an additional  $0.5^\circ$ .

## 2 THEORETICAL ANALYSIS

It is assumed that the impedance of the comb drive is much larger than that of the compensator so that the modulation input to the actuator  $v_m$  when they are coupled together is about the same as the modulation output  $V_{out}$  when the terminal of compensator is open (Fig. 1(c)). The relation between  $V_{out}$  and  $V_{in}$  is as follows:

$$V_{out} = V_{in} \{1 - [1 - \exp(-t/\tau)] R_1 / (R_1 + R_2)\}, \quad (1)$$

$$\tau = C_1 \cdot R_1 R_2 / (R_1 + R_2)$$

Thus the total voltage input to the comb drive is:

$$\begin{aligned} V_B + v_m &\approx V_B + V_{out} \\ &= V_B + V_{in} \{1 - [1 - \exp(-t/\tau)] R_1 / (R_1 + R_2)\} \end{aligned} \quad (2)$$

In simulation,  $R_1 : R_2 = 9 : 1$  and the steady-state voltage gain is only 0.1. For optimization purposes,  $\tau$  is chosen to be  $0.04/w_0$  where  $w_0$  is the undamped natural frequency of the comb drive. A vertical flexure suppresses one vibrational mode. The electric loop and moment equations for the comb drive are:

$$V_B + v_m = iR + V = \frac{dq}{dt} R + \frac{q}{C}, \quad (3a)$$

$$I_0 \frac{d^2 q}{dt^2} = M_{beam} + M_{electric} = -kq + \frac{1}{2} V^2 \frac{\partial C}{\partial q} \quad (3b)$$

where  $I_0$  is the mass moment of actuator inertia,  $V$  is the voltage across it and  $q$  is charge. Equations (2), (3a) and (3b) are combined to solve the time response to a step modulation input. Since the coupled equations are nonlinear, numerical simulation is carried out. The parameters used for simulations in all cases are chosen to be the same, e.g., zero initial overlap of the combs, and  $RC_0 w_0 = 0.57$  ( $C_0$ : capacitance with bias). A complete description of these parameters and their nominal values can be found in [3]. A comparison between the time response with an electrical compensator and that without it has been made and shown in Fig. 3(a) and Fig. 3(b). It shows that an electrical compensator reduces the rise time while the modulated deflection remains the same.

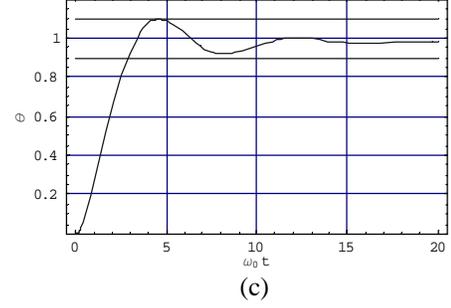
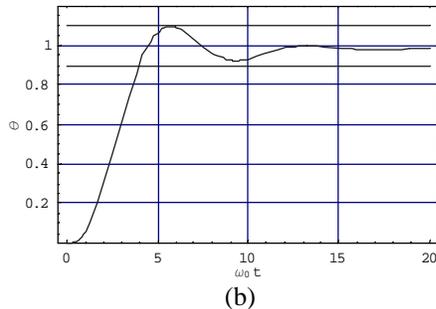
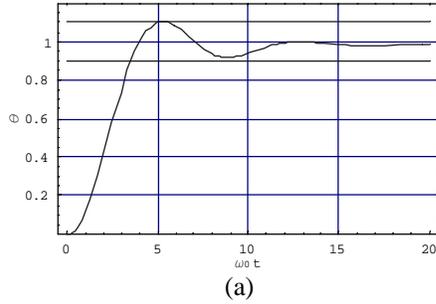


Fig. 3. Normalized (optimized) transient behavior of the comb drive (a) with an electrical phase-lead compensator, (b) comb without any compensator, and (c) with a modified comb shape of Fig. 2(b). The rise time in (a) and (c) is reduced by 10% and 25% from (b).

An alternative compensator uses a narrower gap near the tip of top fingers so that  $\frac{\partial C}{\partial q}$  is larger and the actuator feels a larger modulation force (ideally an impulse) at the initial stage. Two shapes of comb drive are shown in Fig. 2 and their resultant time responses (both without an electrical compensator) to a step modulation voltage are compared in Fig. 3(b) and Fig. 3(c). The new shape design (Fig. 2(b)) reduces the rise time by 25%. We find that this optimal tooth geometry is given by a tooth “width” of  $q_B / 15$  and tooth height such that  $\frac{\partial C}{\partial q}$  increases by a factor of 2.7 with respect to  $\frac{\partial C}{\partial q}$  for  $q < q_B$  and  $q > (1+1/15)q_B$ . In other words, the gap in the toothed region is  $1/2.7$  of the nominal gap.

In the electrical phase-lead compensator, simulation of equation (2) shows that the capacitor  $C_1$  makes the initial modulation voltage across  $R_1$  small and the whole voltage input will be across  $R_2$  and the comb drive, thus the initial electrostatic force is larger than that in steady state. However, the disadvantage is that a larger voltage input (e.g., 10 times of the case without a compensator) is needed to compensate the gain loss (and to achieve the same tilt angle) owing to the steady-state voltage division. No such loss occurs in the shape design.

### 3 CONCLUSION

In this paper, we have demonstrated that two novel easy-to-fabricate designs of a phase-lead compensator can reduce the rise time of a comb drive’s transient response to a step modulation input. Numerical simulations have been carried out to show that the rise time of comb drive with an electrical phase-lead network is reduced by 10% from the comb-drive-only case. More significantly, the rise time of the comb drive with a shape design is reduced by 25%. The control factors to achieve the reduction of rise time are presented.

## ACKNOWLEDGMENT

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