AUTO-REGRESSIVE TIME SERIES MODELLING OF STOCHASTIC SURFACES

B.Naga Prasada Rao and V.S.R.Murti

Department of Mechanical Engineering
University College of Engineering (Autonomous)
Osmania University
Hyderabad-500 007, (A.P.) INDIA
Fax: 040-7017065 E-mail: pnaga@hotmail.com Tel: (O)040- 7018951/317, (R) 7017802

ABSTRACT

Roughness profiles of EDM surfaces were obtained from a perthometer and digitized through AUTO CAD. Fortran program was developed for fitting ARMA (n, n-1) models. A significant observation was that for the sinking electrode type. EDM generally AR (1) or AR (2) models were adequate but for wire cut EDM higher order models, generally ARMA (3,2) and ARMA (4,3) were required. The higher order models indicate additional factors like wire vibration influencing the dynamics of wire EDM process.

Keywords: Time-series, EDM, Stochastic surface profiles.

INTRODUCTION

Surface texture being the imprint left behind by the machining process, which if analysed properly can lead to a better understanding of the basic mechanism of surface generation. In those of the machining processes where the tool is fed at a specific rate relative to the work surface, the final roughness is repetitive or periodic in nature and can be easily modelled mathematically. Such surfaces are called deterministic surfaces. On the other hand some surfaces like those from Electro Discharge Machining, (EDM) and Ultrasonic Machining (USM) are generated by erosion from random attack of electrical sparks or abrasive grains respectively. Such surfaces cannot be modelled and are termed as stochastic surfaces. Their random characteristics require statistical modelling. The roughness ordinates can be estimated in a sequence by digitizing the roughness profiles and treated as a time series, which can be statistically modelled by auto regressive (AR) modelling.

In the present work the EDM surfaces have been selected for time series analysis through auto regressive modelling.

Electro Discharge Machining (EDM) employs high frequency sparks for machining difficult to machine materials and contours. The tool work piece form a pair of electrode separated by about 20 to 200 μm in a liquid dielectric medium through which the spark discharges occur.

There are two forms of EDM. In one, the tool is rigid and preformed to the shape of the desired contour of the machined surface. This process is termed as sinking type EDM (SEDM). The other process employs a flexible wire of less than 0.5mm diameter as the electrode, continuously passing through the machining zone of work piece. The wire path is CNC programmed for travelling wire or wire-cut EDM or simply WEDM. The other difference in these two processes is the dielectric fluid which is kerosene in SEDM and deionised water in WEDM. The difference in the dielectric medium, shape of electrode and tool work interface areas in the two types of EDM is expected to lead to interesting results, analysis and inferences.

TIME SERIES MODELLING

A stochastic process can be modelled by treating the random data (here the roughness defined as a sequence of data in a particular order of time (t-2, t-1, t, t+1, t+2…………..). Time may be replaced by other variables such as space. The technique is termed as auto-regressive modelling since it expresses the dependence of a variable on its previous values. A n_{th} order model [AR(n)] gives dependence of a variable X_t on **n** previous observations [1].

$$X_t=\phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots\ldots\ldots \phi_n X_{t-n} + a_t \qquad (1)$$

$$a_t \approx NID\ (0, \sigma^2_a)$$

where ϕ is auto regressive coefficient and a_t is the disturbance or error with zero mean and σ_a^2, variance. In systems with good memory of the shocks or disturbance, a_t is dependent on a_{t-1}, a_{t-2}…….. In such case the models are called Auto regressive moving average or ARMA models are called Auto regressive moving average or ARMA models and have the following form.

$$\text{ARMA (n,m): } X_t=\phi X_{t-1} + \phi_2 X_{t-2} + \ldots\ldots \phi_n X_{t-n}$$

$$+a_t - \theta_1 X_{t-1} - \theta_2 X_{t-2} \ldots.\theta_{t-m} \qquad (2)$$

The first part of the ARMA model represents the Auto regressive component whereas the second part represents the moving average component.

METHODOLOGY

The modelling procedure and the associated flow chart along with the test of adequacy have been adopted from reference [2].

The roughness profiles of machined specimens (SEDM & WEDM) were obtained using Perthometer with very high magnification. The digitised values of roughness ordinates were treated as a Time series. Appropriate ARMA models were then fitted to each set of data by estimating the AR & MA parameters using inverse function estimates [2].

If the data comes form an ARMA (n,m) model, the orders m & n may not be guessed at all or the guessed order may turn out to be wrong after fitting this model. Thus, the most rational way in which to carry out trial and error is to successively fit all the models order until the suitable one is obtained. Generally an ARMA (n,m) model with m=n-1 is appropriate and ARMA (n,n-1) strategy is the easiest way of arriving at the model.

The technique uses standard non linear least square routines (2) until the residual sum of squares of random disturbances cannot be significantly reduced. EDM roughness profiles were carried out with four belonging to SEDM and six belonging to WEDM. More specimens of WEDM were analysed to study the possible influence of wire tension which has significant effect on wire vibration. Accordingly three values of wire tension: (700, 500 and 300 gm) were selected and replicated. For SEDM 2 levels of current were selected with replication.

RESULTS AND DISCUSSION

Typical roughness profiles of EDM surfaces are shown in fig1.

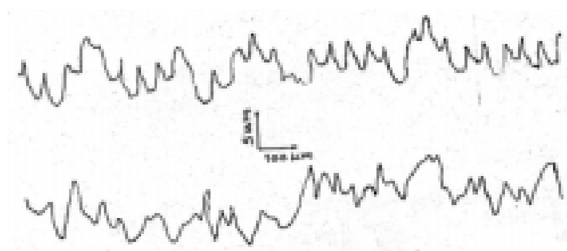

Fig.1 Typical roughness profiles of EDM surfaces.

The computed values of ARMA parameters are listed in table 1 and table 2 respectively for SEDM and WEDM surfaces.

Since the machining conditions were not varied (except for the wire tension) there was not much variation in the order of the ARMA models. Still highly significant observations were recorded.

Generally low order models AR (1) and AR(2) are adequate for SEDM. But for WEDM higher order models ARMA (4,3) and ARMA (3,2) were required. These result are in conformity with those of Williams and Rajurkar [3]. Still higher order models cannot be ruled out with variations in work piece

Table:1

Auto regressive parameters for SEDM surfaces:

S.No	Current	A.R. Parameters	Residual sum of squares	σa^2	Appropriate Model
1.	20	ϕ_1=1.619; ϕ_2= - 0.652;	212.28	0.7100	AR(2)
2.	20	ϕ_1=1.467;	263.73	0.8165	AR(1)
3.	10	ϕ_1=1.438; ϕ_2= - 0.463;	148.46	0.4596	AR(2)
4.	10	ϕ_1=1.595; ϕ_2= - 0.608;	64.76	0.2710	AR(2)

Table:2

Autoregressive parameters for WEDM surfaces:

S.No	Wire tension gm	A.R. Parameters	Residual sum of squares	σa^2	Appropriate Model
1.	700	ϕ_1=1.080; ϕ_2=-0.24;	56.73	0.2280	ARMA (3,2
2.	700	ϕ_1=1.639; ϕ_2=-0.0.775; θ1=0.214	96.09	0.3859	ARMA (2.1)
3.	500	ϕ_1=1.773; ϕ_2=-1.588; ϕ_3=1.100; ϕ_4=-0.422; θ_1=0.421; θ_2 =-0.668 θ_3=0.121	232.74	0.5784	ARMA (4,3)
4.	500	ϕ_1=1.887; ϕ_2=-1.035; ϕ_3=0.106; θ_1=-0.189; θ_2=0.182	91.24	0.3800	ARMA (3,2)
5	300	ϕ_1=0.652; ϕ_2=-0.283; ϕ_3=0.092; ϕ_4=-0.055; θ_1=0.657; θ_2=-0.209; θ_3=0.115	465.99	1.1700	ARMA (4,3)
6.	300	ϕ_1=0.394; ϕ_2=-0.296; ϕ_3=0.155; ϕ_4=--0.081; θ_1=0.917; θ_2=-0.594; θ_3=0.209	323.16	1.0800	ARMA (4,3)

height, pulse on and off times and other possible process variables including work and wire materials. Here only broad conclusions could be derived. The higher order models needed for WEDM can be attributed to wire feed marks and spark induced vibrations in the wire. It is not farfetched to suggest that AR component of ARMA models can be attributed to spark erosion and MA component to the dynamics of wire behaviour particularly its vibration. Low order models are characteristics of non-symmetry and higher order models suggest better symmetry or Kurtosis [4].

The model, AR(2) corresponding to data in row 1 (table1) will be:

$$X_t = 1.691\ X_{t-1} - 0.652\ X_{t-2} + a_t \qquad (3)$$

at ≈ NID (0,0.711)

And the model, ARMA (4,3) corresponding to data in row 6 (table 2) will be:

$$X_t=0.394\ X_{t-1} + 0.296\ X_{t-2} + 0.144\ X_{t-3} - 0.081\ X_{t-4} + a_t + 0.091\ a_{t-1} - 0.594\ a_{t-2} + 0.209\ a_{t-2} \quad (4)$$

$$a_t \approx NID\ (0,1.0800)$$

The first part of the model (eg.4) is AR component whereas the second part is the MA component.

In further evidence of wire vibrations being a definite factor affecting the WEDM surfaces, the effect of wire tension can be seen from the results.

For higher tensions (lower amplitude of wire vibration) lower order models [ARMA (2,1)] were adequate whereas for low tensions (higher amplitude of vibration) higher order models [ARMA (4,3)] were required. The shocks or disturbance caused by wire vibration appear to be remembered longer requiring higher order models.

CONCLUSIONS:

Time series analysis through Auto regressive modelling is an effective technique of evaluating stochastic surfaces. For conventional EDM with sinking electrodes low order models AR(1) and AR(2) are adequate whereas for WEDM surfaces, higher order models, typically ARMA (3,2) and ARMA(4,3) were required. The higher order models indicate additional factors like wire vibration influencing the dynamics of WEDM process. As a corollary, lower wire tensions lead to higher order models compared to the surfaces machined with higher order models compared to the surfaces machined with higher tension.

REFERENCES:

1. Pandit S.M. and Rajurkar, K.P. "DDS Approach to EDM process modelling from surface roughness profiles", Annals of CIRP, V-29, 1980, pp.107-112.

2. Pandit S.M, and Wu,S.M./ "Time series and system analysis with application" John wiley, N.Y. 1983.

3. Williams, R.E. and Rajurkar, K.P. "Study of wire EDM surface characteristics" Journal of material processing technology, V-28, 1991, pp.127-137.

4. Peters,J. et al., "Assessment of surface typology analysis techniques", Annals CIRP, Vol.28/2/79, p.p.539-533.