Techniques for Coupled Circuit and Micromechanical Simulation

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ABSTRACT

This paper details advanced simulation techniques for efficiently determining system behavior of integrated microsystems that contains both circuit elements and micromechanical ones. Techniques for building reduced-order dynamical models for coupled energy domain nonliner (both weakly nonlinear and strongly nonlinear) MEMS devices have been developed. An open-ended and expandable simulation framework that enables complete simulation of integrated microsystems is also implemented.

Keywords: nonlinear, dynamical, model-order reduction, composite simulation.

1 INTRODUCTION

Microelectromechanical systems are a rapidly field with great future potential. Simulation of these systems including both micromechanical devices and electronics will allow prediction and optimization of system performance before costly and time-consuming prototyping. Most micromechanical devices involve some form of nonlinearity. The action of these devices usually involves several physical effects which are coupled together. Direct dynamic simulation based on fully-meshed structures is computationally intensive, making it difficult to use in system-level simulators. In order to perform rapid design prediction and optimization of microelectromechanical systems, it is essential to build accurate and easy-to-use reduced-order dynamical models for these devices [4].

In the past, a lot of efforts have been focused on generating MEMS device reduced-order models using lumped-parameter techniques where devices are approximated as a network of circuit elements [1-2]. However, it is often difficult to accurately model continuous systems as lumped elements [3]. Another approach [5] uses static linear analysis to generate the linear modes of the device and formulate device dynamic behavior in terms of a finite set of the linear modes. However, static linear modes may not adequately capture the dynamic nonlinear behavior of the device [4]. Also when the problem involves dissipation such as the fluid damping effect, which is very important in studying the dynamic behaviors of most of the MEMS devices, this approach becomes substantially more difficult. Recently, in [6], Arnoldi approach is used to automatically generate reduced-order models for a fixed-fixed beam structure. It is very efficient to generate reduced-order models when the beam is operated in the linear regime. However when the beam deflection is large, linearized model deviates from the original nonlinear model significantly suggesting that nonlinear model-order reduction strategies are required.

In this paper, we propose several techniques to automatically generate reduced-order dynamical models for coupled energy domain nonlinear microelectromechanical devices. Specifically, a new method by combining Taylor series expansion and Arnoldi method is proposed for developing reduced-order models for weakly nonlinear MEMS devices. To develop reduced-order models for strongly nonlinear MEMS devices, the Karhunen-Loeve Galerkin's procedure is developed. Simulation results with the reduced-order models for a fixed-fixed beam structure demonstrate good agreement with the data generated from the finite difference model but with an order of magnitude reduction in execution time. These model reduction techniques can be applied to both energy conservation and energy dissipation systems. The reduced device models are represented by a small set of coupled ordinary differential equations and thus can be effectively connected to a circuit simulator for complete system simulations.

With the advent of micromechanical modules to be integrated with electronic circuits, simulation and analysis capabilities that go beyond traditional SPICE type electrical-level simulation to support multi-level and mixed technology simulations are required. For this purpose, a multi-level and mixed-technology simulator iSIMS (illinois simulator for integrated microsystems) has been developed. Various issues when simulating mixed technology systems represented across multi-levels such as system partitioning, event processing and scheduling, mixed relaxation/MNA formulation, and timestep control have been addressed.

2 BEAM EXAMPLE

In order to illustrate the model reduction technique, we examine the example of a fixed-fixed beam structure in a fluid (air) environment which is also studied in [6][8]. Fig. 1 shows the front view of the beam structure. When a voltage is applied, the top plate bends downward due to the electrostatic force. Also when the beam bends, the pressure distribution of the ambient air under the beam increases and this pressure increase produces a backward pressure force which damps the beam motion. The beam can be modeled by coupling the 1D Euler beam equation with the electrostatic force and the 2D Reynolds's squeeze-film damping equation as follows,

$$EI\frac{\partial^4 u}{\partial^4 x} - S\frac{\partial^2 u}{\partial^2 x} = F_{elec} + \int_0^w (p - p_a)dy - \rho\frac{\partial^2 u}{\partial^2 t} \quad (1)$$

$$\nabla \cdot (u^3 p \nabla p) = \frac{12\mu}{1+6K}\frac{\partial(pu)}{\partial t} \quad (2)$$

where $F_{elec} \approx -\frac{\varepsilon_0 w V^2}{2u^2}$ is the electrostatic force, $F_{air} =$

$\int_0^w (p - p_a)dy$ is the mechanical load from the squeezed air, $u(x,t)$ is the height of the beam above the substrate, and $p(x,y,t)$ is the pressure distribution under the beam. Other parameters include beam length $l = 610um$, width $w = 40um$, thickness $t = 2.2um$, undeflected gap $g_0 = 2.3um$, material Young's modulus $E = 149GPa$, density $[\rho/(hw)] = 2330kg/m^3$, air viscosity $\mu = 1.82$ x $10^{-5}kg/(m \cdot s)$, moment of inertia $I = wh^3/12$, residual stress $[S/(hw)] = -3.7MPa$, Kundsen's number $K(x,t) = \lambda/u(x,t)$, and the ambient air pressure $p_a = 1.013$ x 10^5Pa. The mean-free path of air $\lambda = 0.064um$.

Equations (1)-(2) show that simulating the dynamical behavior of the device involves nonlinear squeeze-film damping and also mechanical, electrostatic, and fluid components. The system is nonlinear due to the nonlinear nature of the squeeze-film damping equation and the nonlinear electrostatic force. Direct simulation is very expensive. Instead lower-order models are desired.

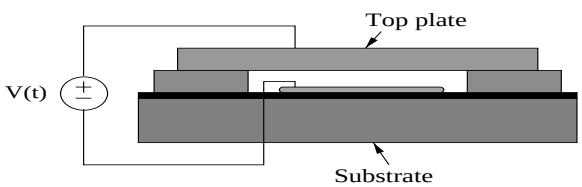

Figure 1: Schematic view of the fixed-fixed beam microstructure.

3 MODEL-ORDER REDUCTION

In this section, we will first discuss model-order reduction for weakly nonlinear MEMS devices by using Taylor series expansion and Arnoldi method. We then discuss model reduction for strongly nonlinear MEMS devices by using the Karhunen-Loeve decomposition and Galerkin's method.

3.1 Taylor Series Expansion with Arnoldi Process

In [6], model-order reduction of the fixed-fixed beam device has been achieved by first linearizing the nonlinear system (1)-(2) and then performing model reduction on the linearized system. It is shown in [6] that when the beam is operated with a very small input voltage, the reduced linear model can represent the original nonlinear beam model accurately. However, when the input voltage becomes larger the reduced linear model derivates from the original nonlinear model very significantly indicating that the nonlinear effects of the beam becomes important and can not be simply neglected.

In order to develop reduced nonlinear models, a new method by combining Taylor series expansion and Arnoldi method is proposed. Let us first discuss deriving reduced 2nd order nonlinear model for the beam device. Similar to [6], we start by performing Taylor series expansion of the device nonlinear equations (1)-(2) around the equilibrium state. Define $\hat{u}(x,t) = \frac{u(x,t)-u_0}{u_0}$ and $\hat{p}(x,y,t) = \frac{p(x,y,t)-p_0}{p_0}$ to be the normalized perturbations with $u_0 = g$ and $p_0 = p_a$. Plugging $\hat{u}(x,t) = \frac{u(x,t)-u_0}{u_0}$ and $\hat{p}(x,y,t) = \frac{p(x,y,t)-p_0}{p_0}$ into (1)-(2) and applying $\frac{1}{(1+\hat{u})^2} = 1 - 2\hat{u} + 3\hat{u}^2 + \ldots$ and $\frac{1}{(1+\hat{u})} = 1 - \hat{u} + \hat{u}^2 + \ldots$, we derive an approximated second-order partial differential equation model for the beam device as

$$EI\frac{\partial^4 u}{\partial^4 x} - S\frac{\partial^2 u}{\partial^2 x} = (2u-1)\frac{\varepsilon_0 w V^2}{2g^3} + \frac{p_a}{g}\int_0^w p dy - \frac{3\varepsilon_0 w V^2}{2g^3}u^2 - \rho\frac{\partial^2 u}{\partial^2 t} \quad (3)$$

$$\left[(1+\frac{6\lambda}{g}) + (1+\frac{6\lambda}{g})p + (2+\frac{6\lambda}{g})u\right]\frac{g^2 p_a}{12\mu}\nabla^2 p = \frac{\partial p}{\partial t} + (1+p-u)\frac{\partial u}{\partial t} \quad (4)$$

where in (3)-(4) we keep all the linear and second order terms of $\hat{u}$ and $\hat{p}$ and neglect the third and higher order terms. Also the $\hat{}$ is dropped to simplify the notation. Equations (3)-(4) are further discretized in space with an $(N+1)$ x $(M+1)$ mesh. We then define a state-vector X of size Q as $X = [u_1 \ldots u_N \ \frac{\partial u_1}{\partial t} \ldots \frac{\partial u_N}{\partial t} \ p_{11} \ldots p_{MN}]^T$ and project it onto the mesh points. By doing this, we convert equations (3)-(4) into a second-order state-space model of size Q as

$$\dot{X}(t) = AX(t) + X^T P X + Bv(t) \quad (5)$$

$$y(t) = C^T X(t) \quad (6)$$

where $A \in \Re^{QxQ}$, B and $C \in \Re^Q$ and are determined by the input and output. Here we choose the output to be the beam center point position and input to be the applied voltage. P can be considered as a Q array of Q by Q matrix and contains all the second-order nonlinear behavior of the original beam model (1)-(2). The main idea of reducing (5)-(6) is to apply a state-vector projection operation to transform the linear and second-order nonlinear terms in (5)-(6) into low dimensional forms (say q with $q \ll Q$). The state vector transformation matrix we used is the column orthogonal matrix V_q generated from the Arnoldi process when it is applied to the linearized beam system. Readers are refer to [6] for a detail description of the Arnoldi process. It can be shown that this transformation not only reduce the linear components of the original model (1)-(2) by matching the dominant moments of its linearized system but also reduce the higher order nonlinear terms. And in the case when q=Q, this transformation ensures that the reduced system to be exactly reproduced. During model reduction, if we increase q, we can expect that the reduced system will get closer to the original system.

It can be easily extend this method to higher order approximated state-space models. In Fig. 2, we compare the original finite difference model, reduced 4th linear model, reduced 4th second-order nonlinear model and reduced 4th third-order nonlinear model simulation results for the fixed-fixed beam structure with a $7.4V$ step input. The beam center point position vs. time is plotted. As can be seen, with $7.4V$ input, the conventional reduced linearized model (also studied in [6]) deviates from the original nonlinear solution significantly. However, the reduced second-order and third-order nonlinear models can follow the original finite difference model with a much better accuracy. Especially the reduced 3rd-order nonlinear model follows the original nonlinear solution very faithfully. The speed up factor for the reduced 2nd model is 15.6 with an error tolerance of 2.26%

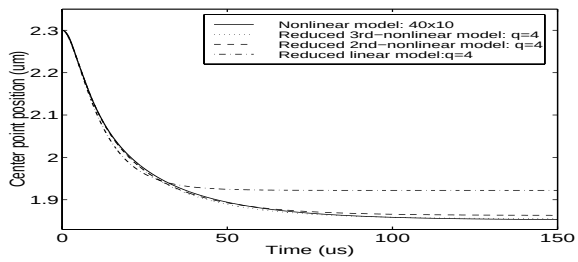

Figure 2: Comparison of original nonlinear model and the reduced models.

and 4.8 for the reduced 3rd model with an error tolerance of 0.49%.

Besides the Arnoldi method, any other linear model reduction techniques can also be used together with the Taylor series expansion to generate reduced nonlinear models, such as in [7], the balancing and the aggregation methods are used. However the Arnoldi method with Taylor series expansion will provide much better results.

3.2 Karhunen-Loeve Galerkin's Procedure

To develop reduced-order models for strongly nonlinear MEMS devices, the Karhunen-Loeve Galerkin procedure is proposed. The reduced-order models are extracted from fully-meshed finite element/finite difference model runs using the Karhunen-Loeve decomposition method. Eigenfunctions obtained from the Karhunen-Loeve decomposition method are then used as basis functions in spectral Galerkin expansions of the device governing partial differential equations to generate the reduced order models. In [8], the singular value decomposition (SVD) method is exploited to generate the global basis functions. Singular value decomposition is a deterministic approach which minimizes the total least-square error between the sampled data of the FEM/FD solution and the reduced model solution. In this research, we use a relatively complicated procedure, the Karhunen-Loeve decomposition method, to generate the global basis functions. Generating the global basis functions by using Karhunen-Loeve decomposition is optimal in the sense that the reduced system will on average contain the most energy of the original system (energy compaction property). The mean-square error between the unknown function and its truncated representation is minimized. Generating the global basis functions through the Karhunen-Loeve decomposition is also optimal in the sense that the number of basis functions in the truncated representation is minimized for a given error. It also permits advanced techniques such as phase-space analysis to be carried out.

In Fig. 3, we compare the simulation results of the finite difference model and the reduced models for the fixed-fixed beam device. Compared to section 3.1, the external applied voltage is larger enough to make the pull-in actual happens. For the finite difference solution, a 40x10 meshing is used, which results in 480 coupled nonlinear ODEs. From the results, we see that when 3 displacement and pressure basis functions are used (a total of 9 ODEs), the reduced model result is almost indistinguishable from the original finite difference solution. The speed up factor is about 25.5 and an error tolerance of less than 0.36%. This drastic reduction in computation time will facilitate the MEMS device models to be effectively used in system-level simulations.

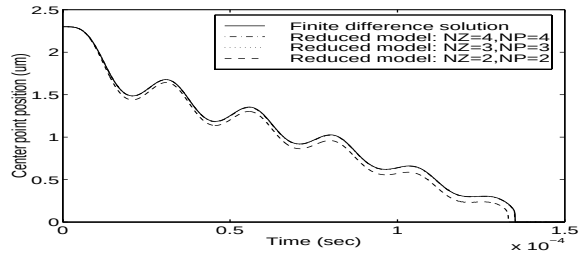

Figure 3: Comparison of the finite difference model and the reduced-order models for a $15V$, 20kHz sine input.

displacement mode	eigenvalue percentage	pressure mode	eigenvalue percentage
1	0.99980885	1	0.97604003
2	0.00018616	2	0.02309392
3	0.00000463	3	0.00053920

Table 1: Eigenvalue percentages of the first three eigenfunctions generated by Karhunen-Loeve decomposition.

Table. 1 shows the Eigenvalue percentages of the first three eigenfunctions generated by the Karhunen-Loeve decomposition method for the pull-in beam device. As can be seen, the first several eigenmodes are sufficient to capture up to 99.9% of the total energy of the system.

In order to capture burst-like behavior which occurs in a very short time interval (thus contain very little energy) but might be dynamically very important, phase-space Karhunen-Loeve analysis is also performed. Fig. 4 shows an example of the beam center point displacement plotted as a function of both time and arclength. As can be seen, when plotted against arclength s, the "bursts" are much broader and occupies a much wider fraction of the total space. This makes it unlikely to miss the "burst" components if one sample snapshots in equally spaced arclength rather then time. However the dynamic behavior of the beam system is not very rich, it is unlikely to generate extremely sharp bursts components. When phase-space Karhunen-Loeve technique is applied, the results are almost identical as that of the time-domain decomposition. But one can imagine situations where the phase space K-L modes will be different from the time domain K-L modes. And this can have an significant effect on deciding which structures are the dominant structures to be used in constructing the lower-order models.

4 COMPOSITE SIMULATION

With the aid of effective MEMS device model reduction techniques, it is possible to perform fast and efficient system-level composite circuit and micromechanical simulations. Fig. 5 illustrates our iSIMS simulator architecture. iSIMS provides a general event-processing and scheduling framework that ties together various simulation algorithms needed for simulation of various classes of devices and systems. In Fig. 5, the iterative timing analysis (ITA) is a relaxation-based algorithm for elec-

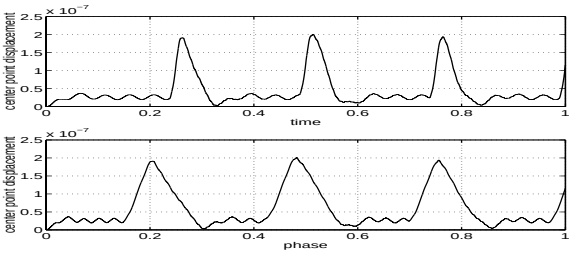

Figure 4: Beam center point position plotted as a function of time and arclength.

trical and functional level simulation. The models associated with it include transistors, capacitors, inductors, resistors and controlled sources. The Logic algorithm processes behavioral descriptions of logic gates. Currently, the simulator includes over 40 models for logic gates. The Analog Behavioral algorithm is developed to handle user defined high-level behavioral blocks. The reduced MEMS device models represented by a small set of nonlinear ODEs can be effectively embedded in the simulator as analog behavioral models. Event-driven, selective trace techniques are used for the simulation. Single timestep is not required.

To demonstrate multi-level and mixed-technology simulations, the beam device has been embedded in a circuit as shown in Fig. 6. A voltage controlled voltage source is applied to the beam device, the output of the beam device is its center point position. The beam center point position is then compared with a reference position, whenever the beam center point position is less than the reference position, a logic LOW signal is generated conditioned that the "Enable" signal is HIGH. The reason we choose this system is that it includes various modeling levels. The modeling levels in this system include MEMS beam behavioral-level model, controlled voltage source function-level model, analog behavioral comparator model, digital gate model, and electrical models of the transistors and resistors.

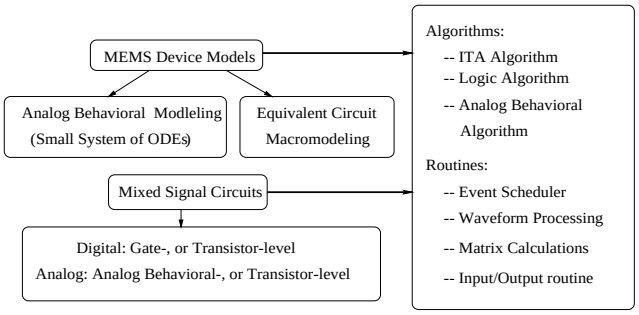

Figure 5: iSIMS simulator architecture.

Digital	Analog	MEMS
Gate	Behavioral	Behavioral
Switch	Functional	Functional
	(ideal/nonideal)	(lumped circuit
Electrical	Electrical	equivalent model)

Table 2: Digital-Analog-MEMS simulation levels in the iSIMS simulator.

Fig. 7 shows the simulation result. The reference position is set to be $1.2um$. The top two plots show the input, a square-wave with varying duty cycles, and the beam center point position response with this input. The bottom plot shows the logic output. As can be seen, whenever the beam center point position is less than $1.2um$, the logic signal will be LOW.

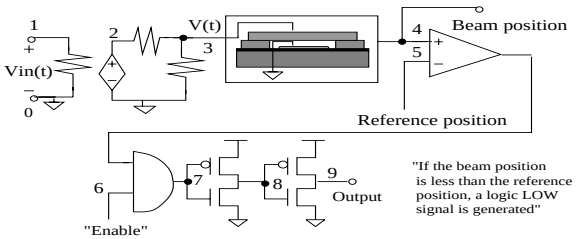

Figure 6: MEMS beam device embedded in a circuit. Various modeling levels are used to model this system.

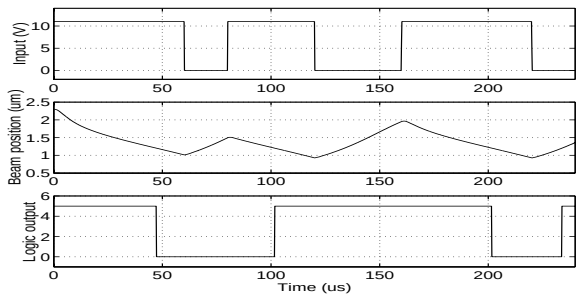

Figure 7: Simulation result of the beam-circuit system.

5 CONCLUSION

We have presented systematic methods for developing low-order dynamical models for coupled domain nonlinear microelectromechanical devices. We also presented iSIMS, a simulation framework that allows composite circuit and micromechanical simulations. The model reduction algorithms and the reduced fixed-fixed beam models have been integrated in the iSIMS simulator. Important issues for developing multi-level and mixed-technology simulation programs are studied.

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