Modeling of Spatial Correlations in Process, Device, and Circuit Variations

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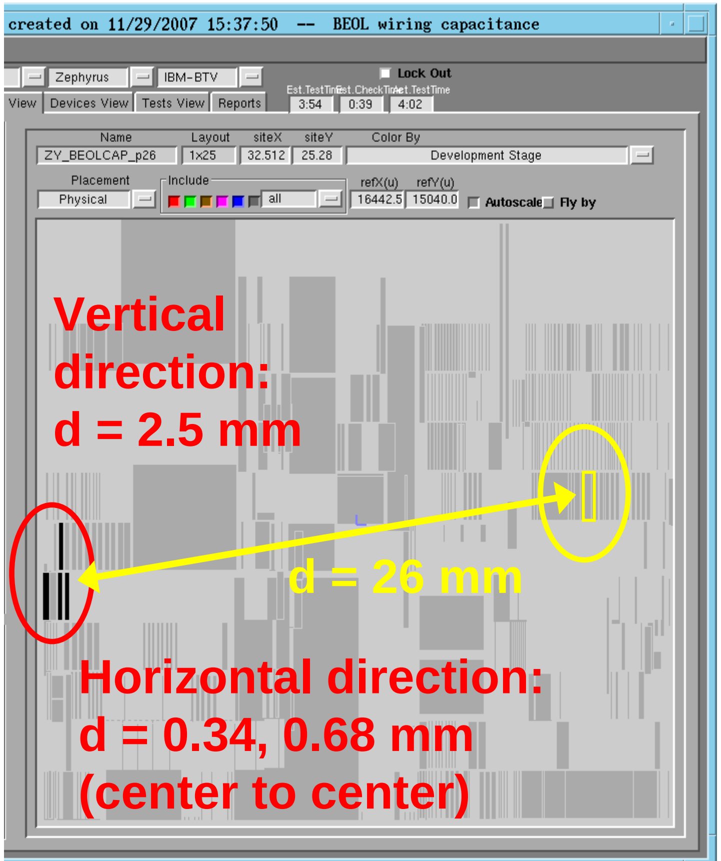
Outline

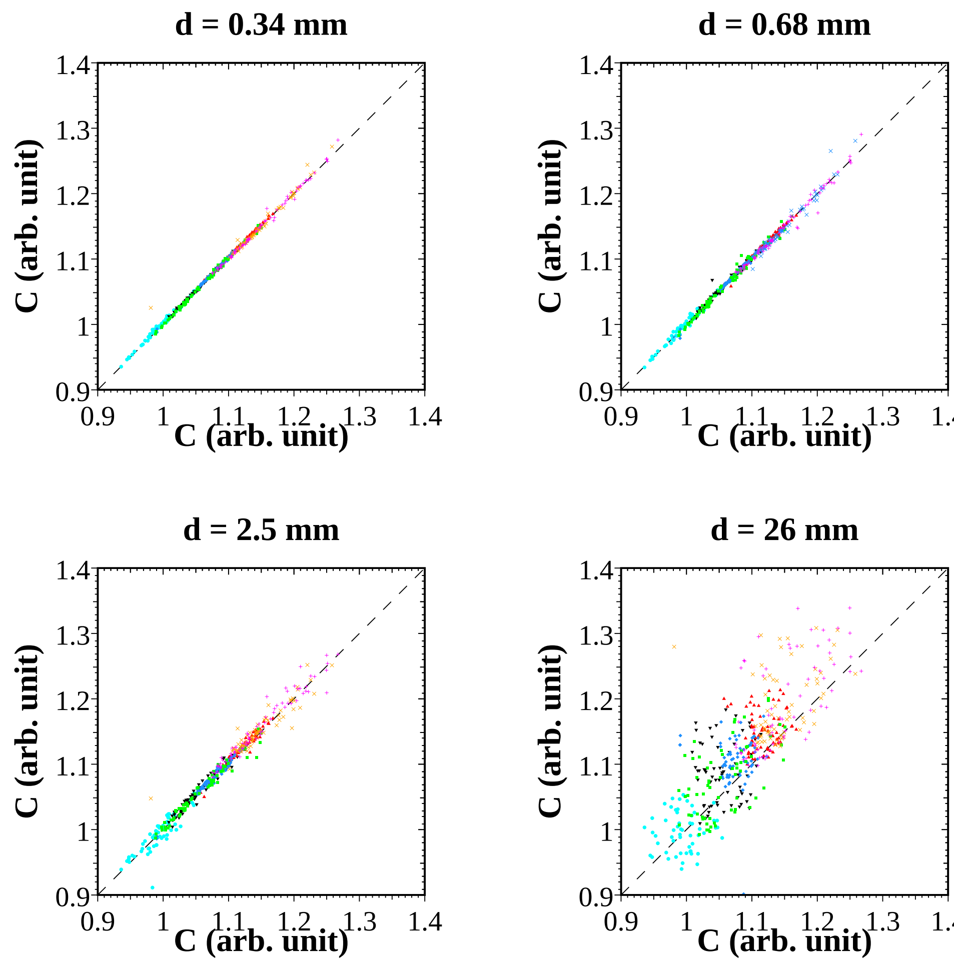
- Spatial Correlations in Measured Data
- Modeling of 1D Spatial Correlations
- Modeling of 2D Spatial Correlations
- Summary

Interconnect Capacitance Data



45 nm technology, 7 wafers from 5 lots

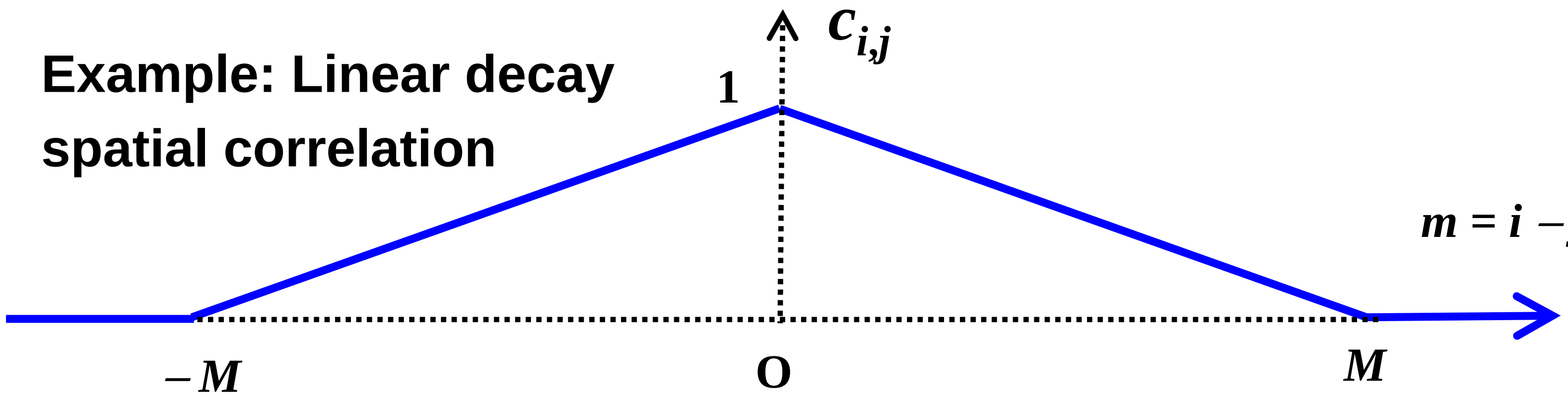
M2 over M1 under M3, min. pitch



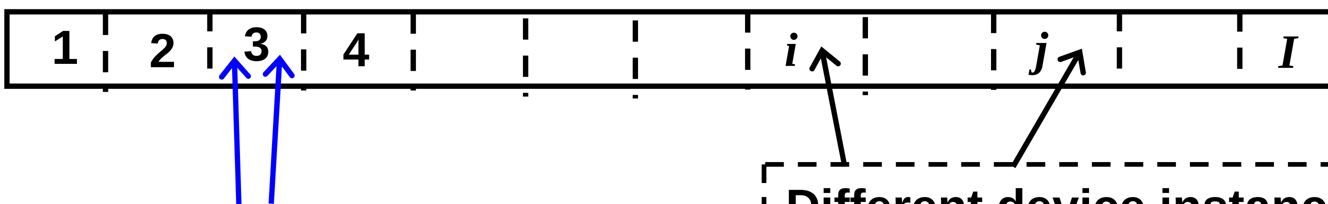
Correlation degree decreases with increasing *d*.

Modeling of 1D Spatial Correlations

Example: Linear decay spatial correlation



Divide a 1D region into I sub-regions (I grid points)



Different device instances within the same sub-region are treated as perfectly correlated.

Different device instances from different sub-regions are treated as either partially correlated or have no correlation.

All instances have the same mean x_0 and same standard dev. σ.

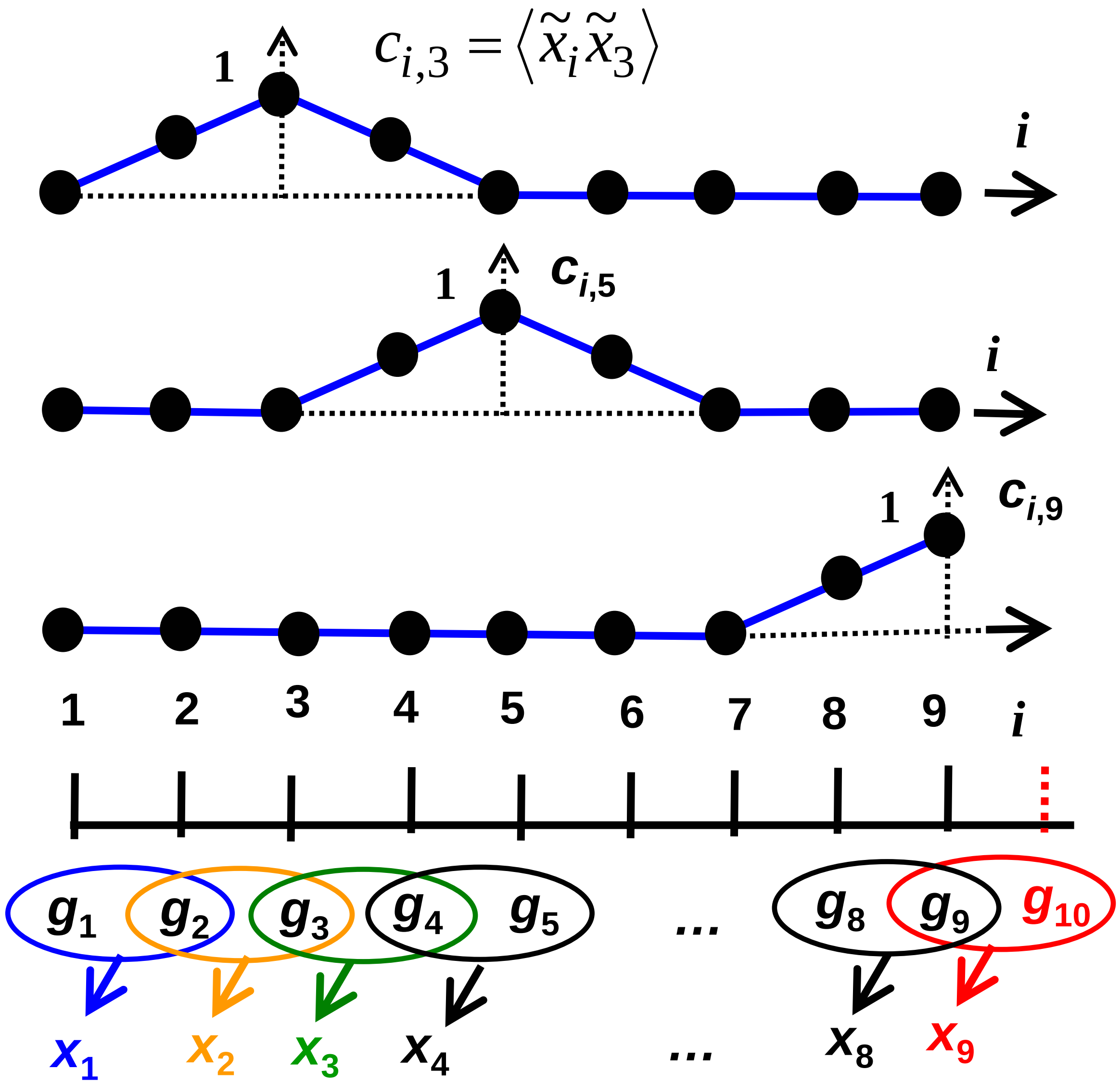
Traditional PCA Approach

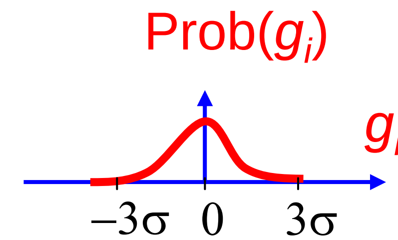
Sub-region	1	2	3	4	5	6	7
1	**1**	.8	.6	.4	.2	0	0
2	.8	**1**	.8	.6	.4	.2	0
3	.6	.8	**1**	.8	.6	.4	.2
4	.4	.6	.8	**1**	.8	.6	.4
5	.2	.4	.6	.8	**1**	.8	.6
6	0	.2	.4	.6	.8	**1**	.8
7	0	0	.2	.4	.6	.8	**1**

- ***I*** **sub-regions (grid points)**
  - ***I*** **correlated instances**
  - **Size of correlation matrix:** $I \times I$
  - **Number of eigenvalues and eigenvectors:** ***I***
  - **Number of independent stochastic variables:** ***I***
  - **Each instance uses all** ***I*** **stochastic variables**
  - **In 2D case, matrix size is very large**

1D Solution Formalism (1)

Nearest neighbor case:





$$\langle g_i \rangle = 0$$

$$\langle g_i^2 \rangle = 1$$

$$\langle g_i g_j \rangle = 0\,, \quad i \neq j$$

Each instance uses only 2 stochastic variables.

1D Spatial Correlation Solution (1)

Nearest neighbor case: 1) Fixed correlation degree

$$c_{ii} = 1, \quad i = 1, 2, \cdots, I \qquad c_{i,i\pm1} = \pm\frac{1}{2}$$

$$c_{ij} = 0, \quad i, j = 1, 2, \cdots, I, \quad |i - j| \geq 2$$

Use (I + 1) independent stochastic variables

to represent I correlated instances.

Solution: $$x_i = x_0 + \frac{\sigma}{\sqrt{2}}(g_i \pm g_{i+1}), \quad i = 1, 2, \cdots, I$$

2) Arbitrary correlation degree: $c_{i,i\pm1} = r, \quad |r| \leq \frac{1}{2}$

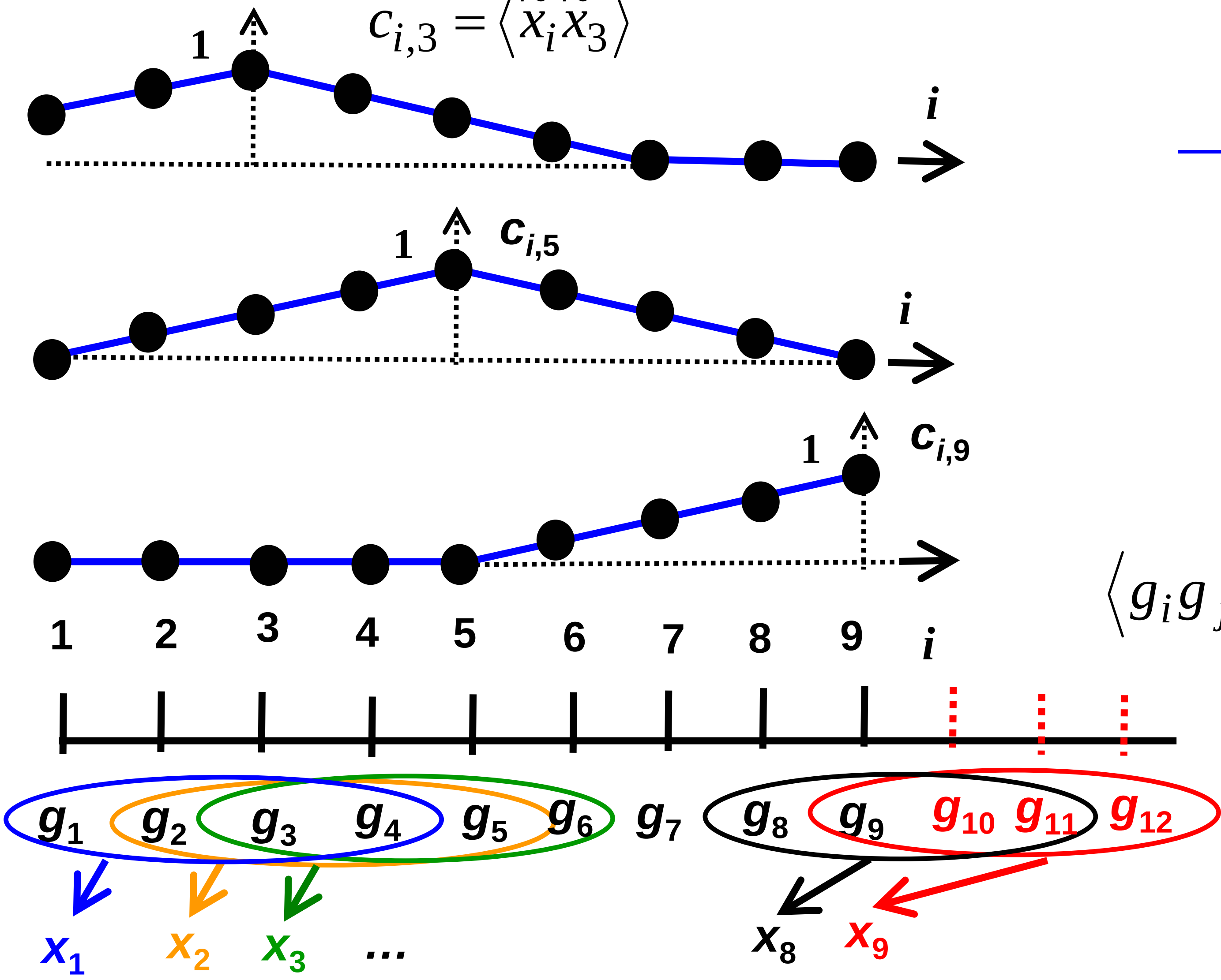
Solution: $$x_i = x_0 + \sigma(a_1 g_i + a_2 g_{i+1}), \quad i = 1, 2, \cdots, I$$

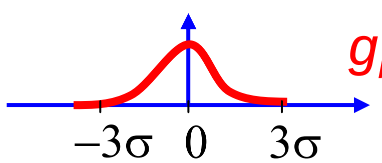
$$a_2 = \mathrm{sgn}(r)\sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - r^2}} \qquad a_1 = \sqrt{1 - a_2^2}$$

1D Solution Formalism (2)

Long distance correlation:

$$c_{i,3} = \langle \tilde{x}_i \tilde{x}_3 \rangle$$





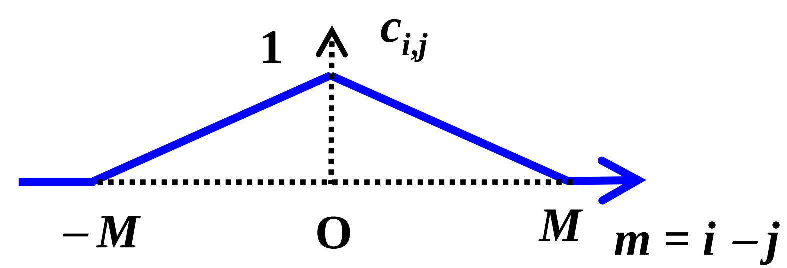
$$\langle g_i \rangle = 0$$

$$\langle g_i^2 \rangle = 1$$

$$\langle g_i g_j \rangle = 0 , \quad i \neq j$$

1D Spatial Correlation Solution (2)

- **Long distance correlation**
- **Linear decay**



$$c_{i,i\pm m} = 1 - \frac{m}{M}, \quad m = 0, 1, \cdots, M$$

$$c_{ij} = 0, \quad |\, i - j \,| \geq M$$

Use (I + M – 1) stochastic variables

to represent I correlated instances.

$$x_i = x_0 + \frac{\sigma}{\sqrt{M}} \sum_{k=1}^{M} g_{i+k-1}, \quad i = 1, 2, \cdots, I$$

Each instance uses *M* stochastic variables.

1D Solution Formalism (3)

- **Long distance correlation**
- **Arbitrary (almost) correlation degree**

$$c_{i,i\pm m} = f(M,m)\,, \quad m = 1, 2, \cdots, M \qquad f(M,0) = 1\,, \quad f(M,M) = 0$$

$$c_{ij} = 0\,, \quad |\,i - j\,| \geq M$$

Solution structure: $$x_i = x_0 + \sigma \sum_{k=1}^{M} a_k g_{i+k-1}\,, \quad i = 1, 2, \cdots, I$$

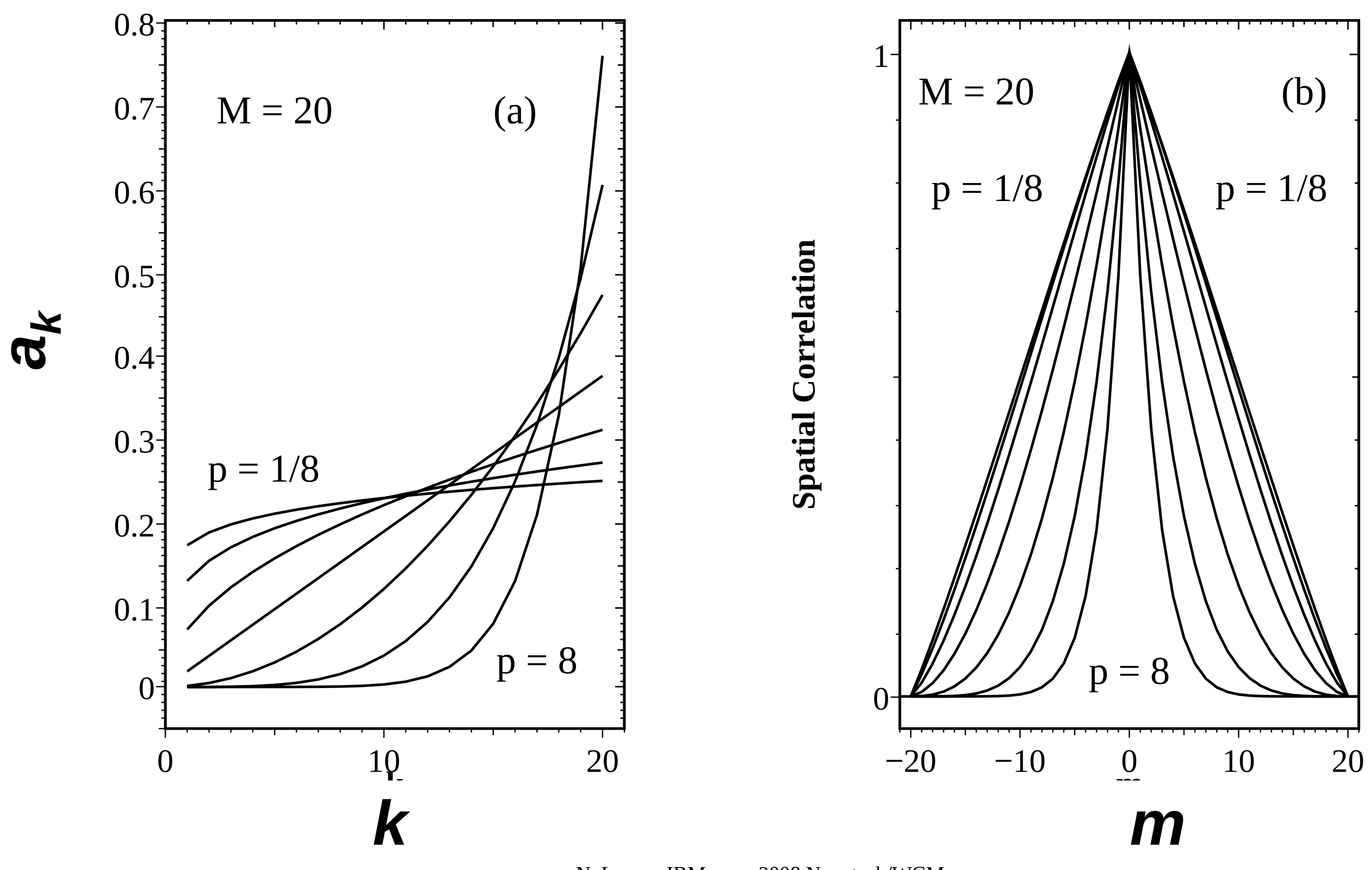
Given *f*(*M*, *m*), need to find a_k's: $$f(M,m) = \sum_{k=1}^{M-m} a_k a_{k+m}\,, \quad m = 0, 1, \cdots, M-1$$

***m* = 0 → Normalization:** $$\sum_{k=1}^{M} a_k^2 = 1$$

1D Spatial Correlation Solution (3)

- **Try a family of a_k curves**
- **Calculate a corresponding family of correlations $f(M, m)$**
- **Pick one of $f(M, m)$ which is closest to the desired correlation function**

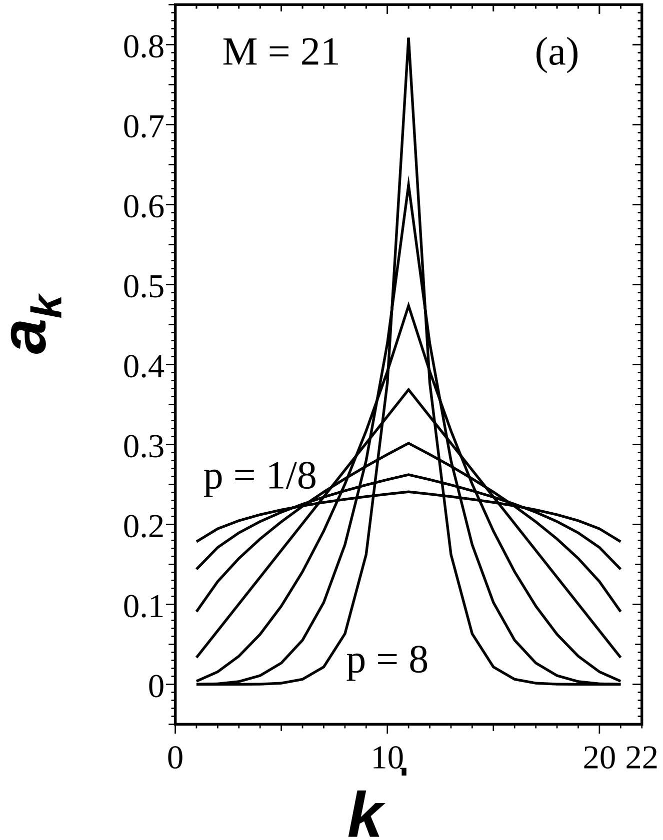
$$a_k \propto k^p, \quad k = 1, 2, \cdots, M$$

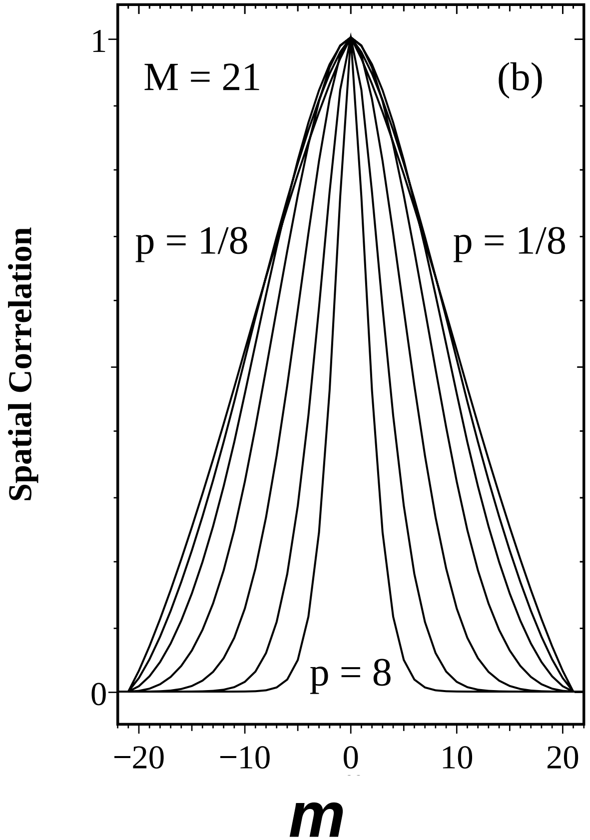


1D Spatial Correlation Solution (4)

- **Another family of a_k curves:**
  - **Each a_k curve is symmetric about its center k_0**
  - **Resulting a family of smoother correlation functions at the center $m = 0$**

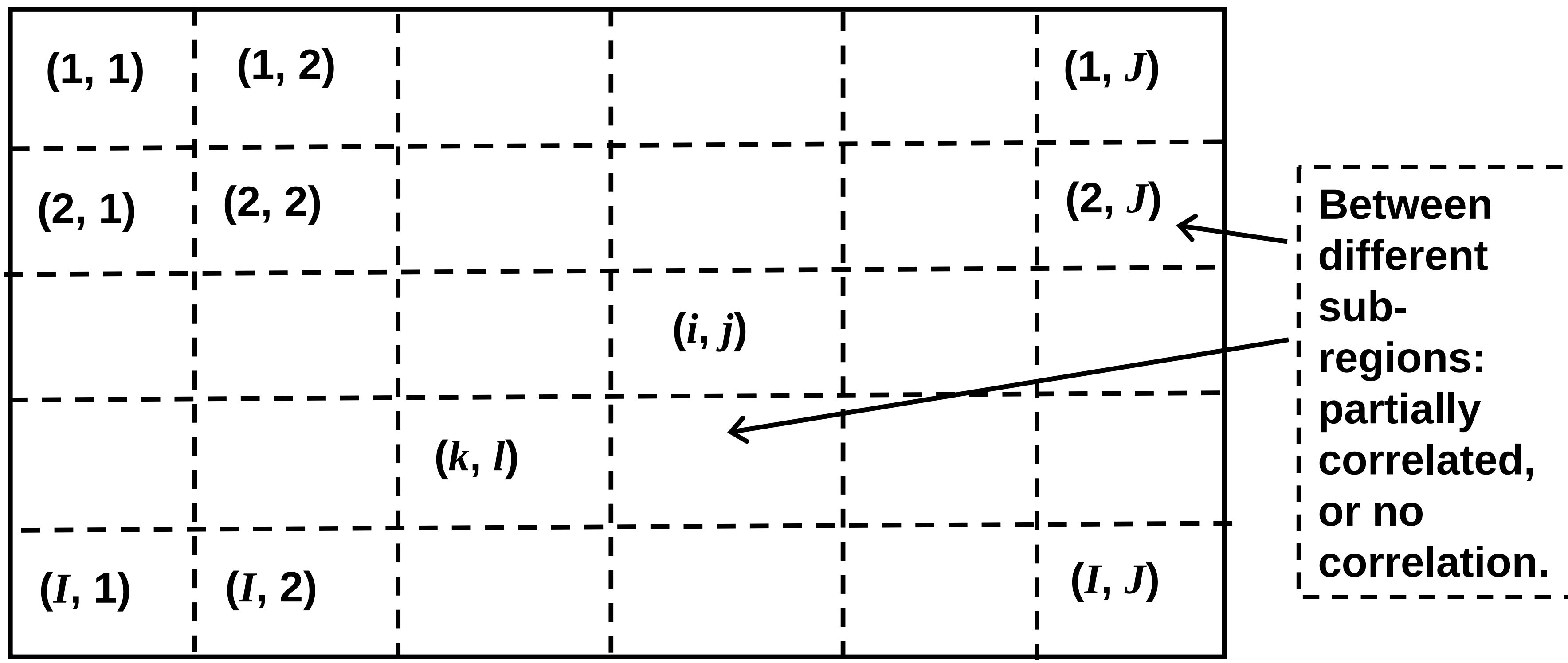
$$a_k \propto (k_0 - |k - k_0|)^p , \quad k_0 = \tfrac{1}{2}(M+1)$$



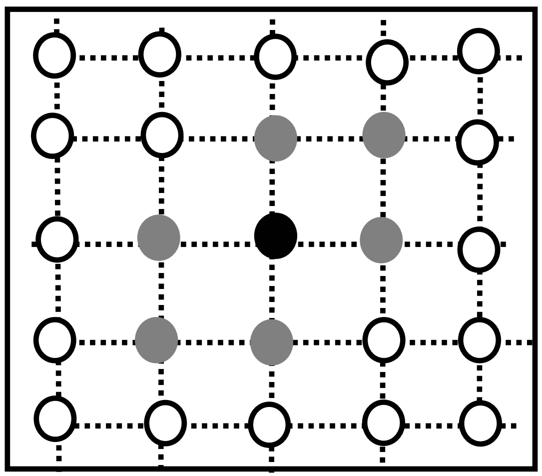


Modeling of 2D Spatial Correlations

A 2D chip region is divided into (I x J) sub-regions, or a (I x J) 2D grid



2D Solution Formalism (1)

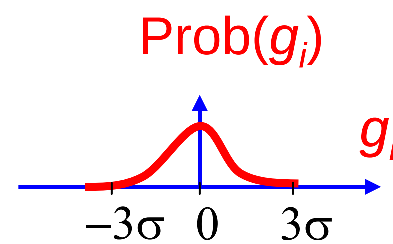


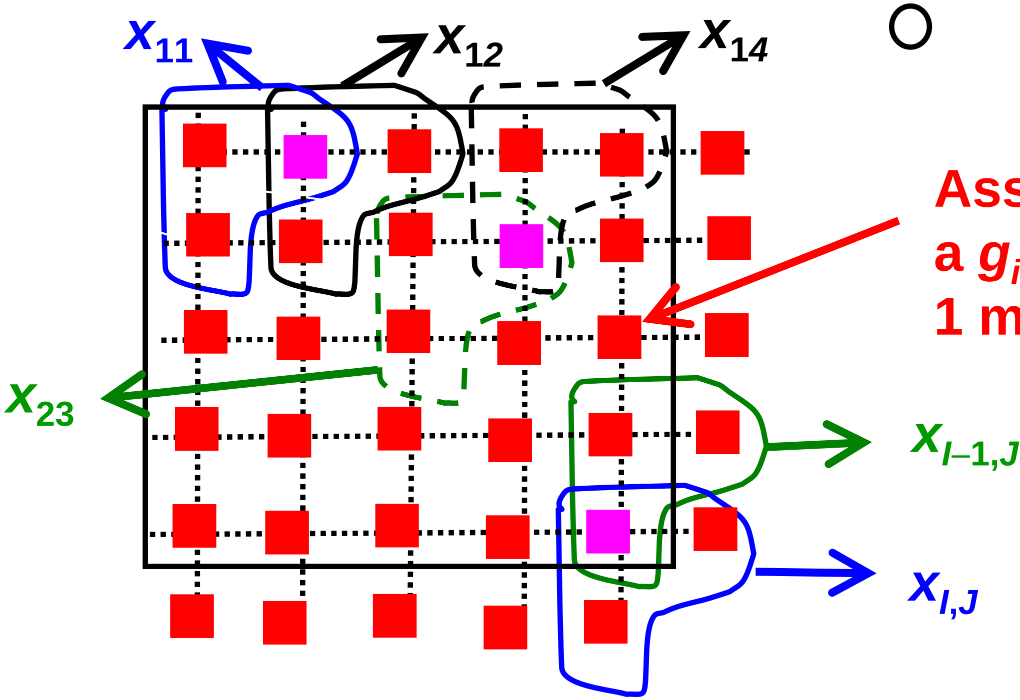
A 2D nearest neighbor case:

X,Y non-separable

2D correlation

- ● **C = 1**
- ● **0 < C < 1**
- ○ **C = 0**





Associate each grid point with a g_i, plus 1 more row of g_i and 1 more column of g_i

Each instance uses only 3 stochastic variables.

2D Spatial Correlation Solution (1)

Example: When a 2D correlation can not be separated as the product of two 1D correlations

$$C(i,\ j;i+1,\ j)=C(i,\ j;i,\ j+1)=C(i,\ j;i+1,\ j-1)=\tfrac{1}{3}$$

A nearest neighbor case:

$$C(i,\ j;i+1,\ j+1)=0$$

Use ($IJ + I + J$) independent stochastic variables

to represent IJ correlated instances

Solution:

$$x_{i,j}=x_0+\frac{\sigma}{\sqrt{3}}(g_{i,j}+g_{i+1,j}+g_{i,j+1})$$

2D Solution Formalism (2)

Correlation range: X: (M – 1) Y: (N – 1)

$$F(M,0;N,0)=1 \qquad F(M,M;N,n)=F(M,m;N,N)=0$$

Use (I + M – 1)(J + N – 1) stochastic variables

to represent IJ correlated instances

$$x_{ij}=x_0+\sigma\sum_{k=1}^{M}\sum_{l=1}^{N}A_{kl}\,g_{i+k-1,\,j+l-1}\;,\quad i=1,\ldots,I\;,\quad j=1,\ldots,J$$

Given $F(M, m; N, n)$, need to find A_{kl}'s:

$$F(M,m;N,n)=\sum_{k=1}^{M-m}\sum_{l=1}^{N-n}A_{k,l}A_{k+m,l+n}$$

$$m=0,1,\cdots,M-1,\quad n=0,1,\cdots,N-1$$

2D Spatial Correlation Solution (2)

When a 2D correlation can be separated as the product of two 1D correlations,

$$F(M, m; N, n) = f_1(M, m)\, f_2(N, n)$$

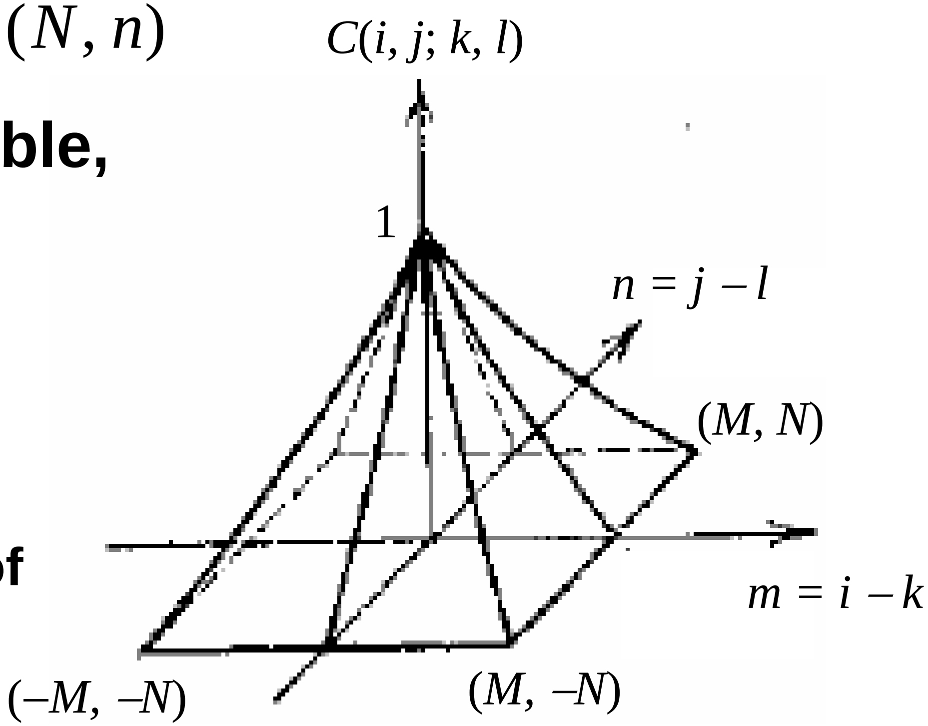
2D solution is also separable,

$$A_{kl} = a_k a_l$$

a_k and $f_1(M, m)$ form a pair of 1D solution and correlation.

a_l and $f_2(N, n)$ form another pair of 1D solution and correlation.

Then, make use of previous found 1D solutions.



2D bilinear decay
←→ (1D linear decay)²

Summary

- Presented a method of obtaining a compact solution to the problem of modeling spatial correlations in process, device, and circuit variations
- Presented some specific compact solutions
- Our solutions are in a higher dimensional space
  - Much simpler than eigen solutions in PCA
- Our solutions are simple and suitable for use in compact models & in SPICE simulations
  - Also suitable for EDA tools, for SSTA