Compact Models for Double Gate (DG) MOSFET with Quantum Mechanical Effects Using Lambert Function

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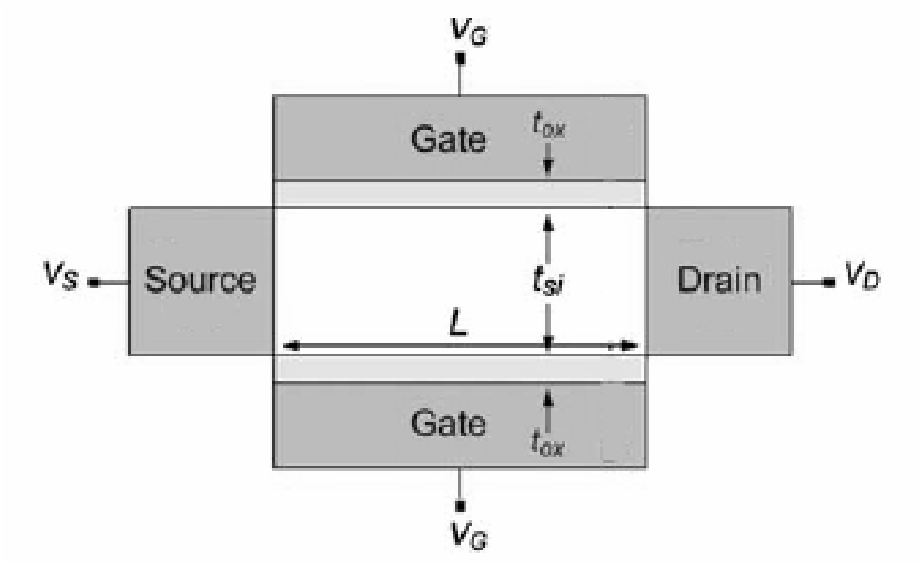
Outline

- DG MOSFET.
- Model equations.

} Review

- Iteration method using Lambert function.
- Compact quantum model for DG.
- Self-Consistent Schrödinger-Poisson (SP) numerical approach.
- Results and comparison.

DG MOSFET



- The above double gate device structure is considered.

DG MOSFET (continues)

- Undoped and symmetric.

Relatively small silicon thickness t_{si}=5nm and t_{si}=20nm.

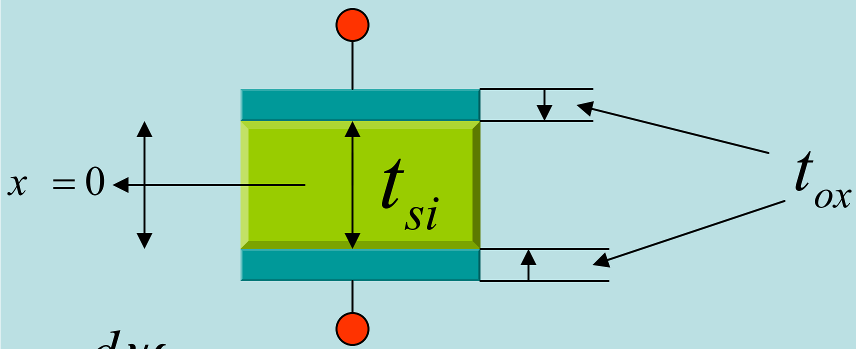
The two gate voltages are considered to be the same, V_g.

- Thin gate oxide t_{ox}=1.5nm

Quantum confinement effect is significant.

Model equations

- Boundary conditions



$$1.\ \frac{d\psi}{dx}\Big|_{x=0} = 0$$

$$2.\ \psi(x=0) = \psi_0$$

$$3.\ C_{ox}(V_g - \Delta\phi - \psi(\pm t_{si}/2)) = \pm\varepsilon_{si}\frac{d\psi}{dx}\Big|_{x=\pm t_{si}/2}$$

$$\text{where } C_{ox} = \varepsilon_{ox}/t_{ox}$$

- Poisson equation

$$\frac{d^2\psi}{dx^2} = \frac{q}{\varepsilon_{si}} n_i e^{q(\psi - V)/kT}$$

Model equations (continues)

- Exact solution using the first two boundary conditions:

$$\psi(x) = V + \psi_0 - \frac{2kT}{q}\ln[\cos(\sqrt{\frac{q^2 n_i}{2\varepsilon_{si}kT}e^{q\psi_0/2kT}}x)]$$

- The surface potential:

$$\psi_s = \psi(t_{si}/2)$$

Model equations (continues)

- The interface boundary condition ⟶ equation for β:

$$\ln \beta - \ln(\cos \beta) + 2r\beta \tan \beta = v$$

$$v = \frac{q(V_g - \Delta\phi - V)}{2kT} - \ln\left(\frac{2}{t_{si}}\sqrt{\frac{2\varepsilon_{si}kT}{q^2 n_i}}\right)$$

$$\text{where } \beta = \frac{t_{si}}{2}\sqrt{\frac{q^2 n_i}{2kT\varepsilon_{si}}}e^{q\psi_0/2kT} \text{ and } r = \frac{\varepsilon_{si}t_{ox}}{\varepsilon_{ox}t_{si}}$$

Model equations (continues)

- The total mobile charge per unit gate area:

$$Q = 2\varepsilon_{si}(d\psi/dx)_{x=t_{si}/2} = 2\varepsilon_{si}(2kT/q)(2\beta/t_{si})\tan\beta$$

- The channel current:

$$I_{ds} = I_{ds0}[\beta\tan\beta - \frac{1}{2}\beta^2 + r\beta^2\tan^2\beta]_{\beta_D}^{\beta_S}$$

where $I_{ds0} = \mu\frac{W}{L}\frac{4\varepsilon_{si}}{t_{si}}(\frac{2kT}{q})^2$ and $0 < \beta < \pi/2$

Iteration method using Lambert function

- Recast $$\ln(\beta \sec \beta) + 2r\beta \tan \beta = v$$

as $$(2rz)e^{2rz} = x$$ where $z = \beta \tan \beta$ and $x = (2r \sin \beta)e^{v}$

to give $$\beta = \Phi[\frac{1}{2r} LambertW(x)]$$ where $z = \Phi \tan \Phi$

LambertW(x) function is the solution of $x = We^{W}$

Iteration method (continues)

$$\Phi(z) = \Phi_0 - \frac{(\Phi_0 \tan\Phi_0 - z)}{\tan\Phi_0 + \Phi_0 \sec^2\Phi_0}$$

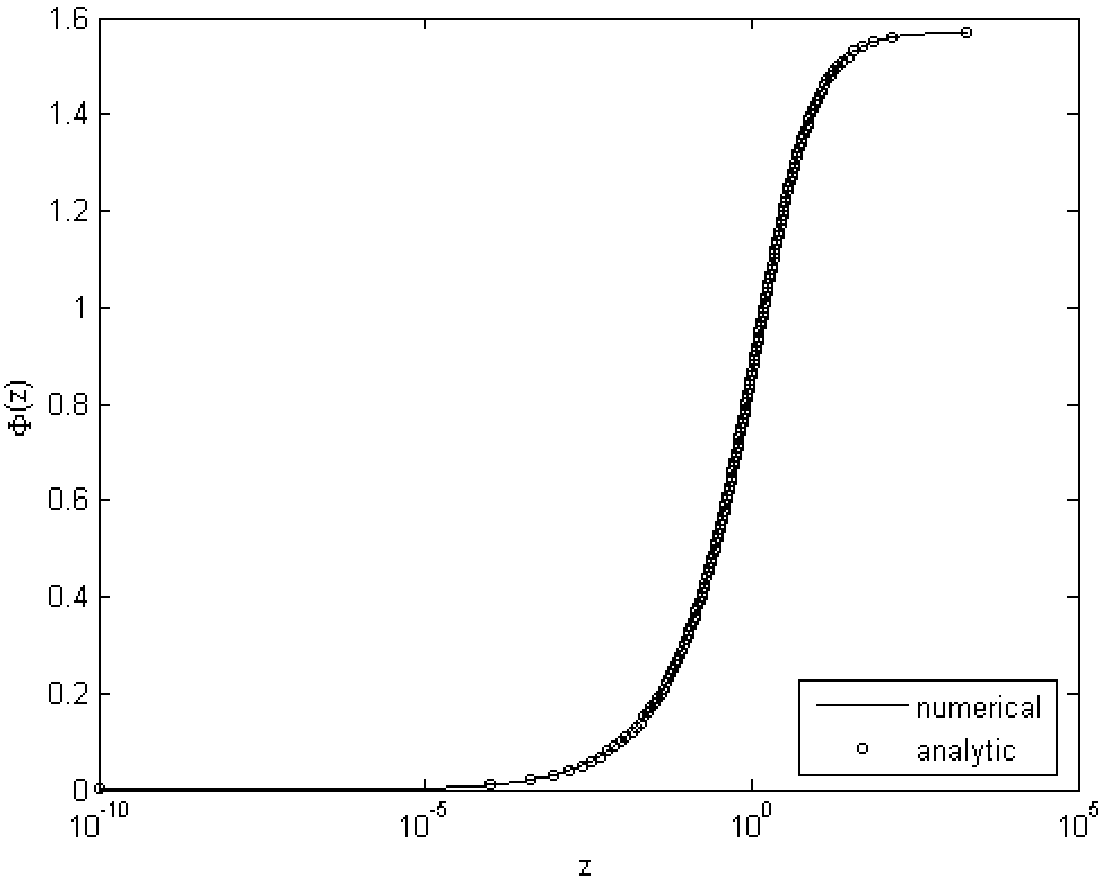
- where

$$\Phi_0 = \sqrt{\frac{(1+z\gamma) - \sqrt{(z\gamma-1)^2 - 4z(z\delta - 1/3)}}{2(z\delta + \gamma - 1/3)}},$$

$$\gamma = \frac{40}{9\pi^2} \text{ and } \delta = \frac{16}{9\pi^4}$$

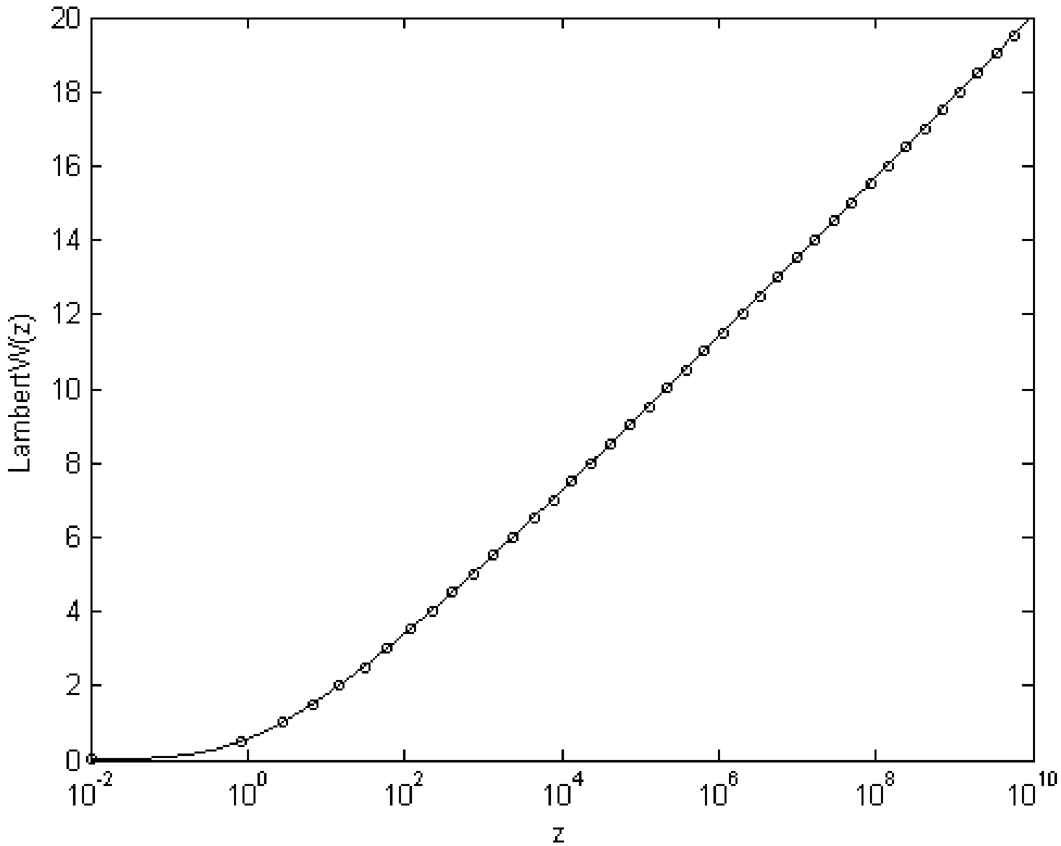
Iteration method (continues)

A comparison of the numerical and analytic solutions for Φ(z)



Iteration method (continues)

A comparison of the numerical and analytic solutions for *LambertW(z)*



Iteration method (continues)

$$\beta_{(n+1)} = \Phi[\frac{1}{2r} LambertW(2r \sin \beta_n e^{\nu})]$$

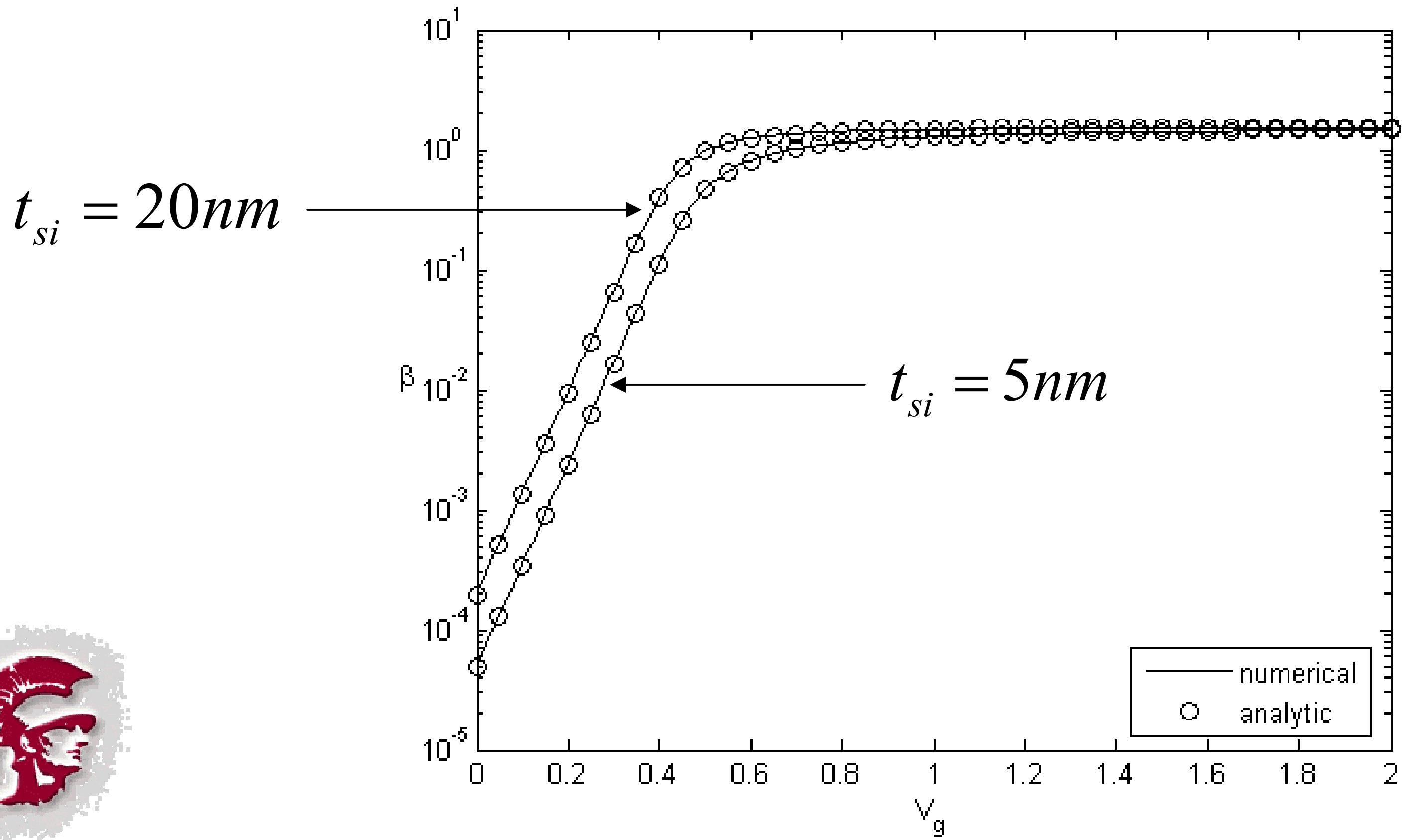
- where *n=0,1,2……* with initial estimate

$$\beta_0 = \frac{e^{-\nu}}{(4r+1)}(-1+\sqrt{1+2(4r+1)e^{2\nu}})$$

- four iterations ⟶ excellent results compared to the numerical

Iteration method (continues)

Comparison of the numerical and iterative solutions for β (four iterations)



Quantum model for DG

- Triangular potential well approximation.
- Considering the Schrödinger equation only along the longitudinal direction.
- Quantum effect on the semiconductor energy band gap and intrinsic density.

Quantum model for DG (continues)

- 1-D Schrödinger equation

$$\frac{d^2\psi_j}{dx^2} + \frac{2m^*}{\hbar^2}[E_j - U]\psi_j = 0$$

- Where $m^*=0.916m_e$ is electron effective mass in the direction perpendicular to the transistor channel surface, $U=-qF_s$ and F_s the surface field.

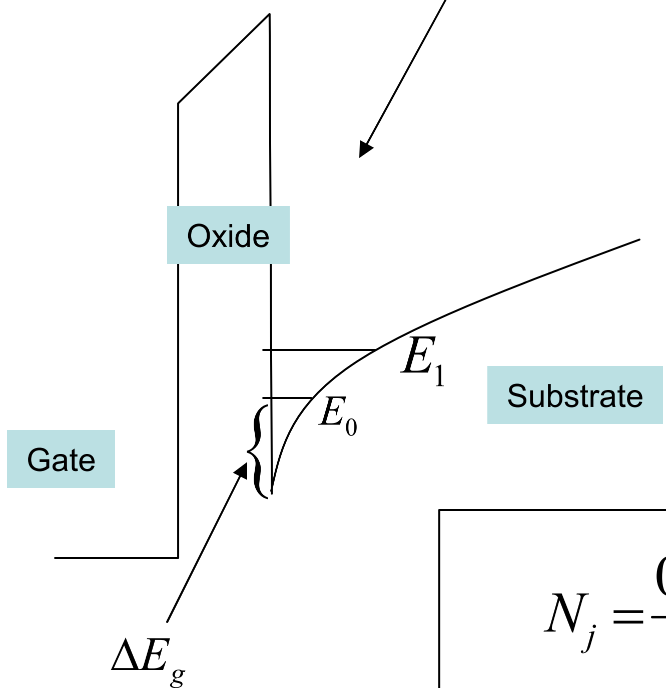
Quantum model for DG (continues)

- Solving the eigenvalue problem gives the ***Airy functions*** as solutions for the Schrödinger equation with energy eigenvalues

$$E_j = \left(\frac{\hbar^2}{2m^*} \right)^{1/3} \left((3/2)\pi q F_s (j + \frac{3}{4}) \right)^{2/3}$$

Quantum model for DG (continues)

•Band structure near the surface



•The electron concentration in j_{th} sub-band:

$$N_j = \frac{0.38 m_e}{\pi \hbar^2} \int_{E_j}^{\infty} \frac{dE}{1 + e^{(E - E_f)/kT}} = 0.38 m_e \left(\frac{kT}{\pi \hbar^2}\right) \ln\left(1 + e^{(E_f - E_j)/kT}\right)$$

Quantum model for DG (continues)

- Quantum effect on the intrinsic charge density

- Classical: →

$$n_i^c = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{\frac{3}{2}} (m_h m_e)^{\frac{3}{4}} e^{-E_g/2kT)}$$

- Quantum: →

$$n_i^q = e^{-\Delta E_g/2kT} n_i^c$$

Quantum model for DG (continues)

- Quantum correction for the energy band gap

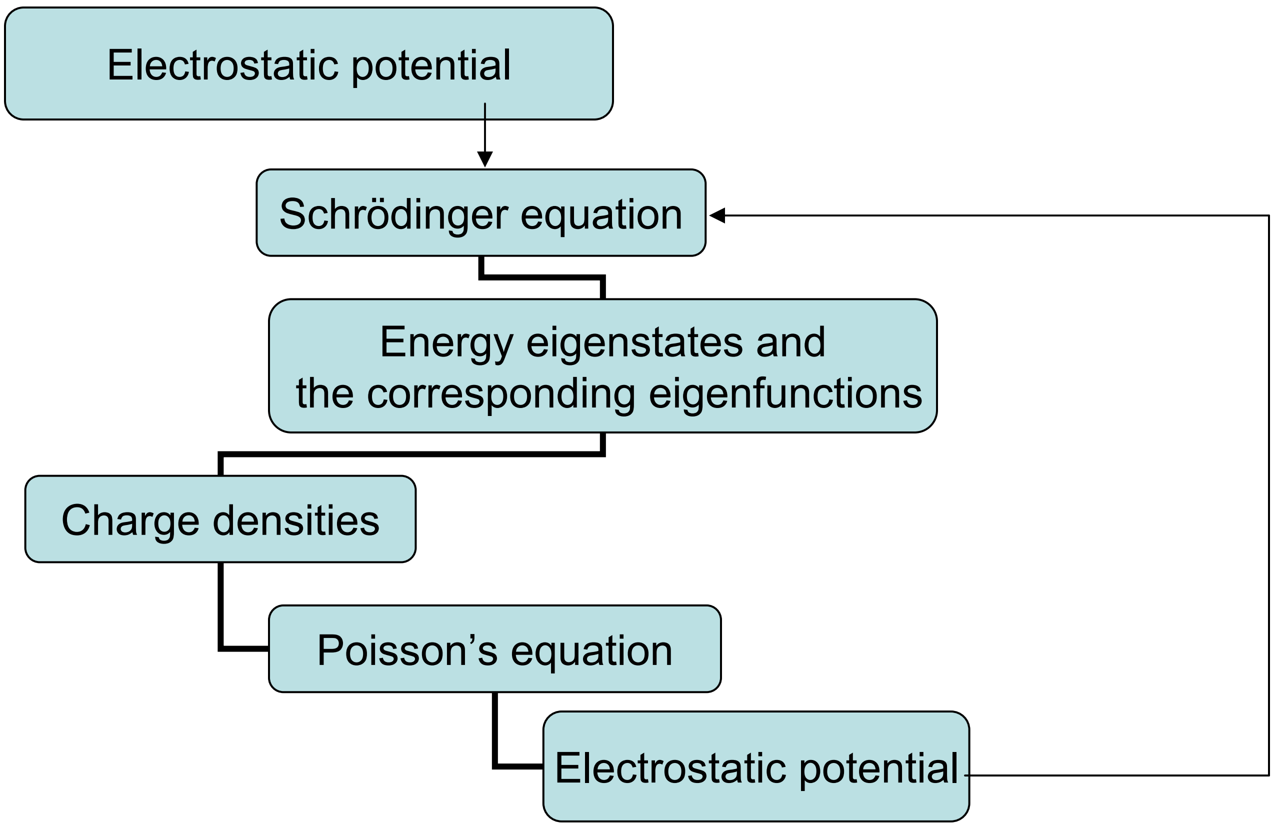
$$\Delta E_g = \frac{13}{9}\beta^*(\varepsilon_{si}/4kT)^{1/3}(F_{seff})^{2/3}$$

where $\beta^* = 6.6X10^{-29}\,J\cdot m, F_{seff} = \eta(V_g + V_t)/t_{ox},$

$$V_t = V_{t0} + 2V_{th}\ln[q(V_g - V_{t0})/4rkT],$$

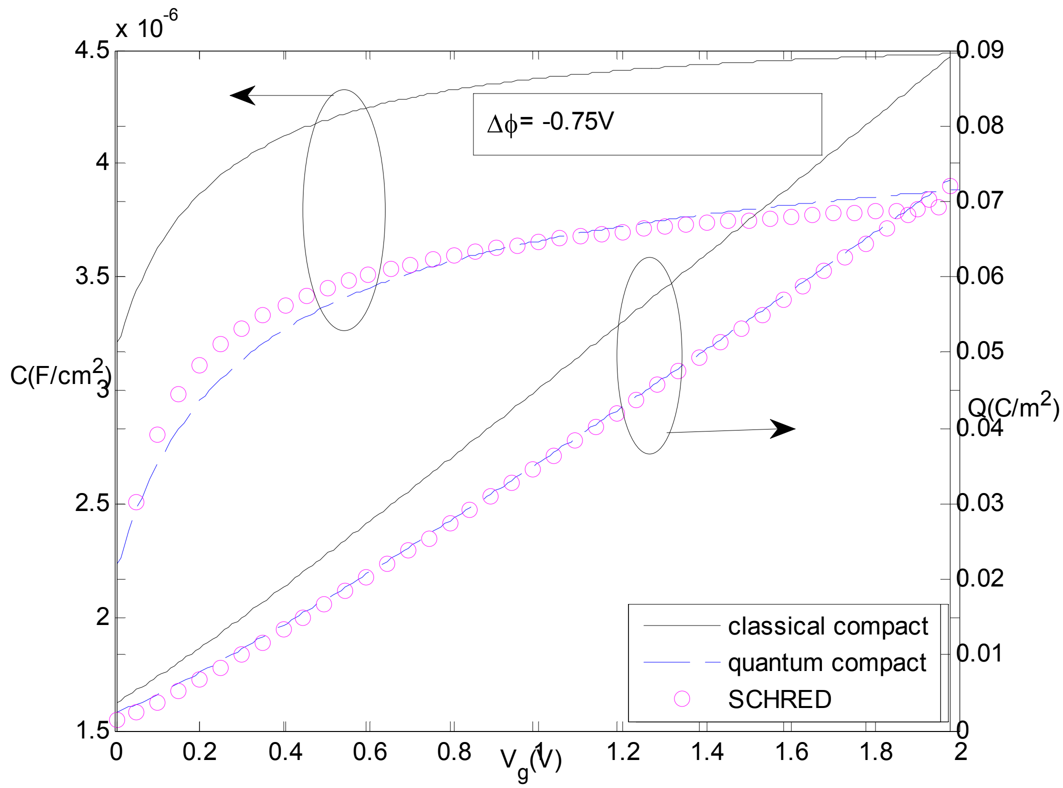
$$V_{t0} = \Delta\phi + 2V_{th}\ln[\frac{2}{t_{si}}\sqrt{2\varepsilon_{si}kT/q^2 n_i^c}\,]\text{ and }\eta = 0.5$$

Self-Consistent Schrödinger-Poisson (SP) numerical approach



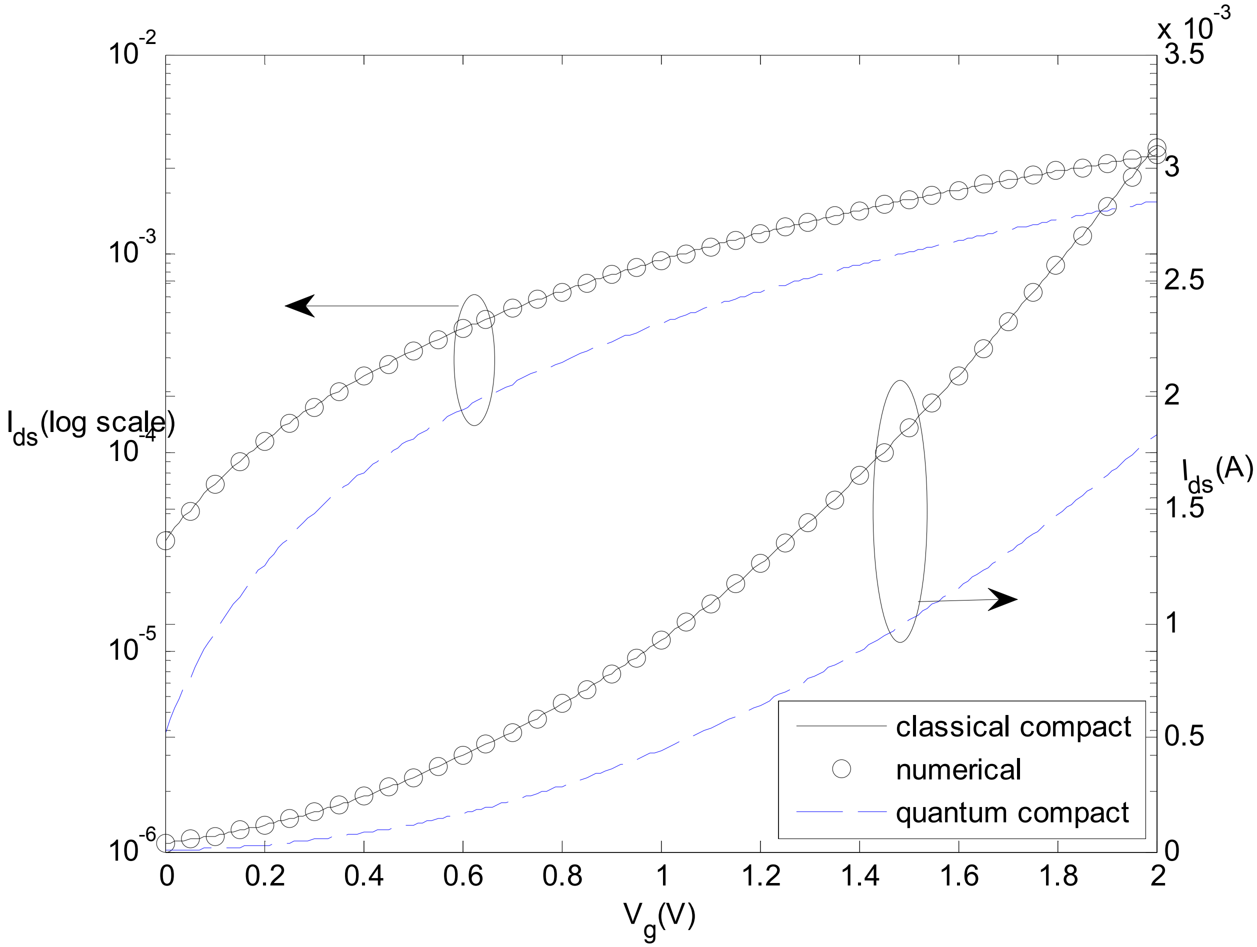
Results and comparison

Total mobile charge and gate capacitance versus gate voltage plots with 5nm silicon, 1.5nm oxide thicknesses.



Results and comparison

- Channel current versus gate voltage with V_{ds}=2V, 5nm silicon,1.5nm oxide thicknesses and constant carrier mobility $\mu = 300\ \mathrm{cm}^2 / V \cdot s$



Results and comparison

Channel current versus source-drain voltage with 5nm silicon and 1.5nm oxide thicknesses