Optimal Skew Corners for Compact Models

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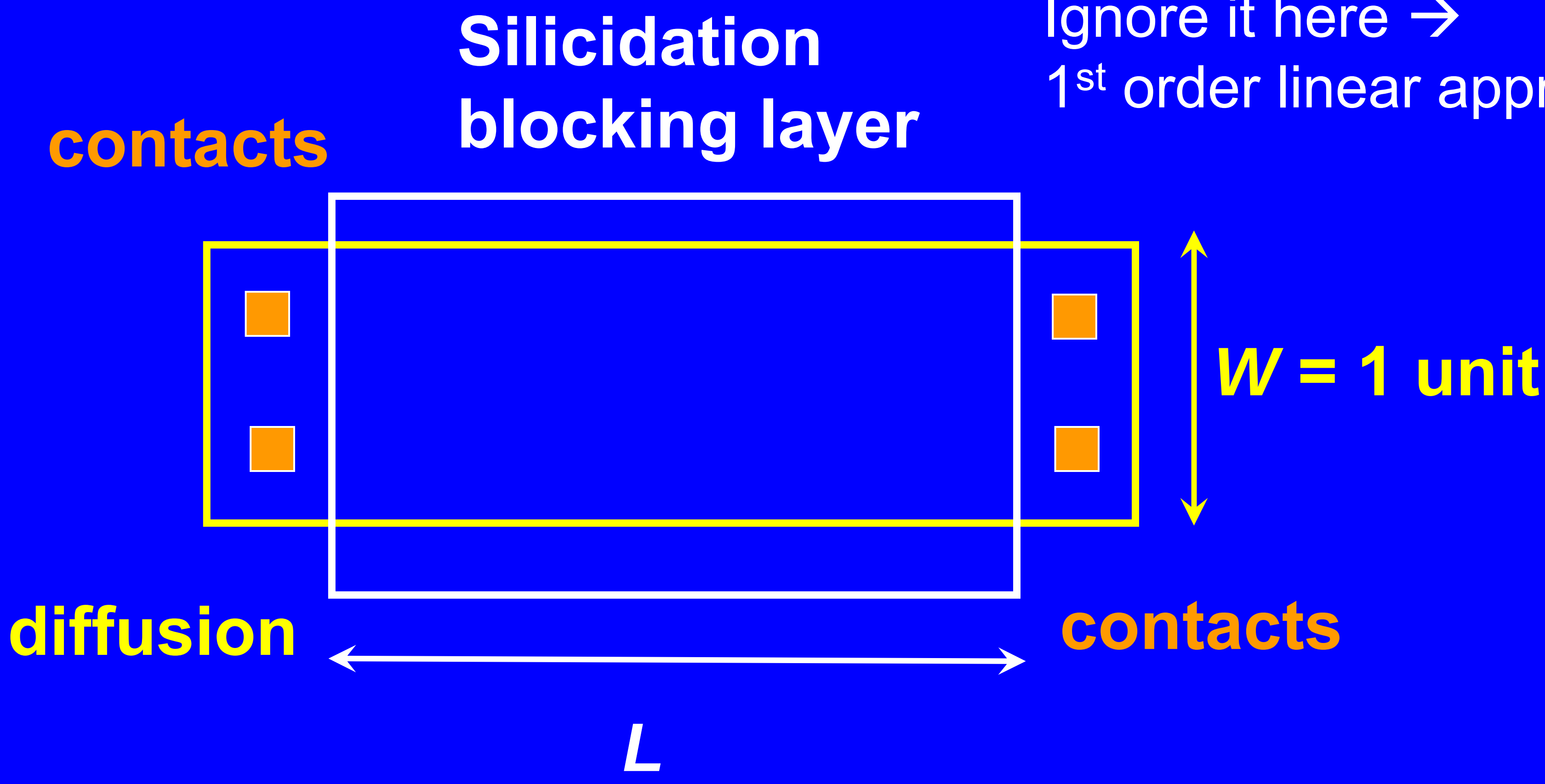
Outline

- Statistical SPICE Model
  - Monte Carlo, Skewing, Corners
- Optimal Corner for a Single Target
- Common/Optimal Corner for Multiple Targets
- Summary

A simple example: diffused resistor

$\Delta w/W$ is small typically.
Ignore it here → 1st order linear approx.



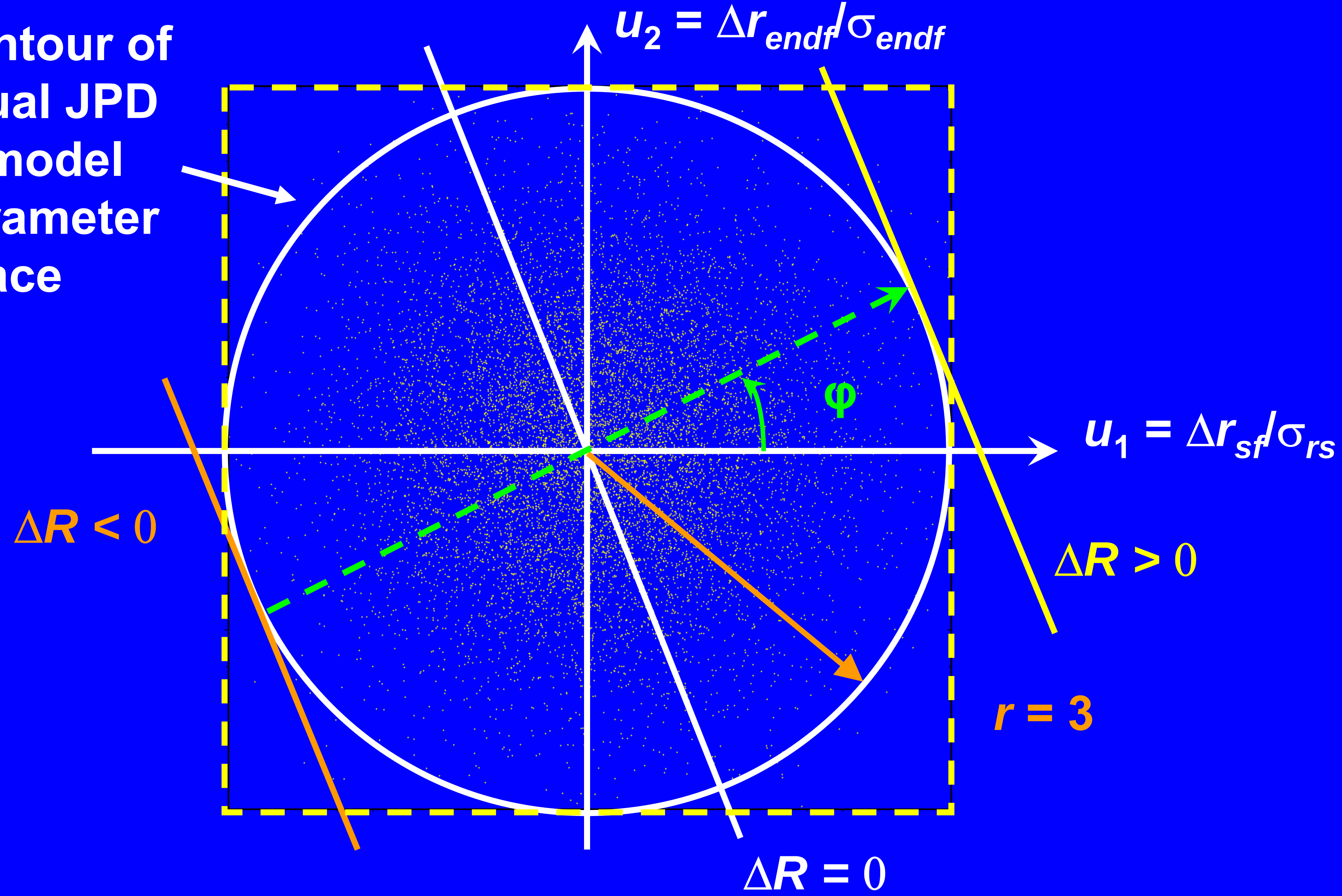
Resistor's statistical model

- $R = L\, r_{sf} + r_{endf} \quad = R_n + \Delta R(u_1, u_2)$
- $r_{sf} = r_{sn} + \sigma_{rs}\, u_1$
- $r_{endf} = r_{endn} + \sigma_{rend}\, u_2$
  - σ_{rs} , σ_{rend} ← Standard deviations
  - u_1, u_2: Independent stochastic variables.
    - Monte Carlo: Normal Gaussian distribution
    - Skewing: Independent corner/adjustable parameter
- Joint probability density (JPD)

$$P(u_1, u_2) = \exp[-\tfrac{1}{2} (u_1^2 + u_2^2)] / (2\pi)$$

In a normalized model parameter space

Contour of equal JPD in model parameter space



Variations

- $R = R_n + \Delta R$
  - Nominal: $R_n = L\, r_{sn} + r_{endn}$
  - Variation: $\Delta R = L\, \sigma_{rs}\, u_1 + \sigma_{rend}\, u_2$

    $= \sigma_R\,(u_1 \cos\varphi + u_2 \sin\varphi)$
- $\sigma_R = \sqrt{(L\, \sigma_{rs})^2 + \sigma_{rend}^{\;2}} = \mathrm{RSS}(s_{11}, s_{12}, \ldots)$

  $\tan\varphi = \sigma_{rend} \,/\, (L\, \sigma_{rs})$

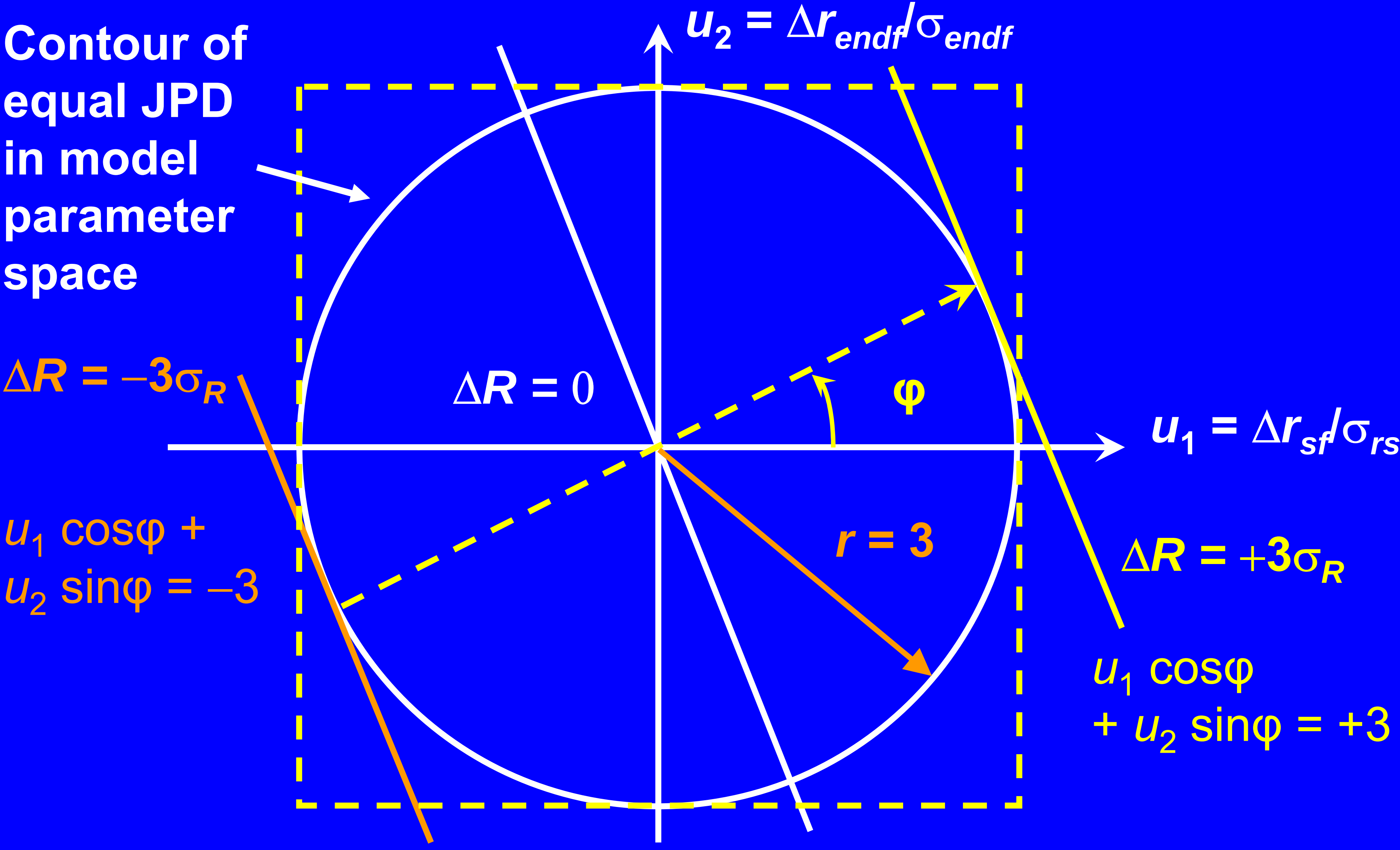
  Sensitivity w.r.t. u_1: $s_{11} = L\, \sigma_{rs} = \sigma_R \cos\varphi$

  w.r.t. u_2: $s_{12} = \sigma_{rend} = \sigma_R \sin\varphi$

  3σ bounds: $\Delta R = \pm 3\sigma_R$

In a normalized model parameter space

Contour of equal JPD in model parameter space



Different Corners

- 3σ bounds: $\Delta R = 3\eta_1 \sigma_R$
  - max: $\eta_1 = +1$; min: $\eta_1 = -1$
- Typical corner (equal skewing):

  $u_1 = u_2 = 3\eta_1\sigma_R / (L\,\sigma_{rs} + \sigma_{rend})$

  RSS(u_1, u_2, …) > 3 when $L\,\sigma_{rs} \neq \sigma_{rend}$

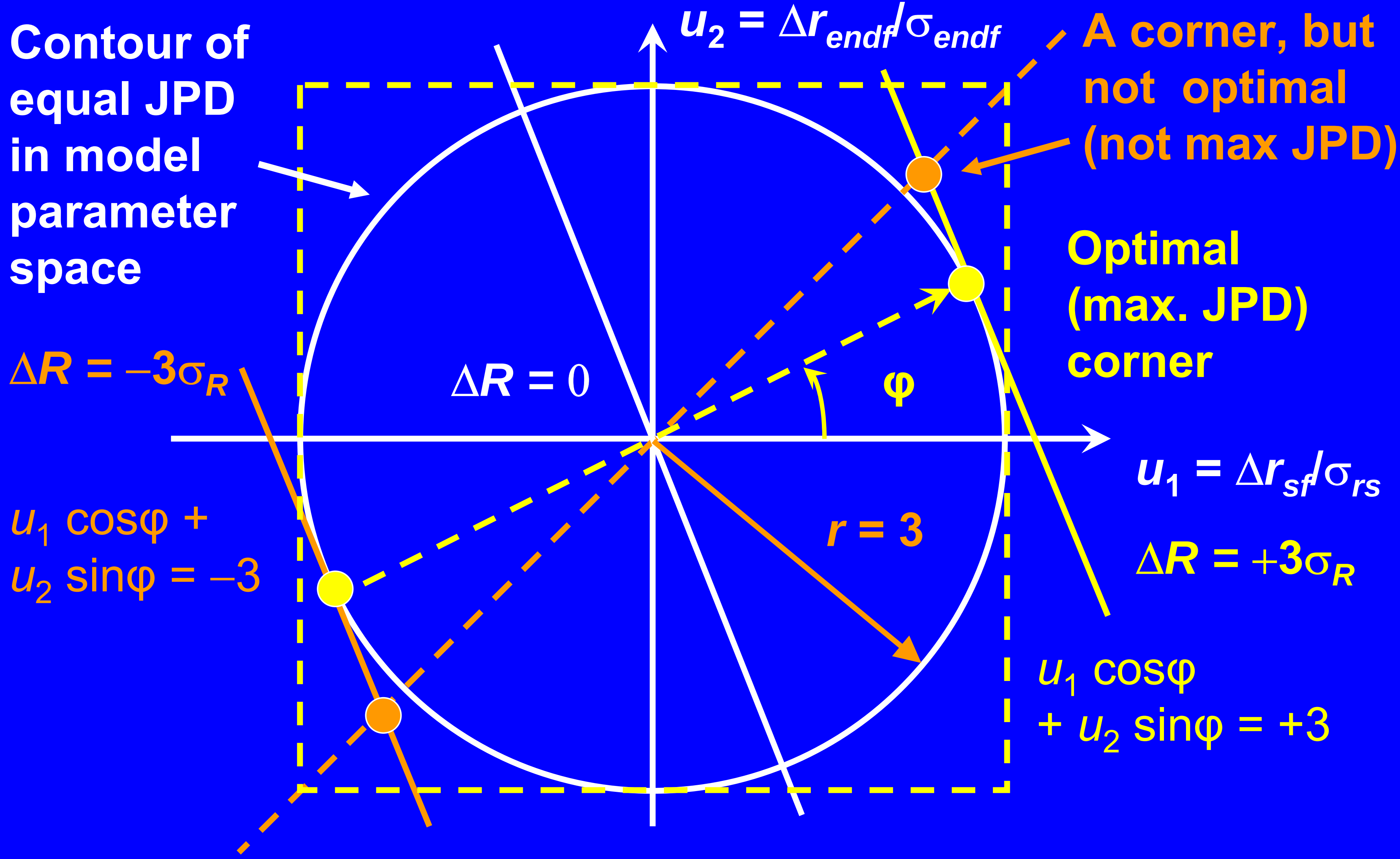
  → Smaller JPD
- Optimal corner (un-equal skewing):

  $u_1 = 3\cos\varphi = 3\eta_1 s_{11} / \sigma_R$

  $u_2 = 3\sin\varphi = 3\eta_1 s_{12} / \sigma_R$

  RSS(u_1, u_2, …) = 3 → Largest JPD

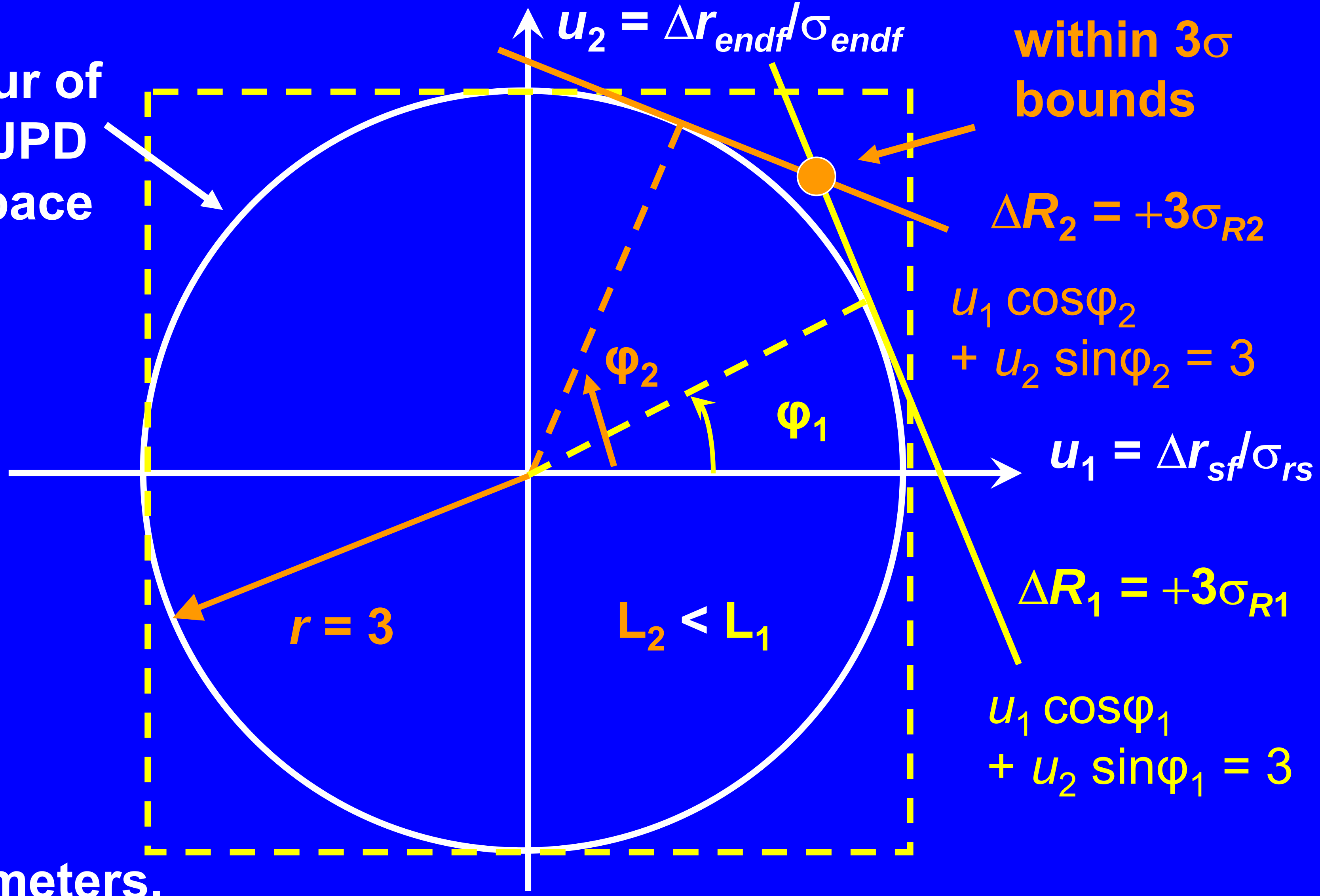
Single target: Various corners



2 resistors: Common corner

Common corner is within 3σ bounds

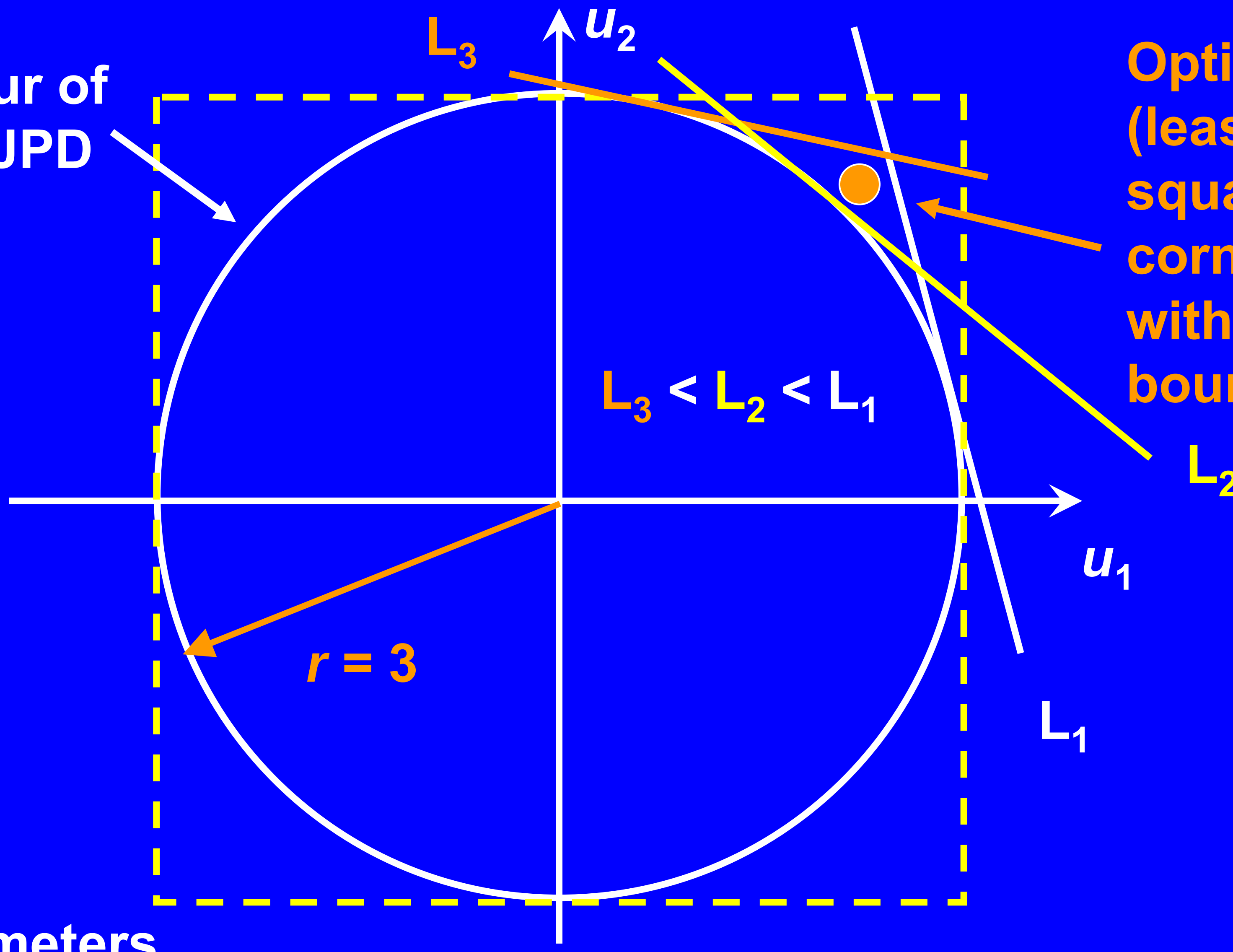
Contour of equal JPD in P space



2 parameters,
2 targets (2 eqs.)

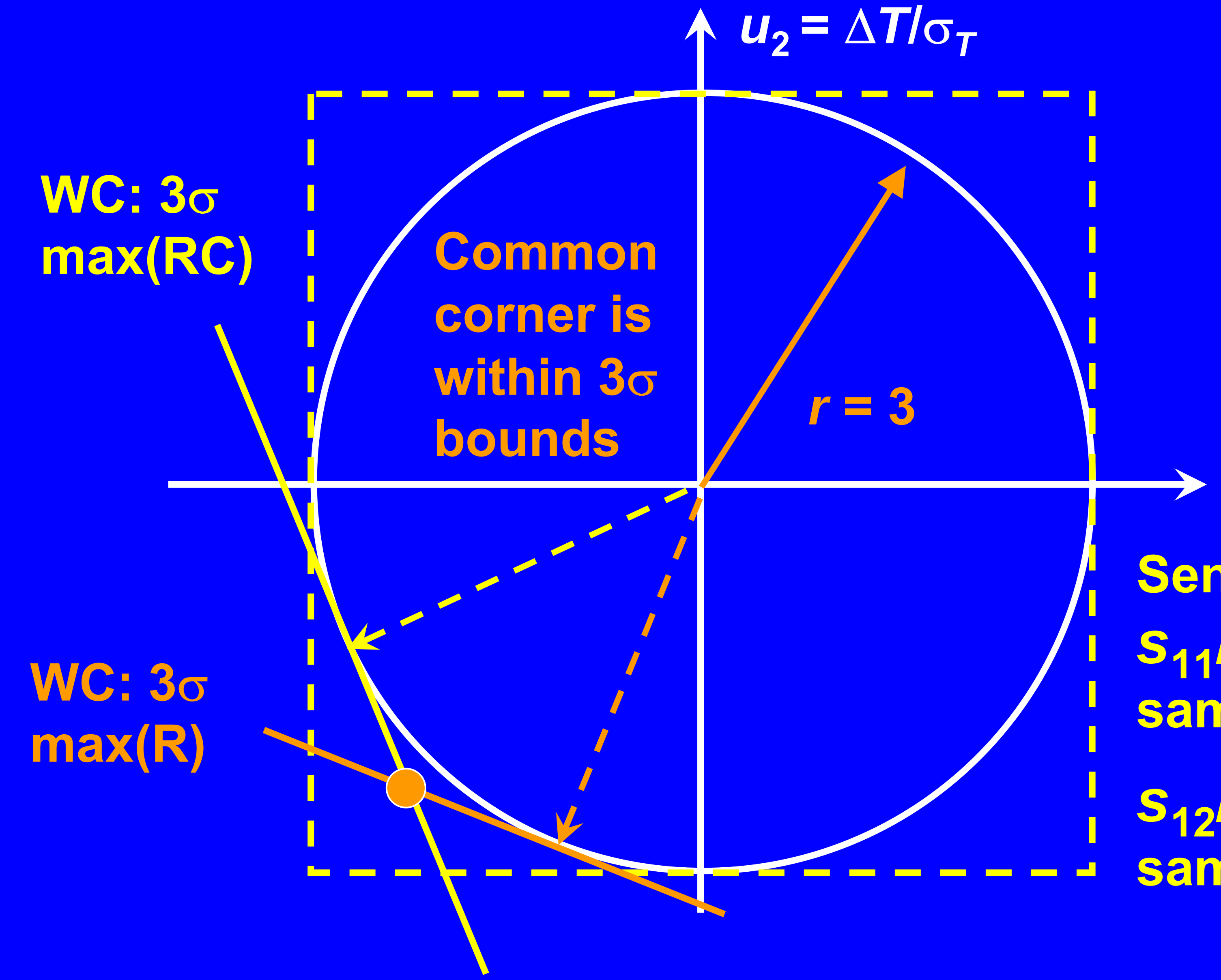
3 resistors: Least-squares corner

Contour of equal JPD



2 parameters,
3 targets (3 eqs.)

2 Different Targets: Interconnect R, RC



Sensitivities:

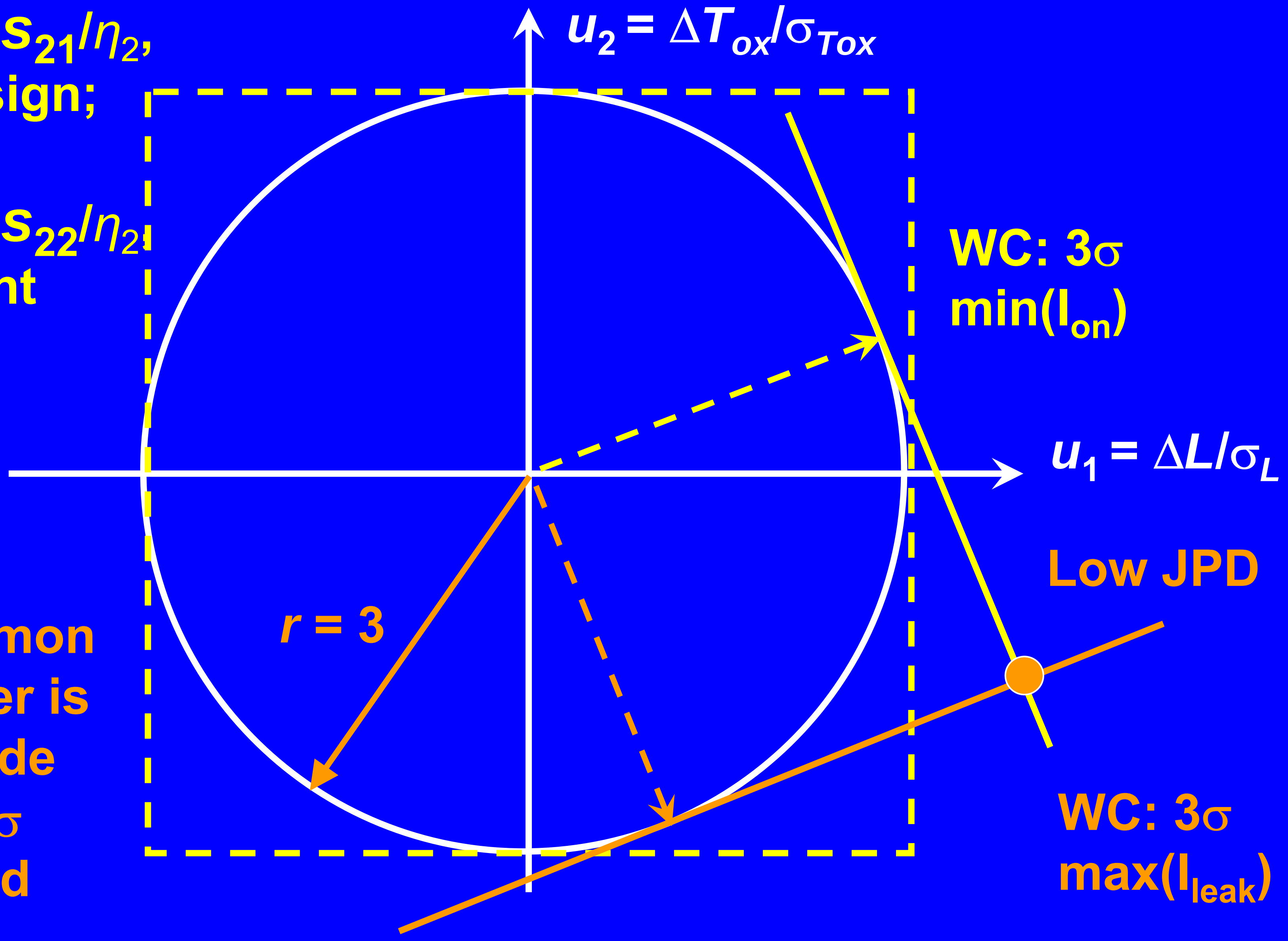
s_{11}/η_1, s_{21}/η_2, same sign;

s_{12}/η_1, s_{22}/η_2, same sign.

2 Different Targets: FET I_{on}, I_{leak}

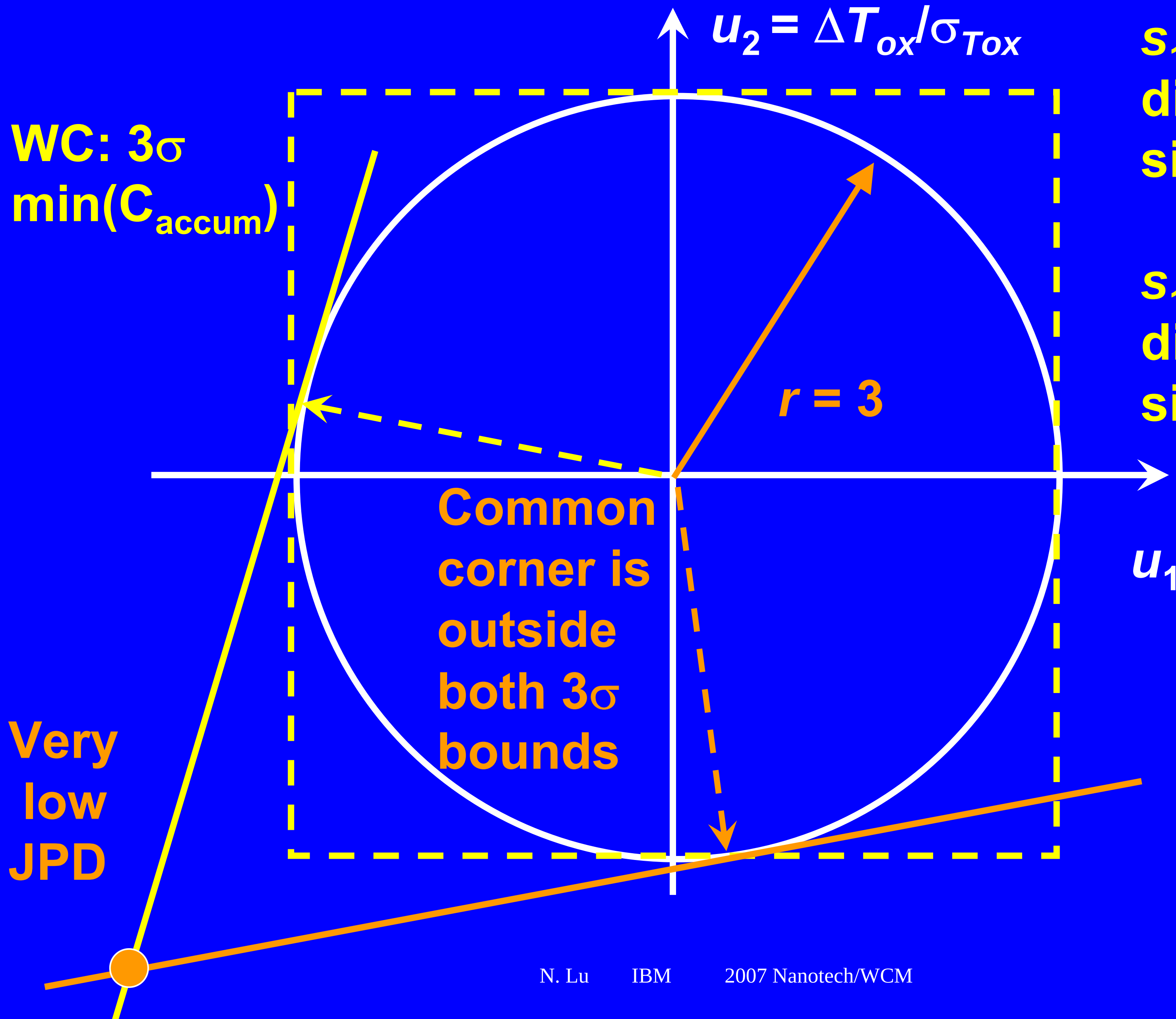
s_{11}/η_1, s_{21}/η_2, same sign;

s_{12}/η_1, s_{22}/η_2, different signs.



Common corner is outside *L*'s 3σ bound

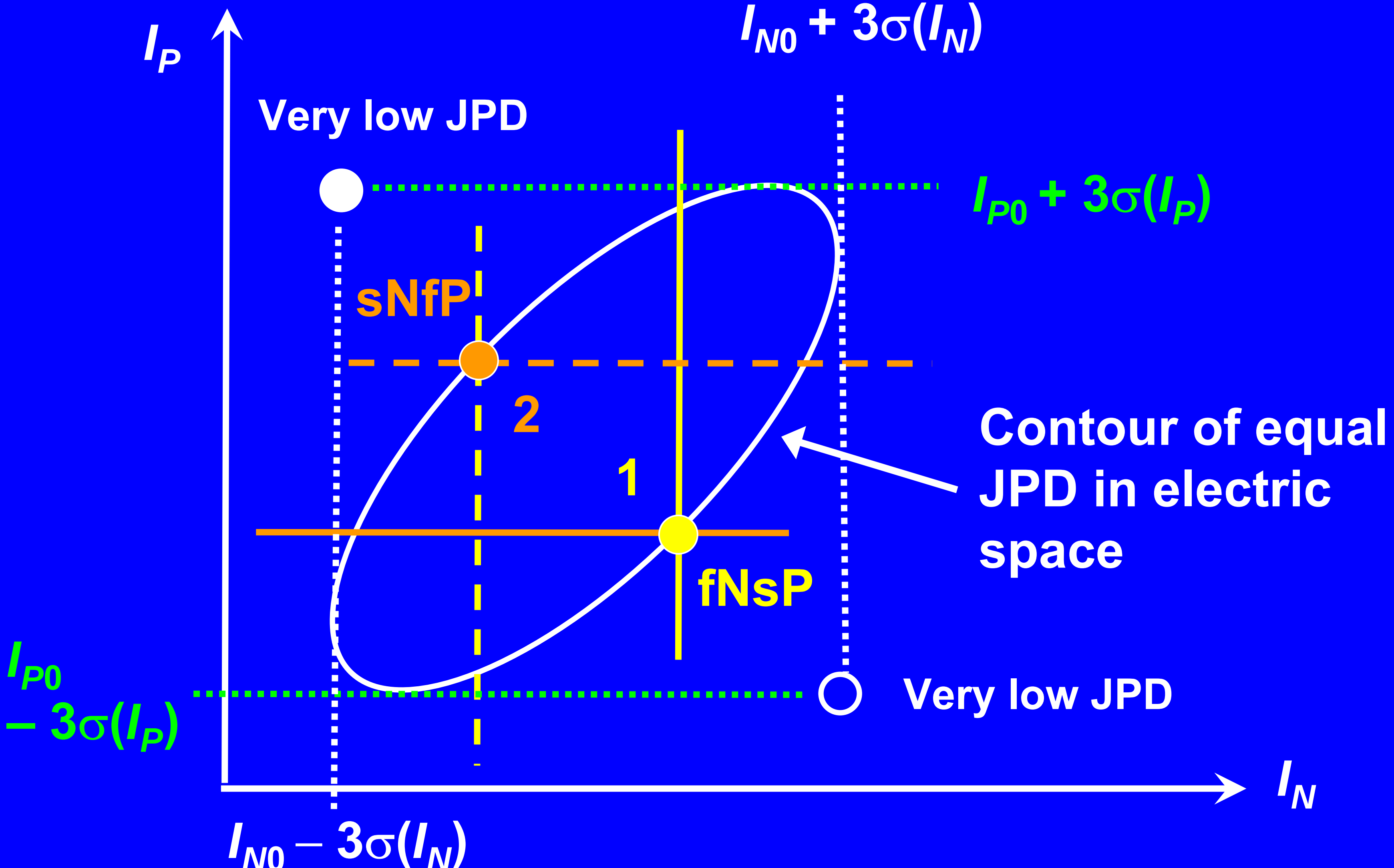
2 Targets: MOS varactor C_{accum}, I_{leak}



s_{11}/η_1, s_{21}/η_2, different signs;

s_{12}/η_1, s_{22}/η_2, different signs.

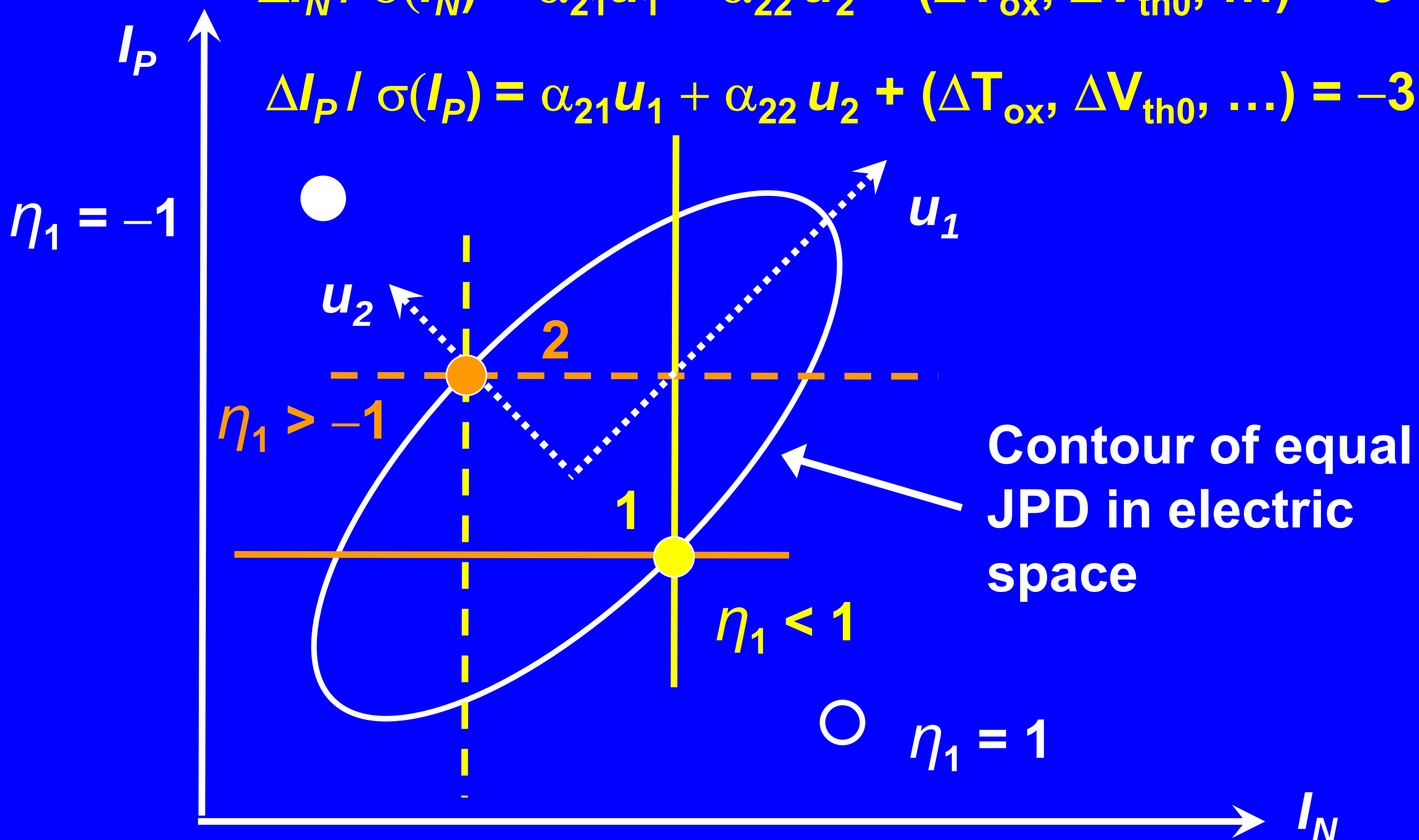
Opposite sensitivities: FET current



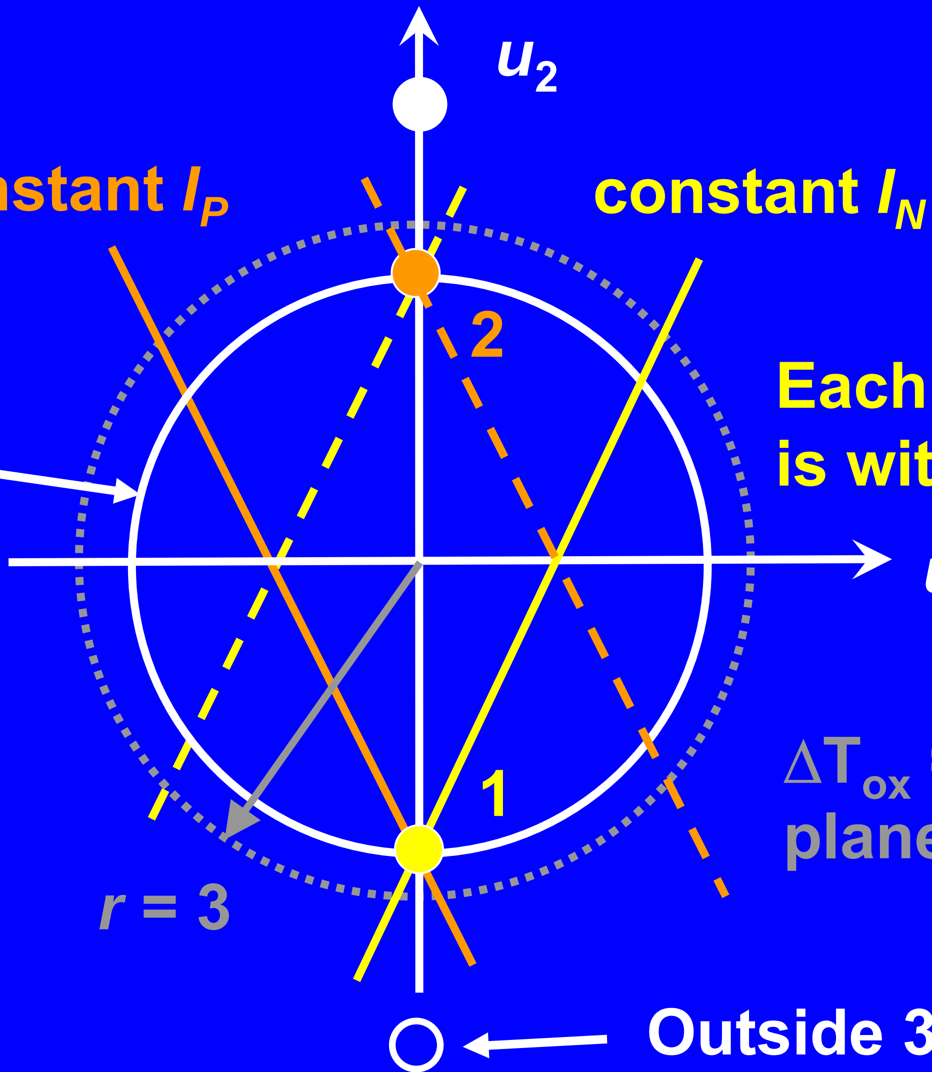
FET fNsP, sNfP corners: PCA in N-P *L*

$$\Delta I_N / \sigma(I_N) = \alpha_{21} u_1 - \alpha_{22} u_2 + (\Delta T_{ox}, \Delta V_{th0}, \ldots) = 3\ \eta_1$$

$$\Delta I_P / \sigma(I_P) = \alpha_{21} u_1 + \alpha_{22} u_2 + (\Delta T_{ox}, \Delta V_{th0}, \ldots) = -3\ \eta_1$$



Opposite sensitivities: Common corner still within bounds



Contour of equal JPD in a plane of constant but non-zero ΔT_{ox}, ΔV_{th0}, …, that passes corners

Each of corner 1, 2 is within 3σ bounds

$\Delta T_{ox} = \Delta V_{th0} = \ldots = 0$ plane

Outside 3σ bounds

Summary

- For a single performance target,
  - There are many corners
  - One of them has the largest JPD ← an optimal corner
- For multiple performance targets
  - Common corner may exit
    - Need to watch out its JPD
    - If too small, separate corners are needed
    - When there are many proper common corners, find max. JPD corner
  - If no common corner, a least-squares corner can be found systematically
- After getting 1st order optimal solution, build a quadratic response surface, then get 2nd order optimal corner solution through iteration