Consistency of Compact MOSFET Models with the Pao-Sah Formulation

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- The Pao-Sah “exact” MOSFET model
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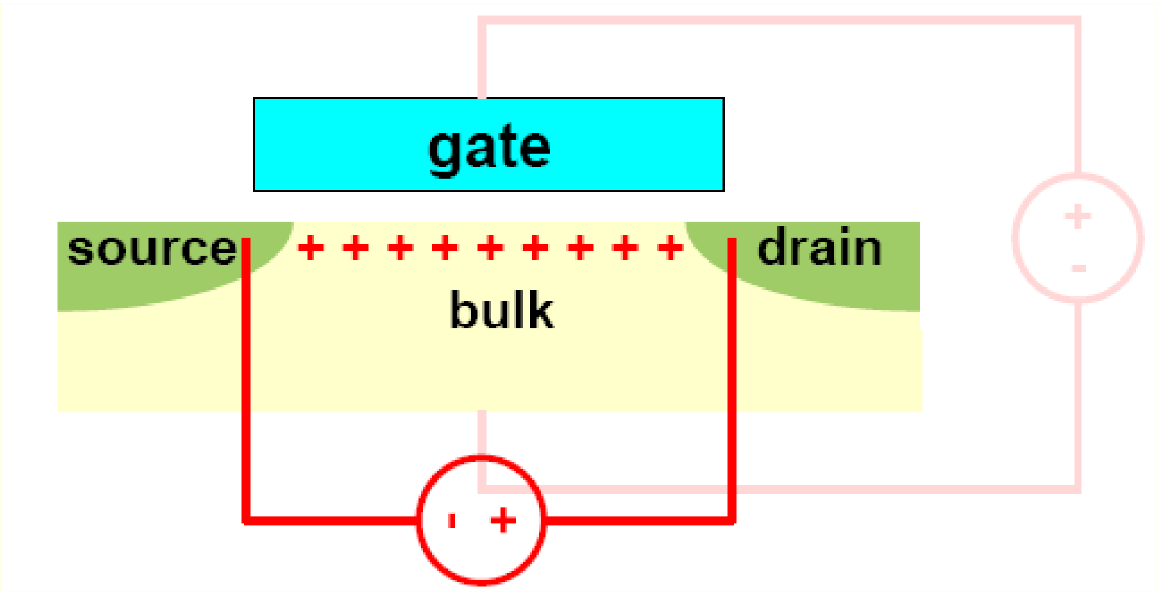
The Pao-Sah model-1

Basic MOSFET Operation *

2-dimensional problem

Approached by separating into 2 1-dimensional problems

- vertical 1-D field electrostatics control conduction charge
- longitudinal 1-D field controls current flow



*Advanced Compact Models for MOSFETs - WCM 2005

The Pao-Sah model-2

$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

⇓

$$I_D = \frac{W}{L}\int_{V_S}^{V_D} \mu\left(-Q_I'\right) dV_C$$

μ: carrier mobility
W : channel width
Q'_I : inversion charge density
$V_S \leq V_C \leq V_D$,
y: distance along the channel

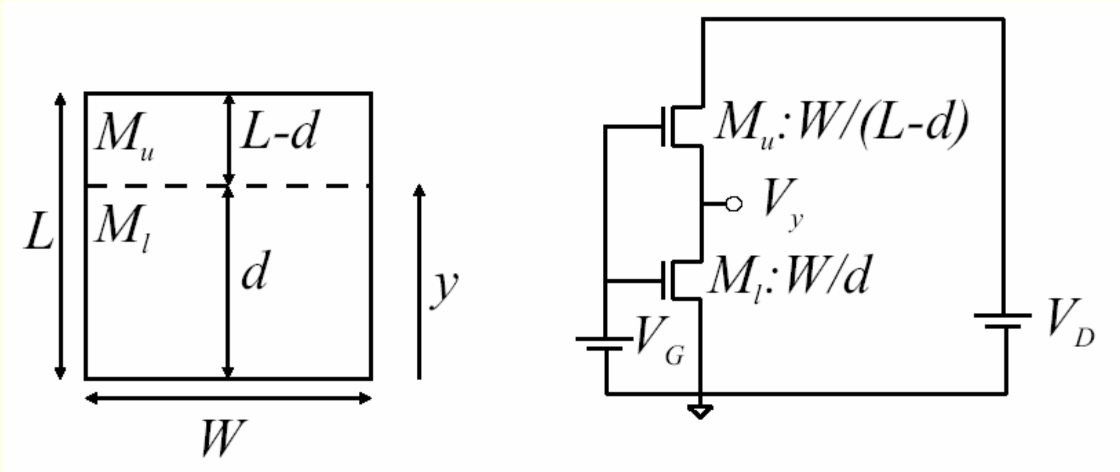
The small-signal output conductance of the transistor is

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G, V_S} = -\frac{W}{L}\mu Q_I'\left(V_D\right)$$

⇑

valid from weak inversion (where the current transport is dominated by diffusion) to strong inversion (where drift dominates).

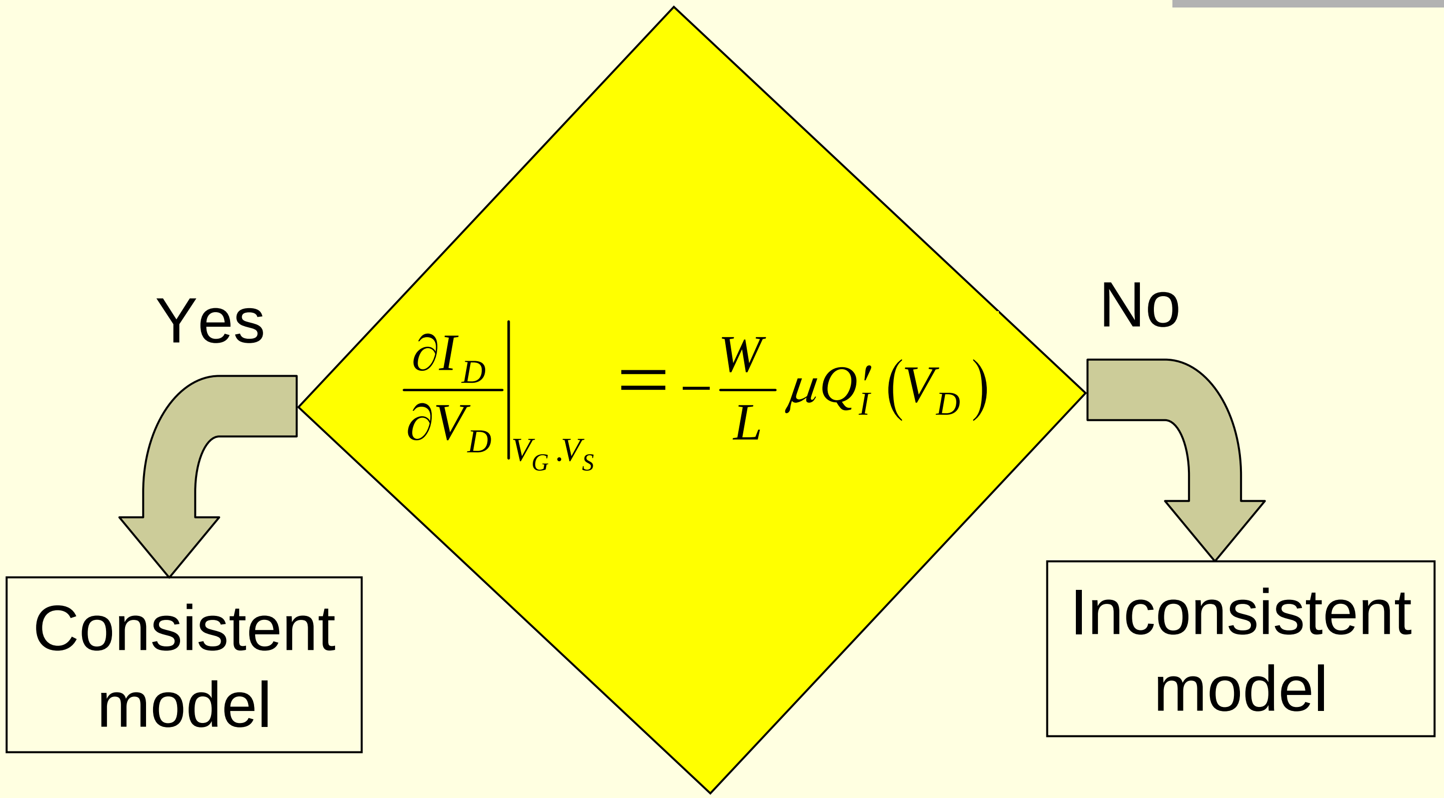
The Pao-Sah model-3:Consistency for the series association



Virtual cutting of a transistor into two parts and its representation as a series association of transistors.

$$I_D = \frac{W}{L}\int_{V_S}^{V_D} \mu\left(-Q_I'\right)dV_C = \frac{W}{d}\int_{V_S}^{V_Y} \mu\left(-Q_I'\right)dV_C = \frac{W}{L-d}\int_{V_Y}^{V_D} \mu\left(-Q_I'\right)dV_C$$

Consistency test



Classical strong inversion model

$$Q_I' = -C_{ox}'\left(V_G - V_{T0} - nV_C\right)$$

- *n* is the slope factor (depletion charge linearization coefficient) and V_{T0} is the equilibrium threshold voltage
- Assuming *n* and μ independent of channel voltage V_C

$$I_D = \mu C_{ox}' \frac{W}{L}\left[V_G - V_{T0} - \frac{n}{2}\left(V_S + V_D\right)\right]\left(V_D - V_S\right)$$

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G, V_S} = \mu C_{ox}' \frac{W}{L}\left(V_G - V_{T0} - nV_D\right)$$

Brews' charge sheet model-1

Charge sheet approximation of the inversion charge

$$Q_I' = -C_{ox}'\left(V_G - V_{FB} - \phi_s - \gamma\sqrt{\phi_s - \phi_t}\right)$$

Charge sheet current expression

$$I_D = I_{drift} + I_{diff} = -\mu W Q_I' \frac{d\phi_s}{dy} + \mu W \phi_t \frac{dQ_I'}{dy}$$

$$I_D = \mu \frac{W}{L} C_{ox}' \left\{ \left(V_G - V_{FB}\right)\left(\phi_{sL} - \phi_{s0}\right) - \frac{1}{2}\left(\phi_{sL}^2 - \phi_{s0}^2\right) - \frac{2}{3}\gamma\left[\left(\phi_{sL} - \phi_t\right)^{3/2} - \left(\phi_{s0} - \phi_t\right)^{3/2}\right]\right\}$$

$$+\mu \frac{W}{L} C_{ox}' \phi_t \left\{ \left(\phi_{sL} - \phi_{s0}\right) + \gamma\left[\left(\phi_{sL} - \phi_t\right)^{1/2} - \left(\phi_{s0} - \phi_t\right)^{1/2}\right]\right\}$$

Brews' charge sheet model-2

$$g_d = \mu \frac{W}{L} C'_{ox} \frac{V_G - V_{FB} - \phi_{sL} - \gamma\sqrt{\phi_{sL} - \phi_t} + \phi_t \left(1 + \frac{\gamma}{2\sqrt{\phi_{sL} - \phi_t}} \right)}{\left. \frac{\partial V_D}{\partial \phi_{sL}} \right|_{V_G}}$$

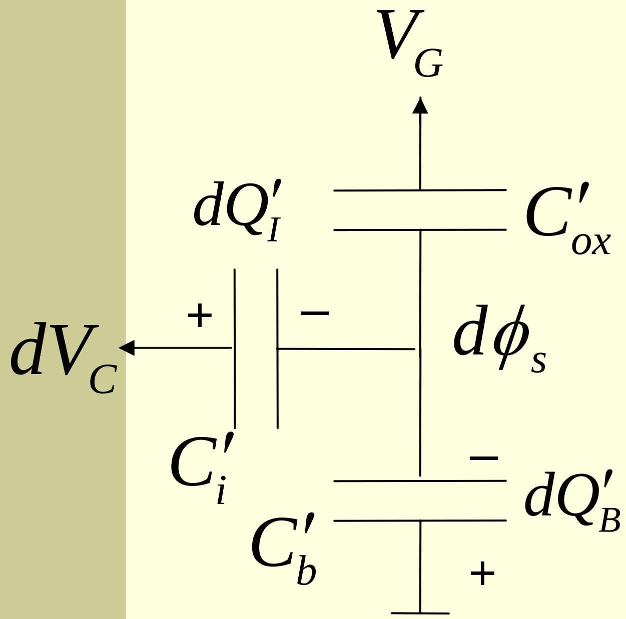
To obtain $g_d = \mu \frac{W}{L} C'_{ox} \left(V_G - V_{FB} - \phi_{sL} - \gamma\sqrt{\phi_{sL} - \phi_t} \right)$

the denominator must be calculated as

$$\left. \frac{\partial V_D}{\partial \phi_{sL}} \right|_{V_G} = 1 + \frac{\phi_t}{V_G - V_{FB} - \phi_{sL} - \gamma\sqrt{\phi_{sL} - \phi_t}} \left(1 + \frac{\gamma}{2\sqrt{\phi_{sL} - \phi_t}} \right)$$

⇐ Physical meaning ?

Capacitive model of the field effect



In the charge sheet model $C'_i = -\dfrac{Q'_I}{\phi_t}$

charge sheet approx

$$\left.\frac{\partial V_C}{\partial \phi_s}\right|_{V_G} = 1 + \frac{C'_b + C'_{ox}}{C'_i} = 1 - \frac{\phi_t}{Q'_I} C'_{ox}\left(1 + \frac{C'_b}{C'_{ox}}\right) =$$

$$1 + \frac{\phi_t}{V_G - V_{FB} - \phi_s - \gamma\sqrt{\phi_s - \phi_t}}\left(1 + \frac{\gamma}{2\sqrt{\phi_s - \phi_t}}\right)$$

charge sheet approx

Baccarani's model (1978)

- First integral of the Poisson (voltage input) equation, neglecting majority carriers

$$\left(V_G - V_{FB} - \phi_s\right)^2 = \gamma^2 \left(\phi_t e^{(\phi_s - 2\phi_F - V_C)/\phi_t} + \phi_s - \phi_t\right)$$

$$\frac{dV_C}{d\phi_s} = 1 + \phi_t \frac{2\left(V_G - V_{FB} - \phi_s\right) + \gamma^2}{\left(V_G - V_{FB} - \phi_s\right)^2 - \gamma^2\left(\phi_s - \phi_t\right)}$$

$$I_D = -\mu \frac{W}{L} \int_{\phi_{s0}}^{\phi_{sL}} Q'_I\left(\phi_s\right) \frac{dV_C}{d\phi_s} d\phi_s$$

currrent is not charge sheet but *Q'*

$$I_D = \mu \frac{W}{L} C'_{ox} \int_{\phi_{s0}}^{\phi_{sL}} \left(V_G - V_{FB} - \phi_s - \gamma\sqrt{\phi_s - \phi_t}\right) d\phi_s +$$

$$+ \mu \frac{W}{L} C'_{ox} \phi_t \int_{\phi_{s0}}^{\phi_{sL}} \frac{2\left(V_G - V_{FB} - \phi_s\right) + \gamma^2}{V_G - V_{FB} - \phi_s - \gamma\sqrt{\phi_s - \phi_t}} d\phi_s$$

very complicated I_D expression

The ACM model-1: Linearization of the depletion charge variation

Charge sheet approximation of the inversion charge

$$Q'_I = -C'_{ox}\left(V_G - V_{FB} - \phi_s\right) - Q'_B$$

- For constant V_G, it follows that

$$dQ'_I = C'_{ox} d\phi_s - dQ'_B = (C'_{ox} + C'_b) d\phi_s = nC'_{ox} d\phi_s$$

In the ACM model the bulk capacitance is calculated neglecting the channel carriers

$$n = 1 + \frac{C'_b(V_G)}{C'_{ox}} = n(V_G)$$

The ACM model-2

$$\left.\begin{aligned} I_D &= -\mu W Q_I' \frac{d\phi_s}{dy} + \mu W \phi_t \frac{dQ_I'}{dy} \\ dQ_I' &= nC_{ox}' d\phi_s \end{aligned}\right\} \quad I_D = \frac{\mu W}{L}\left[\frac{Q_{IS}'^{\,2} - Q_{ID}'^{\,2}}{2nC_{ox}'} - \phi_t\left(Q_{IS}' - Q_{ID}'\right)\right]$$

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G, V_S} = \frac{\mu W}{L}\left[\frac{-Q_{ID}'}{nC_{ox}'} + \phi_t\right]\frac{dQ_{ID}'}{dV_D} = -\frac{W}{L}\mu Q_I'\left(V_D\right)$$

$$dQ_{ID}'\left(\frac{1}{nC_{ox}'} - \frac{\phi_t}{Q_{ID}'}\right) = dV_D$$

depletion charge linearization

charge sheet approximation

The ACM model-3

$$dQ'_I\left(\frac{1}{nC'_{ox}}-\frac{\phi_t}{Q'_I}\right)=dV_C \quad \textbf{(I)}$$

Integrating (I) between V_C and V_P yields the Unified Charge Control Model (UCCM)

$$\frac{Q'_{IP}-Q'_I}{nC'_{ox}}+\phi_t \ln\left(\frac{Q'_I}{Q'_{IP}}\right)=V_P-V_C$$

$$\frac{\partial Q'_{IS(D)}}{\partial V_{S(D)}}=nC'_{ox}\frac{Q'_{IS(D)}}{Q'_{IS(D)}-nC'_{ox}\phi_t}$$

$$\frac{\partial Q'_I}{\partial V_G}=-\frac{1}{n}\frac{\partial Q'_I}{\partial V_C}$$

$$\frac{\partial Q'_I}{\partial V_B}=-\frac{n-1}{n}\frac{\partial Q'_I}{\partial V_C}$$

Calculation of the capacitve coefficients

Consistency conditions for charge sheet models

Model	Equation	Charge sheet approx	Depletion charge linearization	n slope factor
Brews	input	yes	no	x
Brews	output	yes	no	x
ACM	input	yes	yes	$n(V_G)$
ACM	output	yes	yes	$n(V_G)$

Conclusions

- Classical strong (and weak) inversion as well as Baccarani´s models can be derived directly from Pao-Sah formula.

- All-region charge sheet compact MOSFET models consistent with the Pao-Sah formula use the same approximations in the input (electrostatic) and output (transport) equations