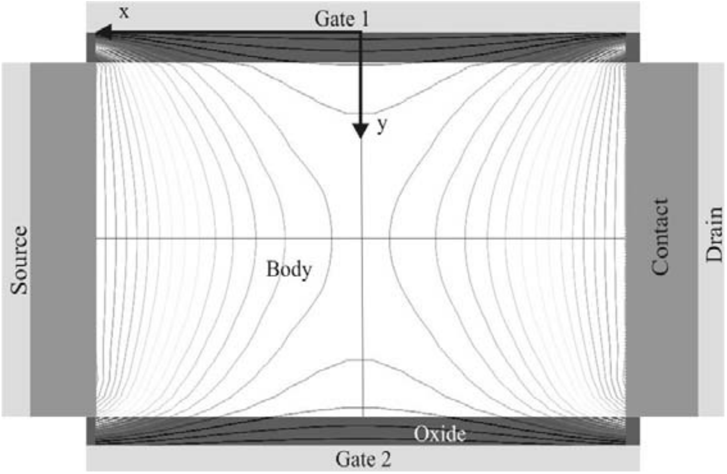
Precise 2D Compact Modeling of Nanoscale DG MOSFETs Based on Conformal Mapping Techniques

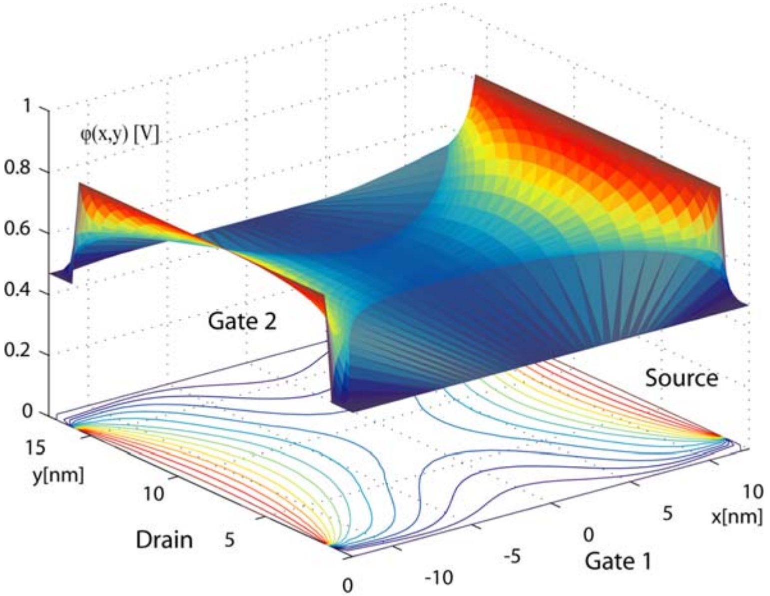
T. A. Fjeldly, S. Kolberg
University Graduate Center (UNIK)
Norwegian University of Science and Technology, Kjeller, Norway

B. Iñiguez
Departament d'Enginyeria Electronica, Electrica i Automatica,
Universitat Rovira i Virgili, Tarragona, Spain

SiNANo Silicon-based Nanodevices

Overview





Objective:

Establish framework for precise, compact 2D modeling of short-channel nanoscale MOSFETs

Here: *Double Gate (DG) MOSFET*

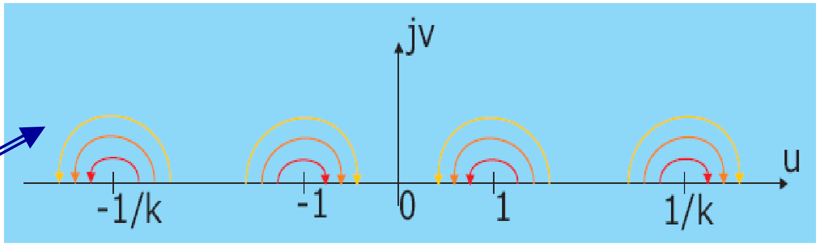
- **Part 1**: *Subthreshold and near threshold* conditions where capacitive coupling between contacts dominant electrostatics

- **Part 2**: *Strong inversion* where effects of electrons dominant electrostatics. Must be treated self-consistently.

- Determine 2D barrier topography
- Calculate electron distribution
- Find short-schannel effects including DIBL
- Calculate current – drift-diffusion and ballistic

Modeling procedure

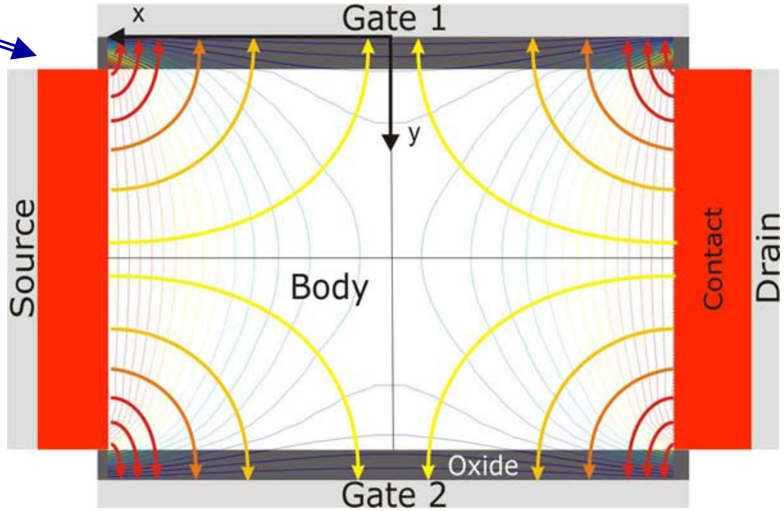
- Include gate oxide in extended device body: $t'_{ox} = t_{ox}\varepsilon_{Si} / \varepsilon_{ox}$
- Map body into upper half plane of transformed (u,iv)-plane using ***conformal mapping*** (Schwartz-Christoffel transform).
- ***Subthreshold***: Solve *Laplace's equation* in (u,iv)-plane (low-doped body)
  → 2D potential distribution for capacitive coupling. Map back to (x,y)-plane
- ***Near threshold***: Determine *total potential distribution self-consistently* by including electrostatic effects of electrons.
- ***Strong inversion***: Use *long-channel solution*. Treat 2D capacitive coupling as perturbation.
- Calculate ***current*** (drift-diffusion and ballistic)
- ***Verify*** against 2D numerical simulations.



Schwartz-Christoffel transformation

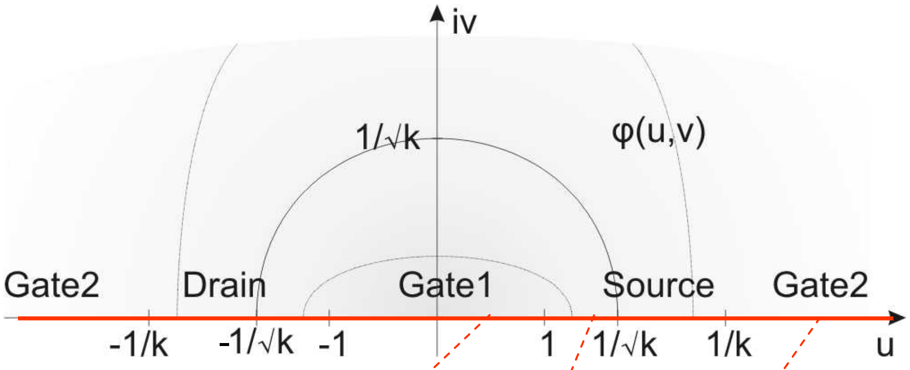
$$z = x + iy = \frac{L}{2}\frac{F(k,u+iv)}{K(k)}$$

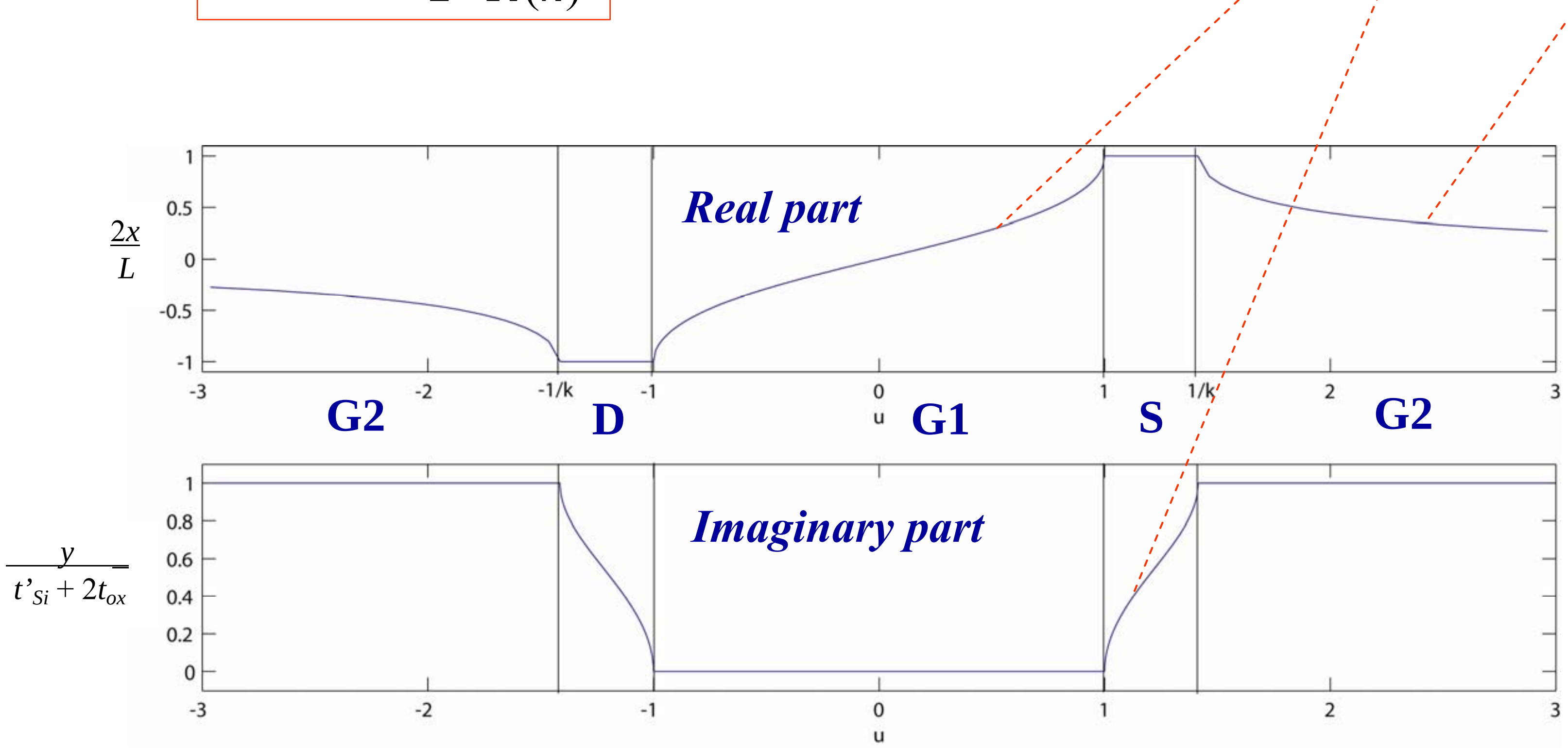
$$F(k,w) = \int_0^w \frac{dw'}{\sqrt{(1-w'^2)(1-k^2w'^2)}}, \quad K(k) = F(k,1)$$



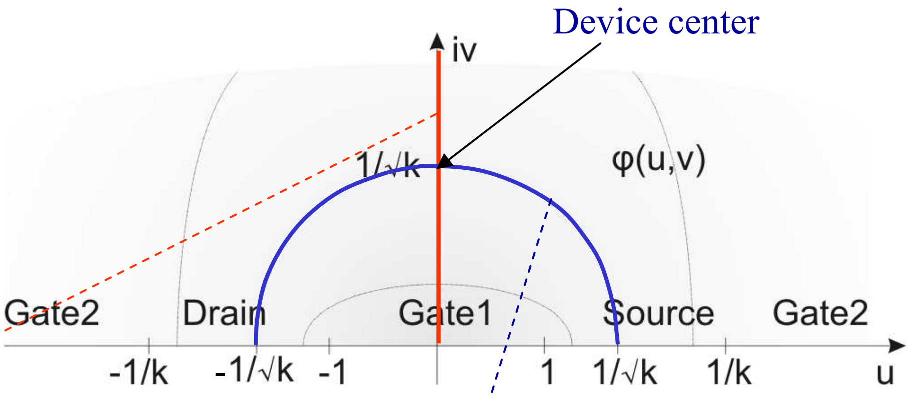
Mapping of the body boundary

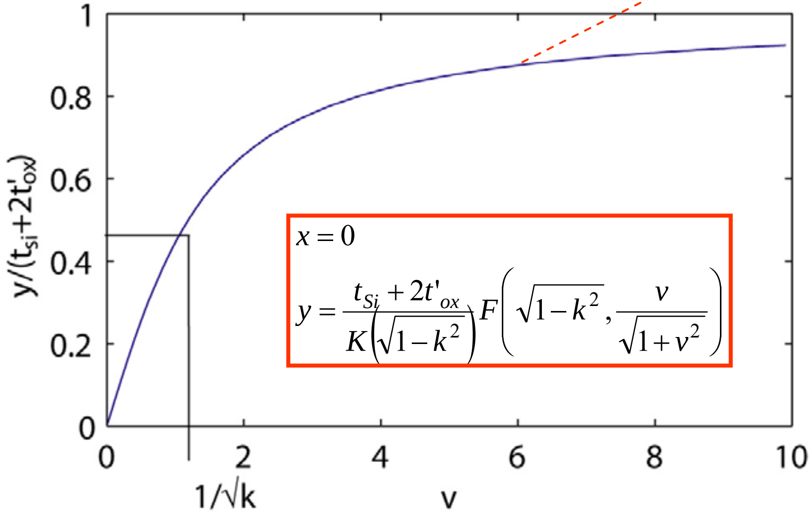
$$z = x + iy = \frac{L}{2}\frac{F(k,u)}{K(k)}$$





Mapping along symmetry lines

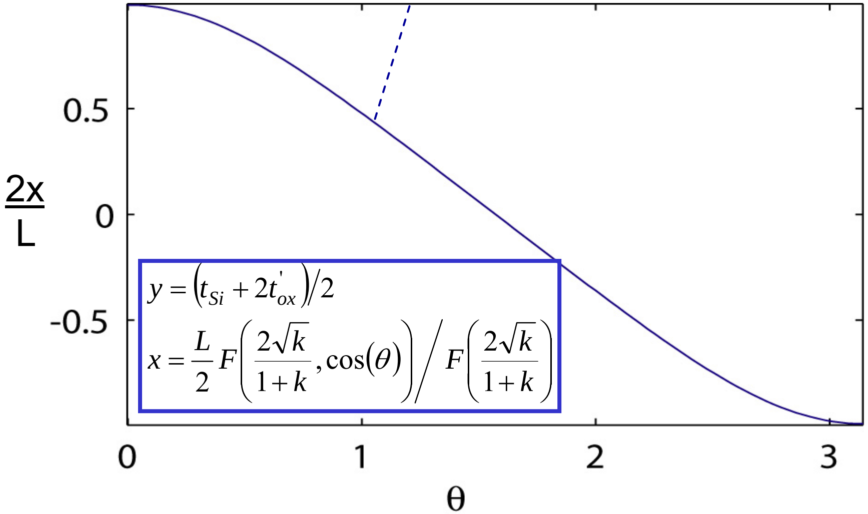




$$x = 0$$

$$y = \frac{t_{Si} + 2t'_{ox}}{K\left(\sqrt{1-k^2}\right)} F\left(\sqrt{1-k^2}, \frac{v}{\sqrt{1+v^2}}\right)$$

Gate-to-gate symmetry line



$$y = \left(t_{Si} + 2t'_{ox}\right)/2$$

$$x = \frac{L}{2} F\left(\frac{2\sqrt{k}}{1+k}, \cos(\theta)\right) \Big/ F\left(\frac{2\sqrt{k}}{1+k}\right)$$

Source-to-drain symmetry line

DG MOSFET and regimes of operation

SINANO template device:

- Double gate *n*-channel SOI MOSFET with aluminum gates

- Specifics: L = 25 nm, t_{si} = 12 nm, t_{ox} = 1.6 nm, ε_{ox} = 7, body doping N_d =10^{15} cm^{-3}, contact doping 10^{19} cm^{-3}, extended body thickness: $t_{si} + 2t'_{ox}$

Operating regimes:

⟶ Part 1: In ***subthreshold***, dopant and carrier charges can be neglected in electrostatics when n or $N_d \leq 10^{18}$ cm^{-3}. Potential and electron distributions dominated by the 2D capacitive coupling between the contacts (source, drain and gates)

⟶ Part 2: ***Above threshold*** requires self-consistent analysis

- 2a) *Near threshold*: Use approximate barrier profile
- 2b) *Strong inversion*: Electrons dominate, treat capacitive coupling as a perturbation

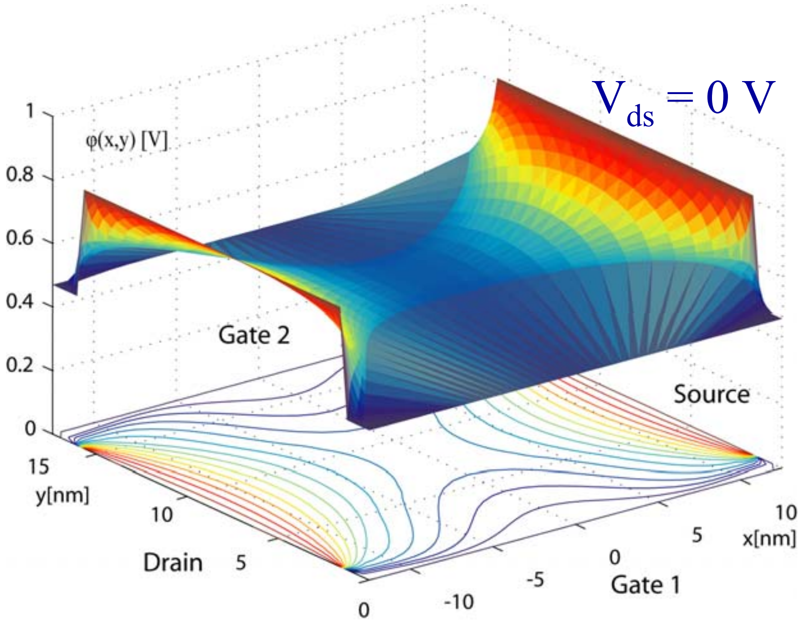
Part 1: Subthreshold

– body potential

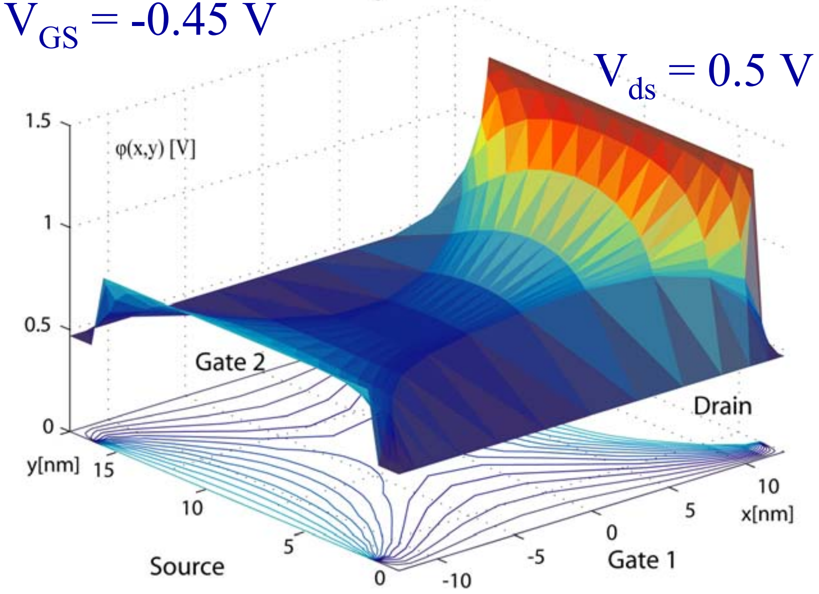
Solution of Laplace's equation in (u,iv)-plane (thin oxide approx):

$$\varphi(u,v)=\frac{v}{\pi}\int_{-\infty}^{\infty}\frac{\varphi(u')}{(u'-u)^2+v^2}du'$$

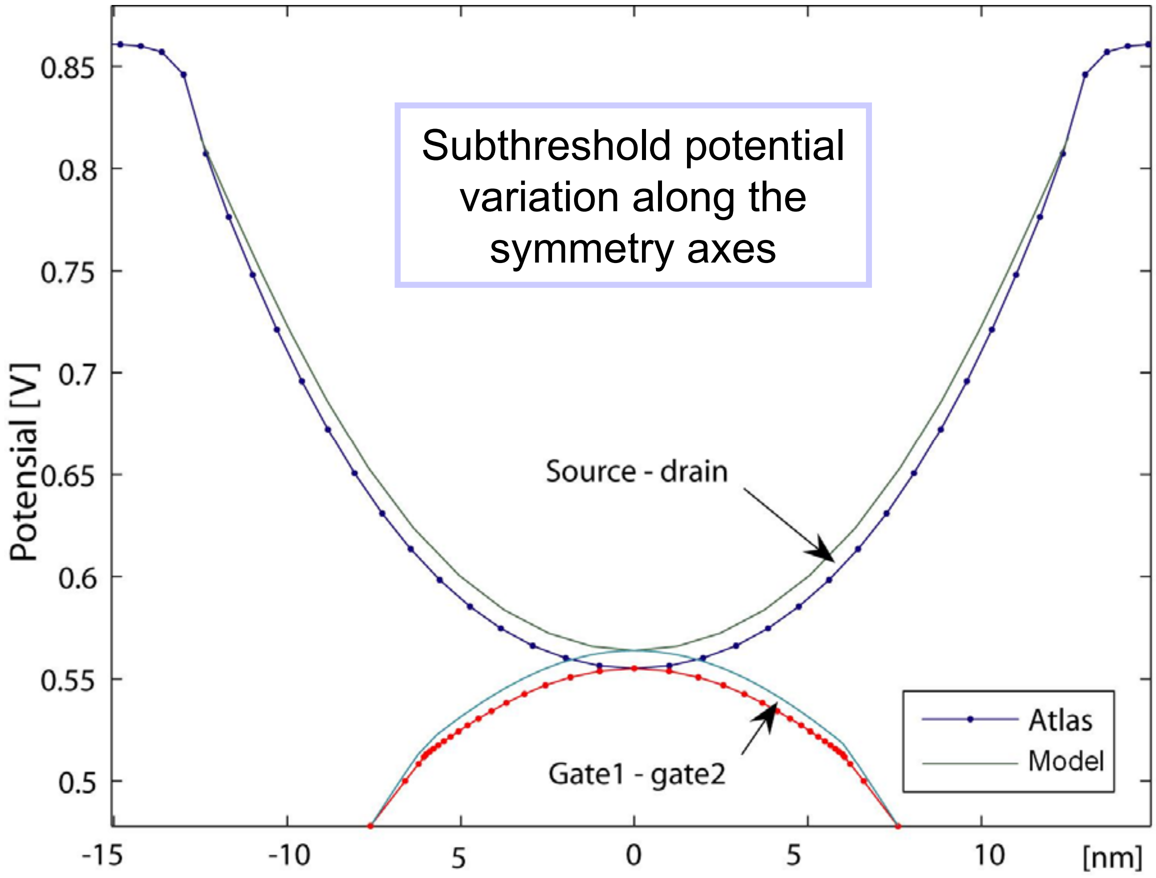
$$=\frac{1}{\pi}\left\{\begin{array}{l}(V_{GS2}-V_{FB})\left[\pi-\tan^{-1}\left(\frac{1-ku}{kv}\right)-\tan^{-1}\left(\frac{1+ku}{kv}\right)\right]\\ +(V_{GS1}-V_{FB})\left[\tan^{-1}\left(\frac{1-u}{v}\right)+\tan^{-1}\left(\frac{1+u}{v}\right)\right]\\ +V_{bi}\left[\tan^{-1}\left(\frac{1-ku}{kv}\right)-\tan^{-1}\left(\frac{1-u}{v}\right)\right]\\ +(V_{bi}+V_{DS})\left[\tan^{-1}\left(\frac{1+ku}{kv}\right)-\tan^{-1}\left(\frac{1+u}{v}\right)\right]\end{array}\right\}$$



V_{GS} = -0.45 V



Comparison with numerical simulations



$V_{GS} = -0.47$ V, $V_{DS} = 0$ V

Charge density distribution

– gate-to-gate symmetry line

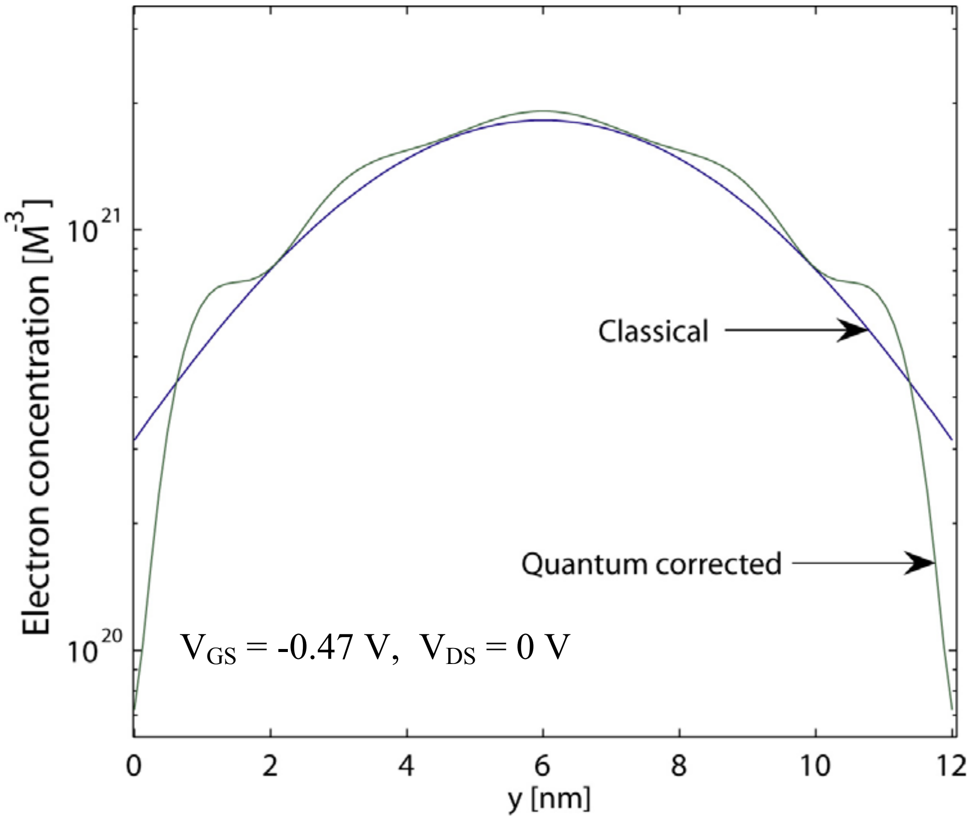
- Classic electron distribution:

$$n_c(y) = 2\left(\frac{m_n k_B T}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{E_g/2 + q[\varphi_b - \varphi_{SS}(y)]}{k_B T}\right)$$

- QM electron distribution (deep well):

$$n_q(y) = \sum_{j=1}^{l} n_{sqj} \left|\psi_j(y)\right|^2$$

Combine subband states from parabolic well (lowest subbands) and square well potentials (higher subbands).



Threshold voltage and DIBL

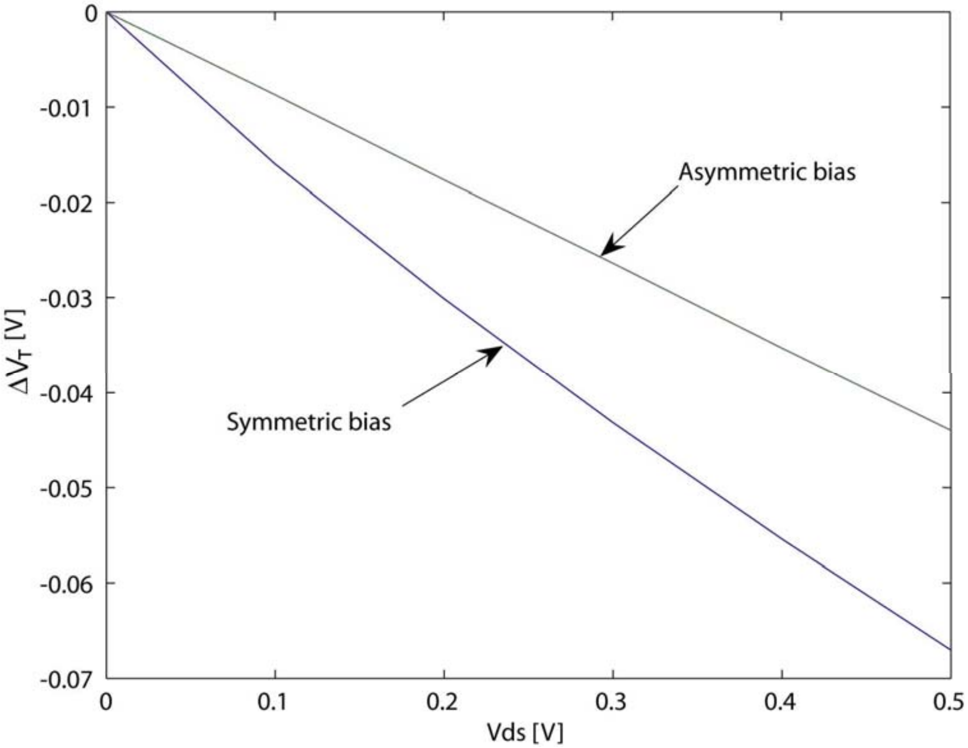
'Classical' threshold voltage definition:
Gate voltage V_T where band bending at barrier is $2q\varphi_b$ at V_{DS} = 0 V

For symmetric gate bias: V_T = -0.47 V

Drain induced barrier lowering (DIBL):
Shift in V_T with applied V_{DS}

Asymmetric gate biasing:
Apply fixed negative bias at Gate 2 relative to Gate 1. Pro/con effects:

+ reduced short channel effect
+ adjustable thereshold voltage
- effective channel width halved
- added small-signal capacitance



***Part 2: Strong inversion** – self-consistency*

Consider energy barrier at gate-to-gate symmetry axis

$\varphi_1(y)$: 1D potential from electrons

$\varphi_2(y)$: 2D capacitive coupling potential between contacts

Superposition principle: $\varphi(y) = \varphi_1(y) + \varphi_2(y)$

$$\varphi_1(y) = -\int_0^y E_1(y) = -K_o \times \begin{cases} y\int_{t'_{ox}}^{(t_{Si}+2t'_{ox})/2} \exp\left(\dfrac{\varphi(y')}{V_{th}}\right) dy' & \text{for } y < t'_{ox} \\ t'_{ox} \int_{t'_{ox}}^{(t_{Si}+2t'_{ox})/2} \exp\left(\dfrac{\varphi(y')}{V_{th}}\right) dy' & \\ \quad + \int_{t'_{ox}}^{y} dy' \int_{y'}^{(t_{Si}+2t'_{ox})/2} \exp\left(\dfrac{\varphi(y'')}{V_{th}}\right) dy'' & \text{for } t'_{ox} \le y < (t_{Si} + 2t'_{ox})/2 \end{cases}$$

$$\varphi_2(v) = V_{GS} - V_{FB} + \frac{2}{\pi}\left[\tan^{-1}\left(\frac{1}{kv}\right) - \tan^{-1}\left(\frac{1}{v}\right)\right](V_{bi} - V_{GS} + V_{FB})$$

2a Near operational threshold

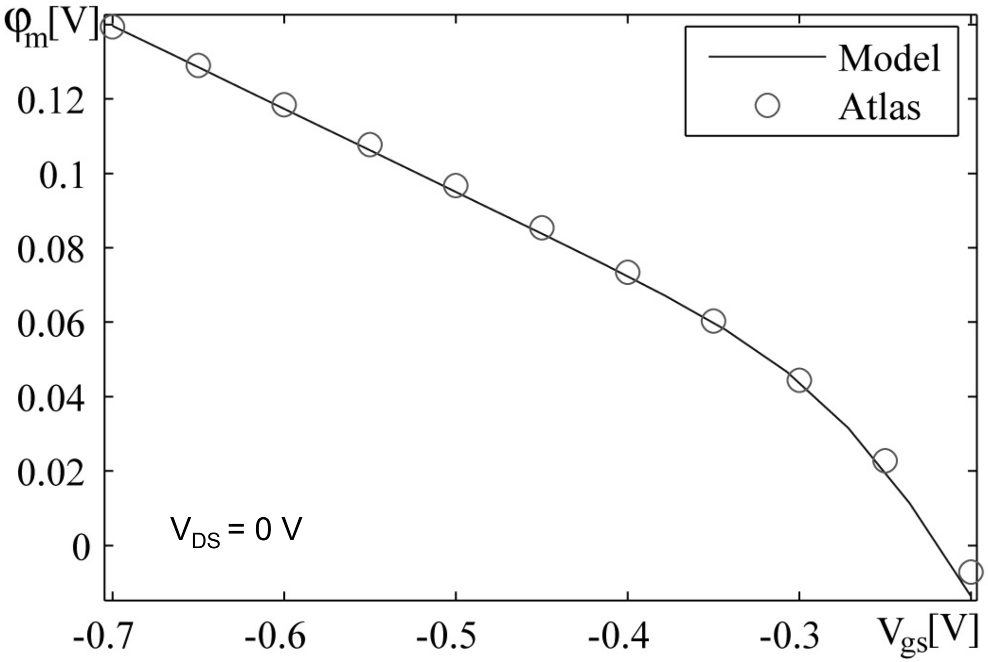
Assume that the total gate-to-gate potential $\varphi(y)$ has a parabolic form, which agrees very well with numerical simulations (Atlas):

$$\boxed{\varphi(y) \approx \varphi_p(y) = V_{GS} - V_{FB} + \varphi_m \left[1 - \left(1 - \frac{2y}{t_{Si} + 2t'_{ox}} \right)^2 \right]}$$

Parameter φ_m is determined by:

- substituting $\varphi_p(y)$ into expression for $\varphi_1(y)$
- performing integrations in expression for $\varphi_1(y)$
- substituting $\varphi_1(y)$ into total potential $\varphi(y)$
- obtaining self-consistent expression for φ_m by considering device center

Parameter φ_m



$$\varphi_m = \left(V_{bi} - V_{GS} + V_{FB}\right)\left[\frac{4}{\pi}\tan^{-1}\left(\frac{1}{\sqrt{k}} - 1\right)\right] - \left(\frac{m_n k_B T}{2\pi\hbar^2}\right)^{3/2} \frac{q\left(t_{Si} + 2t'_{ox}\right)^2}{4} \exp\left(\frac{V_{bi} - V_{GS} + V_{FB} - \varphi_b - E_g/2}{V_{th}}\right)$$
$$\times\left\{ sign(\varphi_m)\sqrt{\frac{\pi V_{th}}{\varphi_m}}\,\mathrm{erf}\left(\sqrt{\frac{\varphi_m}{V_{th}}\left[1 - \frac{2t'_{ox}}{\left(t_{Si} + 2t'_{ox}\right)}\right]}\right) + \frac{V_{th}}{\varphi_m}\left[\exp\left(-\frac{\varphi_m}{V_{th}}\left(1 - \frac{2t'_{ox}}{\left(t_{Si} + 2t'_{ox}\right)}\right)^2\right) - 1\right]\right\}$$

2b Strong inversion

Gate-to-gate barrier:

Use long-channel potential $\varphi_1(y)$ for electrons from 1D Poisson's equation (Taur 2001), adjusted to include effects of body doping:

$$\boxed{\varphi_1(y) = \varphi_c - 2V_{th}\ln\left[\cos\left(\sqrt{\frac{qn_i}{2\varepsilon_s V_{th}}}\exp\left(\frac{\varphi_c - \varphi_b}{2V_{th}}\right)\left(y - t'_{ox} - t_{Si}/2\right)\right)\right]}$$

Treat capacive coupling potential $\varphi_2(y)$ as a perturbation.

Find self-consistent total potential by first-order correction using 1D Poisson's equation

Describes how DIBL-effect fades in strong inversion

Current modeling – subthreshold and near threshold

Drift-diffusion transport:

Use barrier shape derived self-consistently. Assume that the current is ’small’. Then we have:

$$I_{DD} = -qW\mu_n n_s(x)\frac{dV_F(x)}{dx} = \frac{qW\mu_n V_{th}\left(1-e^{-V_{DS}/V_{th}}\right)}{\int_0^L \frac{dx}{n_{so}(x)}}$$

$n_{so}(x)$: surface concentration of electrons in all cross-sections from source to drain.

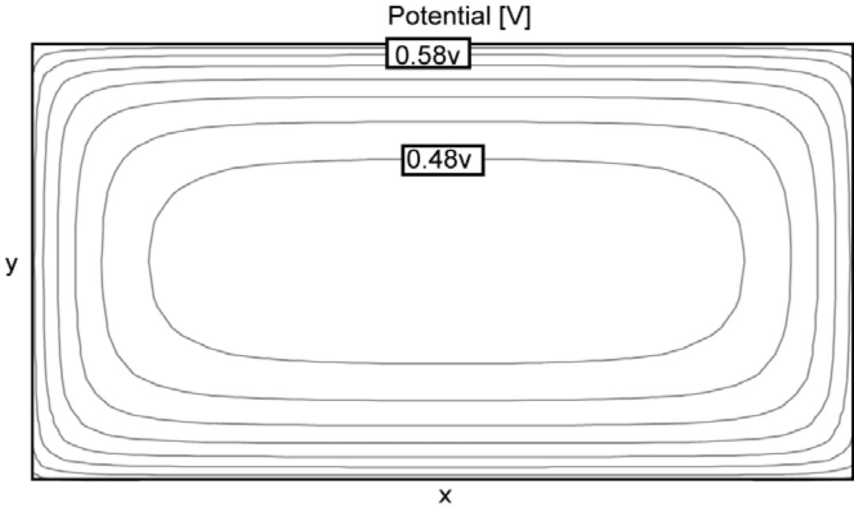
Ballistic transport:

- Current consists of free-flight electrons with sufficient energy to fly over barrier
- Electrons ’filtered’ through available quantum states at barrier position
- Current limited by ability of source and drain to supply high-energy carriers fast enough

Natori formalism (1994, 2002):

$$I_{BT} = q(F^{+}\text{-}F^{-}) \quad \text{where} \quad F^{\pm} = \frac{(2k_BT)^{3/2}}{\pi^2\hbar^2}\sum_{valleys}\sum_{j}\sqrt{m_i}\,F_{1/2}\left(\frac{E_{Fs}-E_{ij}-qV^{\pm}}{k_BT}\right)$$

Current modeling – strong inversion



Simulated (Atlas) strong inversion potential contour plot
$V_{GS} = 0.1$ V, $V_{DS} = 0$ V

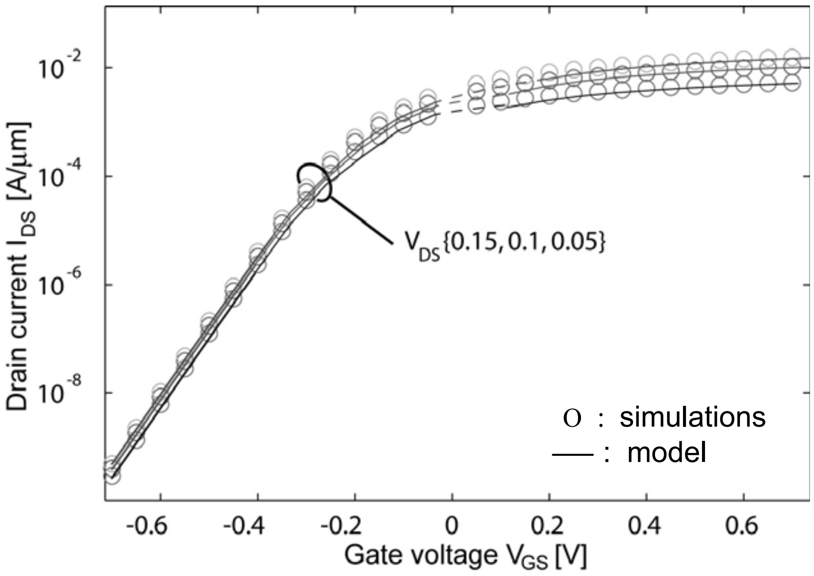
Current flows primarily near the Si/SiO_2 interfaces.
Use classical long-channel expressions for I_{DD}.

Strong inversion threshold voltage (Taur et al. 2002, adjusted for body doping) :

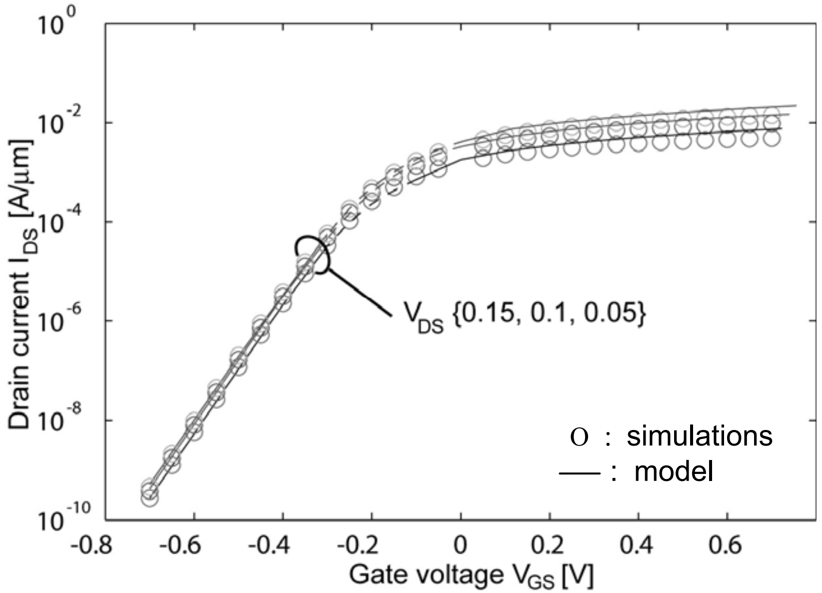
$$V_T = V_0 + 2\varphi_B \quad \text{where} \quad V_0 = \phi_{ms} + \frac{2k_BT}{q}\ln\left(\frac{\varepsilon_{ox}t_{Si}\left(V_{GS} - V_0\right)}{2t_{ox}qn_i}\right)$$

DD and ballistic current – comparision with simulations

Drift-diffusion current



Ballistic current



Observations:

- Good correspondence between modeled and simulated currents
- Quite similar results for drift-diffusion and ballistic current
- Some more work needed to model ’knee’ region)

Summary

Compact 2D modeling framework established for short-channel, nanoscale DG MOSFETs by means of conformal mapping techniques

Precise description of:

- *2D device barrier topography*
- *Short-channel effects including DIBL*
- *Self-consistency at moderate and high electron densities*
- *Fading of DIBL in strong inversion*
- *Drift-diffusion and ballistic transport*

Model verified by numerical simulations

SiNANO Silicon-based Nanodevices