Charge-Based Formulation of Thermal Noise in Short-Channel MOS Transistors

V. C. Pahim, C. Galup-Montoro, and M. C. Schneider

Electrical Engineering Department,

Federal University of Santa Catarina,

Brazil

Basic dc Equations

$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

$$\frac{dV_C}{dy} = \frac{dQ_I'}{dy}\left(\frac{1}{nC_{ox}} - \frac{\phi_t}{Q_I'}\right)$$

$$\mu = \frac{\mu_0}{1-\frac{E}{E_{cr}}} = \frac{\mu_0}{1-\frac{\mu_0 E}{v_{\lim}}}$$

$$dQ_I' = nC_{ox}' d\phi_S$$

Allows analytical integration for I_D

$$i_d = \frac{\left(q_{IS}' + q_{ID}' + 2\right)}{1+\varepsilon\left(q_{IS}' - q_{ID}'\right)}\left(q_{IS}' - q_{ID}'\right)$$

$$q_I' = \frac{Q_I'}{\left(-nC_{ox}'\phi_t\right)}, \qquad i_d = \frac{I_D}{I_S}$$

$$I_S = \frac{\mu_0 C_{ox}' n\phi_t^2}{2}\frac{W}{L}, \qquad \varepsilon = \frac{\mu_0\phi_t / L}{v_{\lim}}$$

ε: short-channel factor

Current division

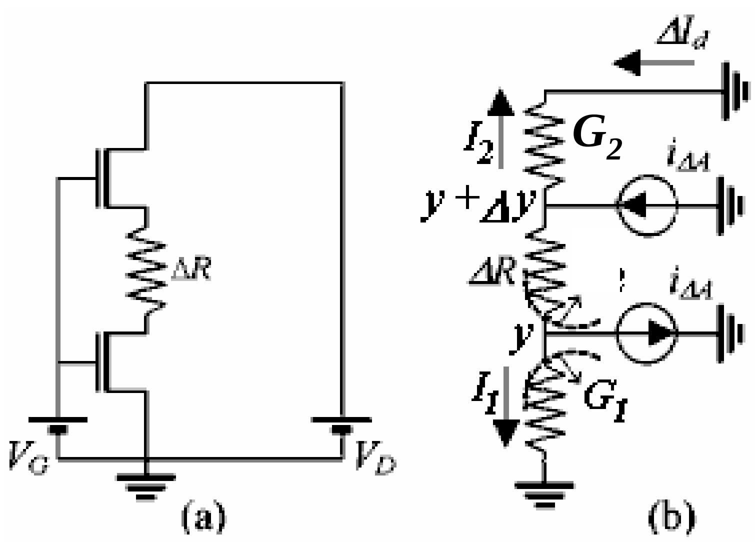

Figure 1: Transistor partition for calculating the contribution of elementary noise sources

$$\frac{G_2}{G_1+G_2}=\frac{\frac{y}{L}+\varepsilon\left(q'_{IS}-q'_I\right)}{1+\varepsilon\left(q'_{IS}-q'_{ID}\right)}$$

$$\Delta I_d==\frac{\frac{\Delta y}{L}\left(1-\varepsilon L\frac{dq'_I}{dy}\right)}{1+\varepsilon\left(q'_{IS}-q'_{ID}\right)}i_{\Delta A}$$

Drain Current Noise

$$\overline{i_{\Delta A}^2} = -4qDQ_I'\left(y\right)\frac{W}{\Delta y}\Delta f \qquad D = D_0 = \mu_0\phi_t$$

Elementary noise sources assumed to be uncorrelated

Integration along the channel results in

$$\overline{I_d^2} = \overline{I_{d,equ}^2} + \overline{I_{d,heat}^2}$$

Drain Current Noise

$$\frac{\overline{I_{d,equ}^2}}{4kTG_0\Delta f} = 2\frac{\left[\frac{2}{3}\frac{\left(q_{IS}^{'}+1\right)\left(1+\eta+\eta^2\right)}{\left(1+\eta\right)}-1\right]}{1+\varepsilon\left(q_{IS}^{'}+1\right)\left(1-\eta\right)}$$

→ Normalized long-channel charge

→ Current reduction factor

$$\eta = \frac{q'_{ID}+1}{q'_{IS}+1}$$

$$G_0 = \frac{I_S}{\phi_t}$$

$$\frac{\overline{I_{d,heat}^2}}{4kTG_0\Delta f} = \frac{\varepsilon}{\left[1+\varepsilon\left(q_{IS}^{'}-q_{ID}^{'}\right)\right]^2}\left\{q_{IS}^{'}{}^2 - q_{ID}^{'}{}^2 + \varepsilon i_d\left(q_{IS}^{'}-q_{ID}^{'}\right) - \varepsilon i_d\left(1-\frac{\varepsilon i_d}{2}\right)\ln\left(\frac{q_{IS}^{'}+1-\frac{\varepsilon i_d}{2}}{q_{ID}^{'}+1-\frac{\varepsilon i_d}{2}}\right)\right\}$$

Noise short channel MOSFET > Noise computed using expression of total charge

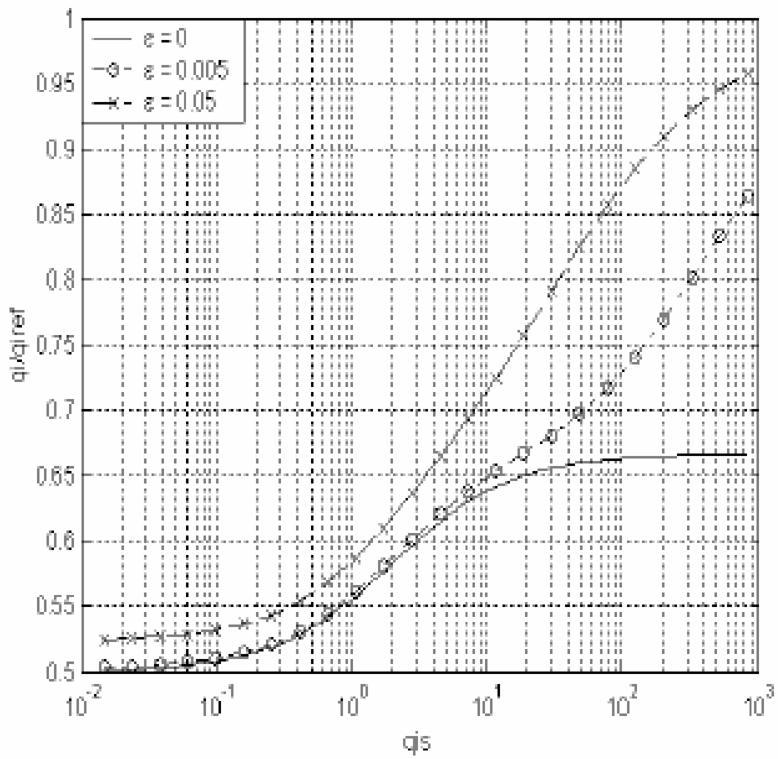

Figure 2 : Channel inversion charge (normalized to the inversion charge for $V_{DS} = 0$) vs. inversion charge density at source for short-channel coefficients equal to 0 ($L \rightarrow \infty$), 0.005 (L=1µm) and 0.05 (L=0.1µm).

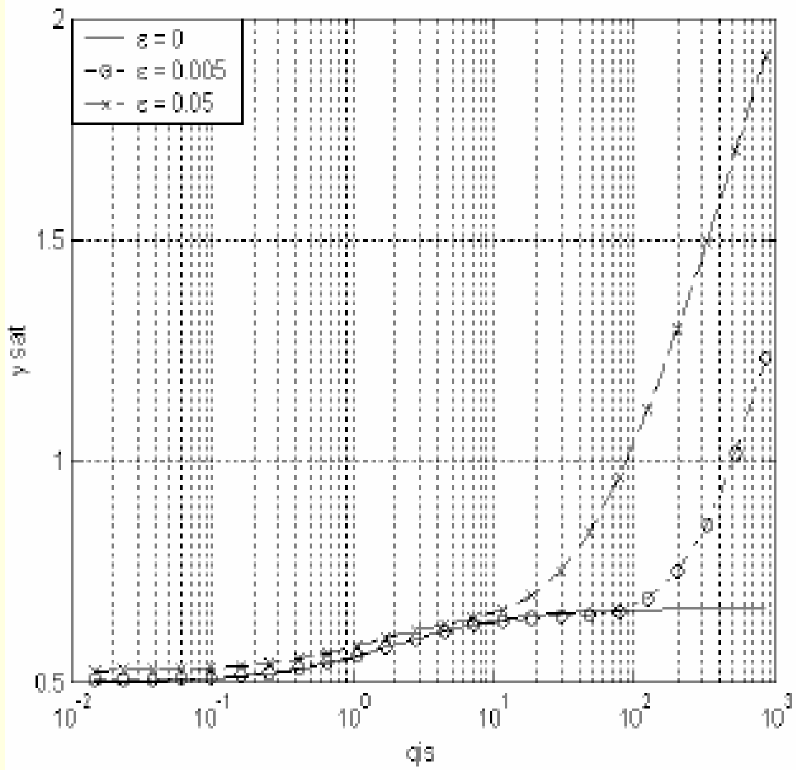

Figure 3 : Gamma factor vs. inversion charge density at source for short-channel coefficients equal to 0 ($L \rightarrow \infty$), 0.005 (L=1µm) and 0.05 (L=0.1µm).