

Compact Modeling of Spiral Inductors for RF Applications

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Outline

- **Background**
- Nonuniform Current Distribution in Metal Line
- Modeling of Inductors (single-ended inductor, differential inductor, transformer/balun)
- Conclusions

Motivations

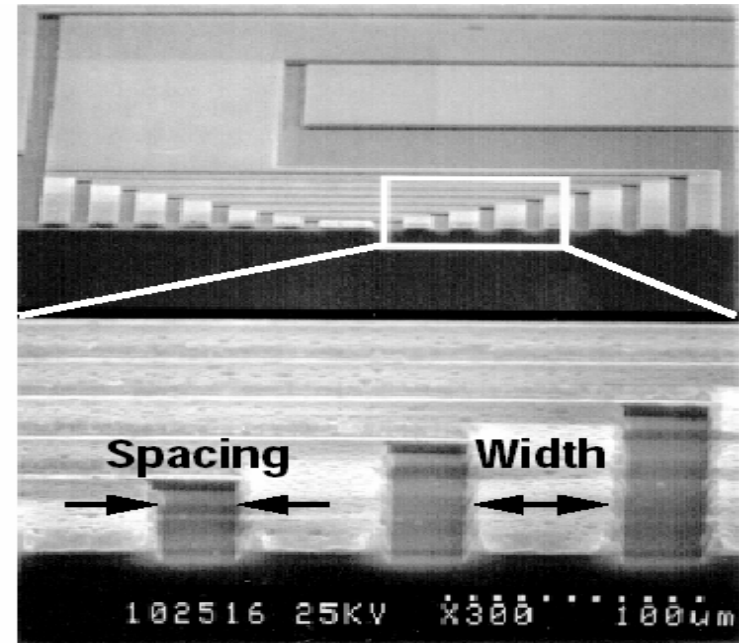
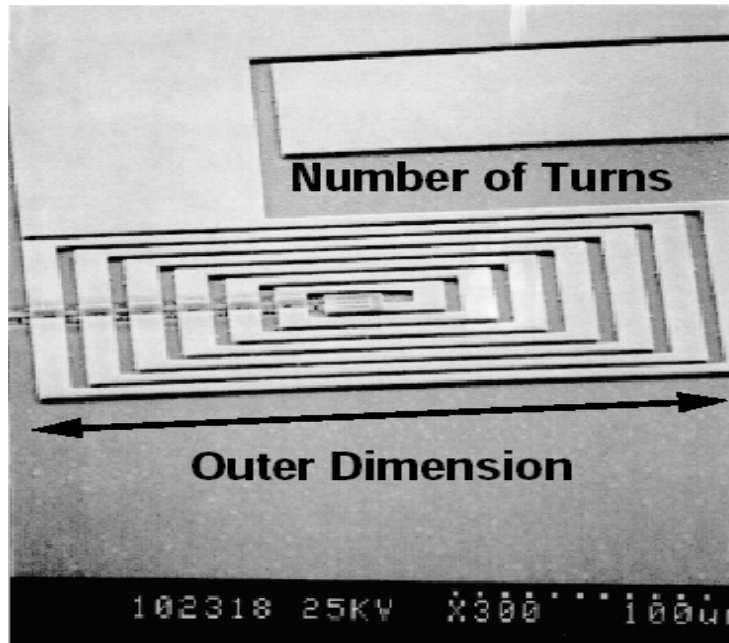
- On-chip passive components are necessary and imperative adjuncts to most RF electronics. These components include inductors, capacitors, varactors, and resistors
- For example, the Nokia 6161 cellphone contains 15 IC's with 232 capacitors, 149 resistors, and 24 inductors
- Inductors in particular are critical components in low noise amplifiers, oscillators and other tuned circuits
- The lack of an accurate and scalable model for on-chip spiral inductors presents a challenging problem for RF IC's designers

Challenges

While inductors are simple in physical structure, their design and modeling encounter the following challenges:

- Electromagnetic field effect → nonuniform current distribution in the metal lines (i.e., skin and proximity effects)
- Model components are bias as well as frequency dependent
- Significant substrate coupling, particularly for coils placed on a lossy substrate such as silicon
- A wide range of dimensions and patterns → complex mutual inductance and capacitance due to adjacent metal lines
- Different topologies (i.e., coils placed on top, inside, or bottom of the wafer)

Typical Square Shaped Spiral Inductor built on Si Substrate

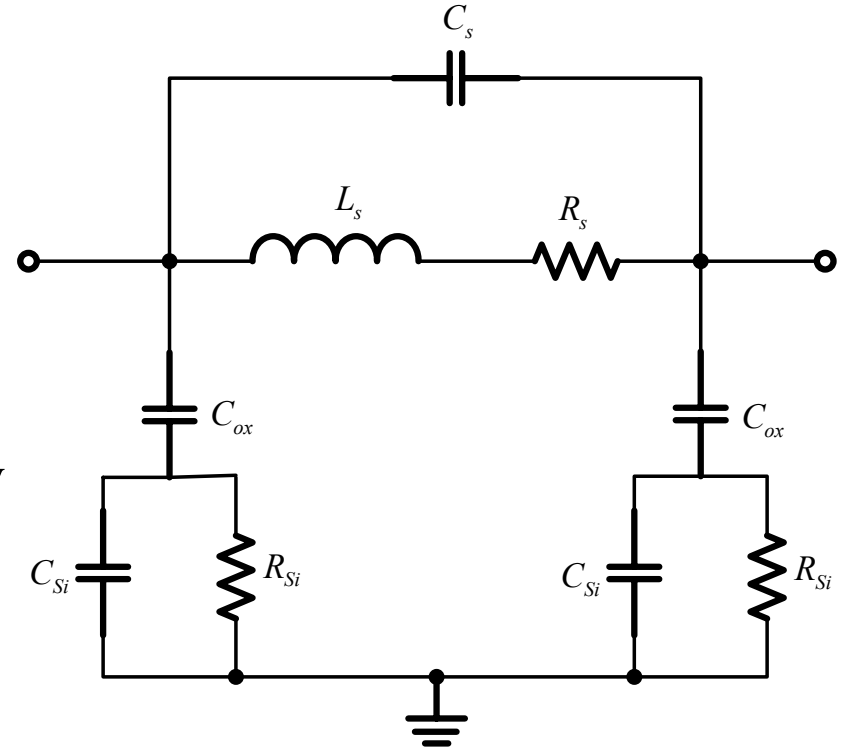


On-chip spiral inductors are used when a relatively small inductance (i.e., several nH) is needed. Otherwise off-chip inductors are used

Performance of the spiral inductor depends on the number of turns, line width, spacing, pattern shape, number of metal layers, oxide thickness and conductivity of substrate

Conventional Compact Model

- **Advantages:**
 - Compact
 - Suitable for SPICE simulation
- **Disadvantages:**
 - Frequency-independent components (i.e., lumped components) used
 - Parameters are normally extracted from S-parameters at the operating frequency (no predictability and not scalable)
 - Parasitics associated with the overlaps and underpasses are not accounted for

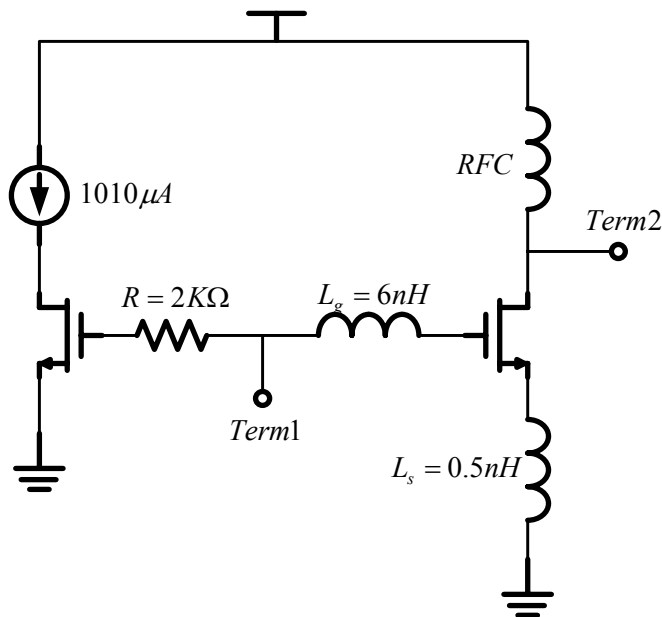


L_s consists of the self inductance, positive mutual inductance, and negative mutual inductance, C_s is the capacitance between metal lines, R_s is the series resistance of the metal line, C_{OX} is the capacitance of oxide layer underneath the spiral, and R_{Si} and C_{Si} are the coupling resistance and capacitance associated with the substrate

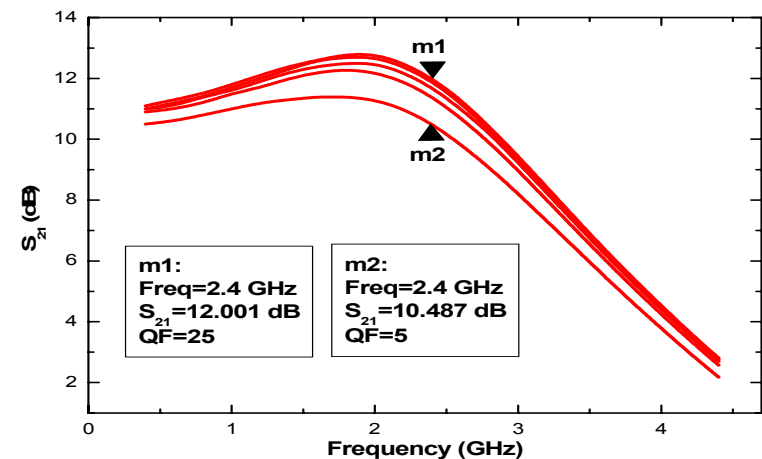
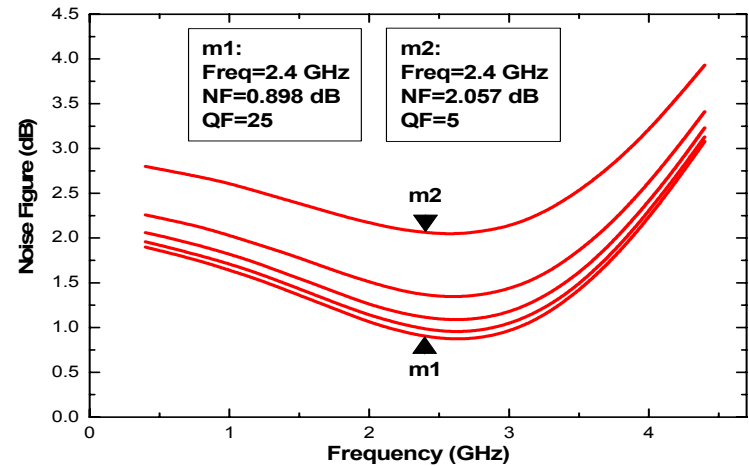
Effect of Inductor Q factor on Circuit Performance

$$Q = \omega \cdot \left(\frac{\text{Energy Stored}}{\text{Average Power Dissipated}} \right)$$

It describes how good an inductor can work as an energy-storage element



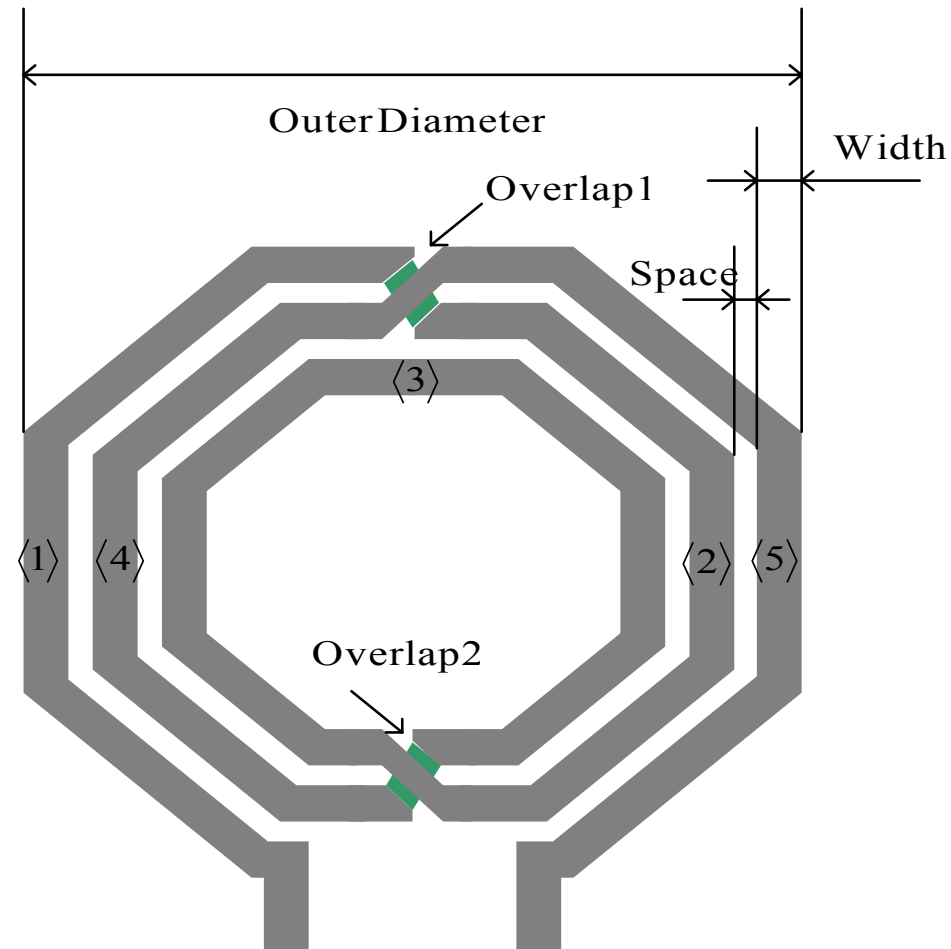
Low Noise Amplifier



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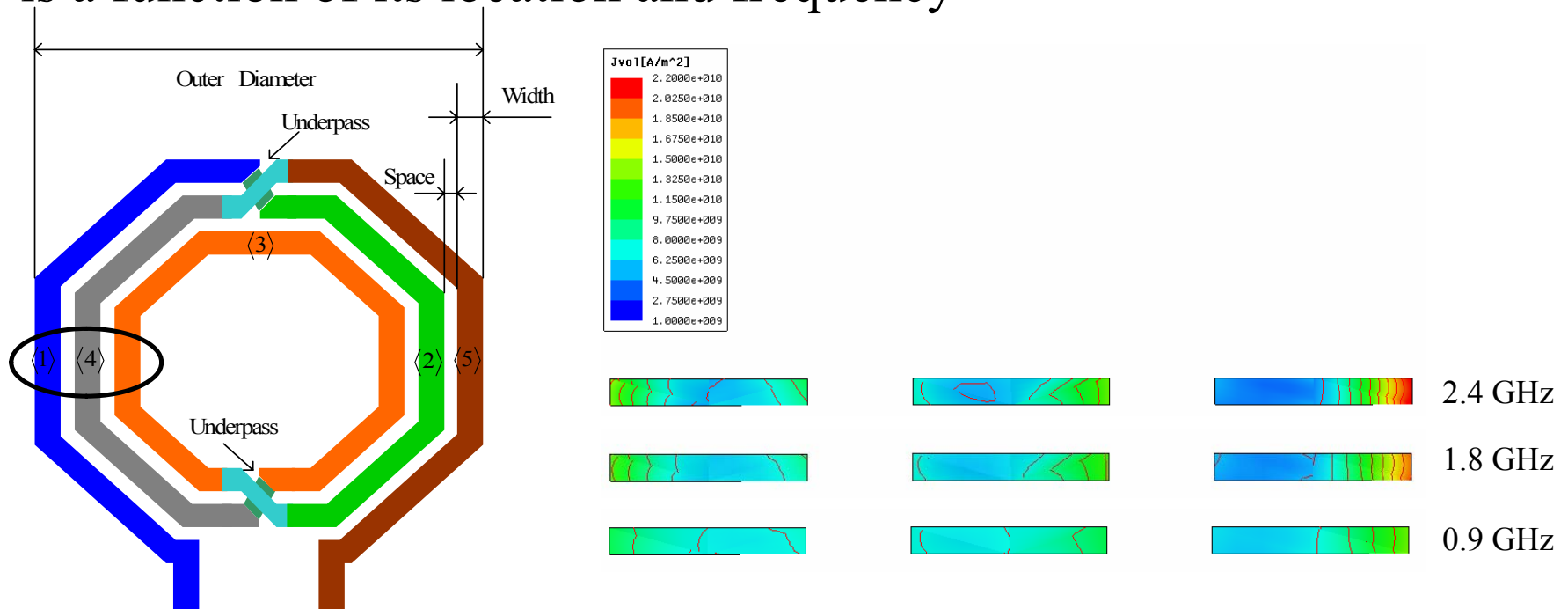
Inductor (Single-Ended) Structure



An octagonal, 3-turn, and symmetrical spiral inductor (on Si substrate)

Nonuniform Current Distribution in Metal Line

Due to the EM fields, the current in a metal line is not uniform, but is a function of its location and frequency



The current crowding cannot be explained by the conventional skin effect, which suggests a skin depth of 1.67/1.92/2.72 μm for all lines for frequencies of 2.4/1.8/0.9 GHz (the line width is 15 μm and thickness is 2 μm). The skin depth is the distance in a metal where the current is decreased exponentially to one time-constant from its max

value at the line edge:

$$\delta = \sqrt{\frac{\rho_m}{\pi \cdot \mu_0 \cdot f}}$$

Effective Metal Line Width

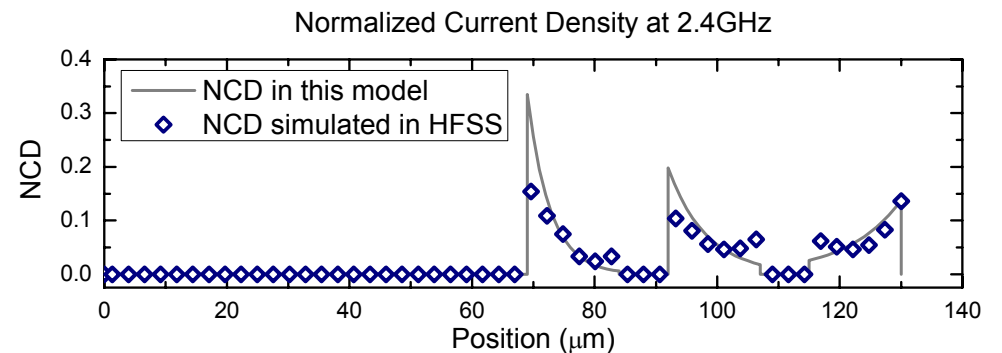
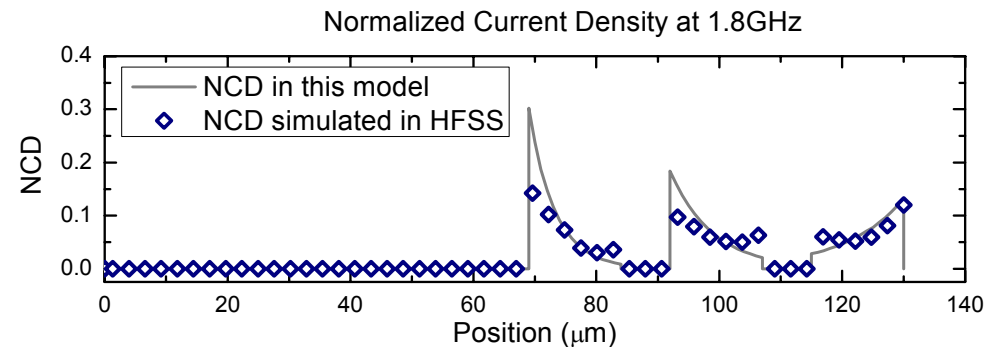
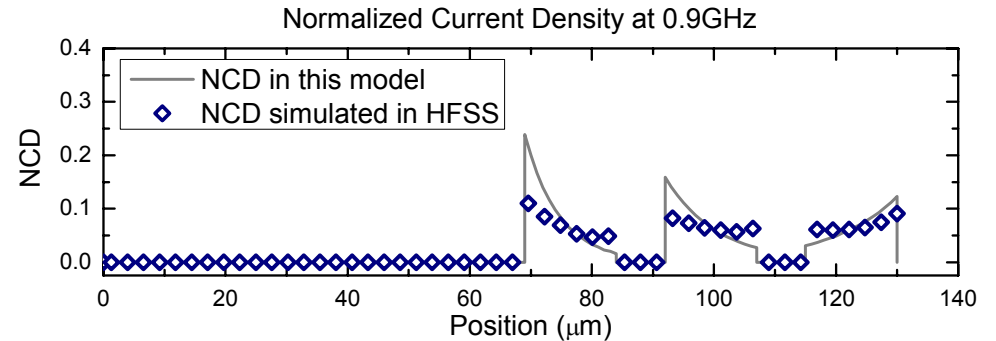
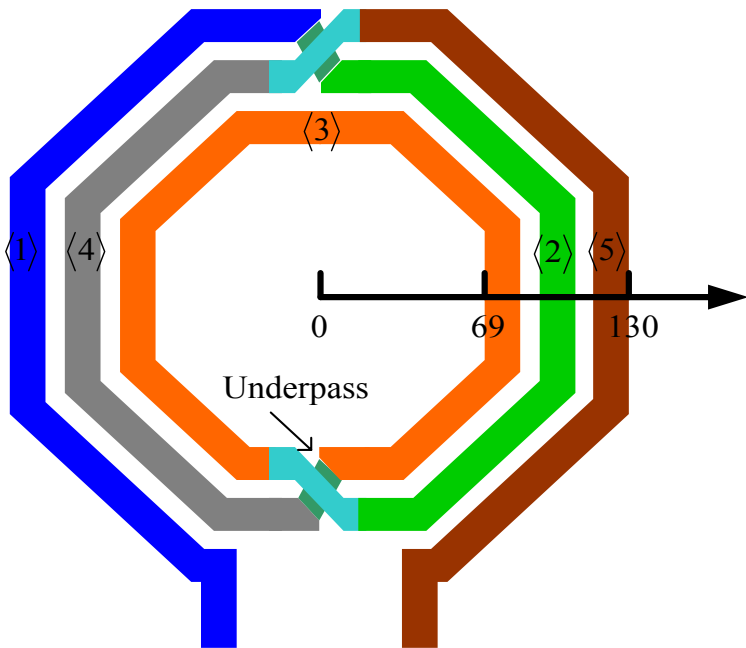
Current distribution in a metal line exhibits an exponential decay from the inner edge (side of metal line closer to the center of spiral) to the outer edge (side of metal line farther away from the center of spiral). Furthermore, this exponential-decay distribution is more prominent in the inner turns (i.e., segments 2, 3, and 4), and the distribution becomes more uniform in the outer turns (i.e., segments 1 and 5). This current crowding phenomenon can be described by the effective line width W_{eff} :

$$w_{eff} = W_{0,i} \left(1 - \exp \left(- \frac{w}{W_{0,i}} \right) \right) \quad W_{0,i} = c1 \cdot c2^{i-1} \sqrt{\frac{1}{f}}$$

W is the line width, f is frequency, i is the turn index, and $c1 = 0.653$ and $c2 = 0.53$ are obtained from the following function that produces the smallest averaged error D :

$$D = \sqrt{\sum_l \left(\frac{Q_{measure}(f_l) - Q_{model}(f_l)}{Q_{measure}(f_l)} \right)^2 + \sum_l \left(\frac{L_{measure}(f_l) - L_{model}(f_l)}{L_{measure}(f_l)} \right)^2}$$

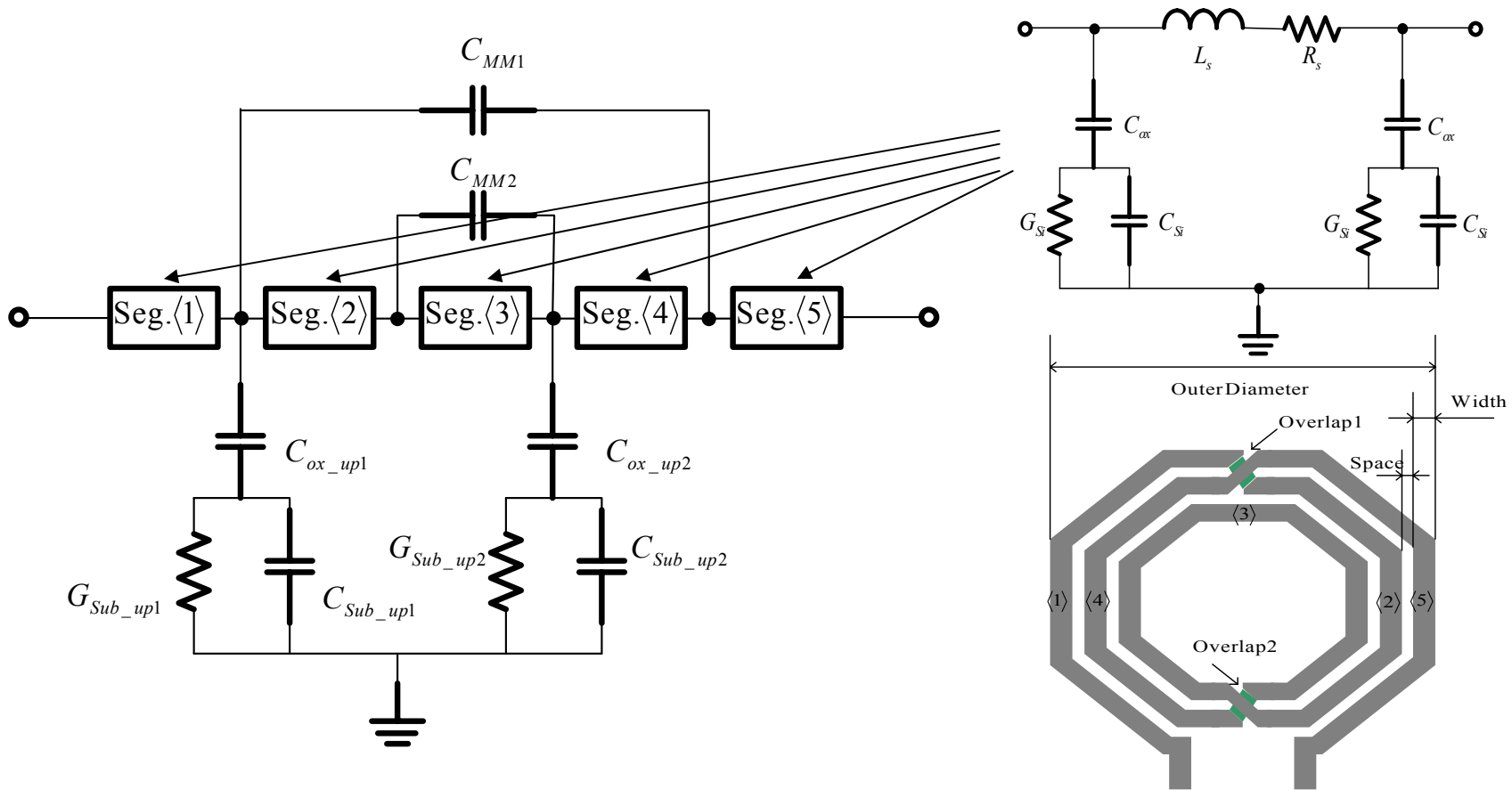
Comparison of Modeled and Simulated Current Distributions in Metal Lines



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Model of Single-Ended (Two-Terminal) Inductor



- There are five segments in the inductor, separated by the overlaps, and each is represented by the conventional lumped model on the right-hand side
- All other components in the left-hand side equivalent circuit represent the parasitics associated with the two overlaps (subscripts 1 and 2 denote overlaps 1 and 2, respectively)
- Parasitics associated with the Si substrate are modeled with G_s and C_s in the lumped circuit

Modeling the Segment Box (Inductance)

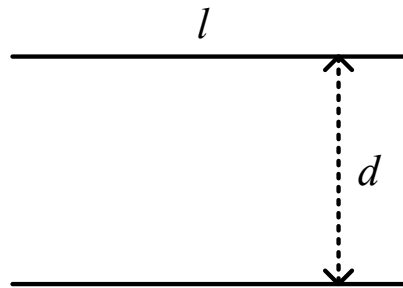
The metal track in each segment can be further divided into several straight metal lines (for example, 5 straight metal lines for segment 4), so that the inductance L_{S_lines} of each straight metal line can be expressed as the self inductance L_{line_self} plus the mutual inductance M from all other metal lines:

$$L_{S_line} = L_{line_self} + \sum M$$

$$L_{line_self} = 2l \left(\ln \frac{2l}{w_{eff} + t} - 0.5 \right)$$

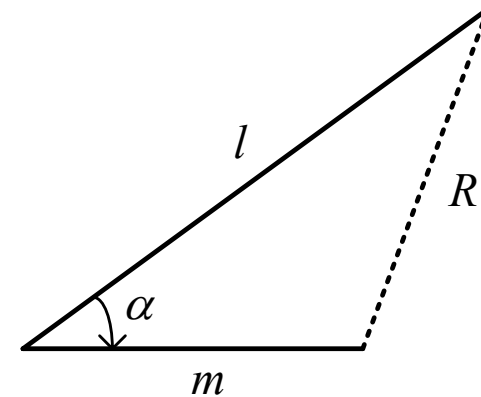
Mutual inductance depends on the relationship of two metal lines

Modeling the Segment Box (Mutual inductance)

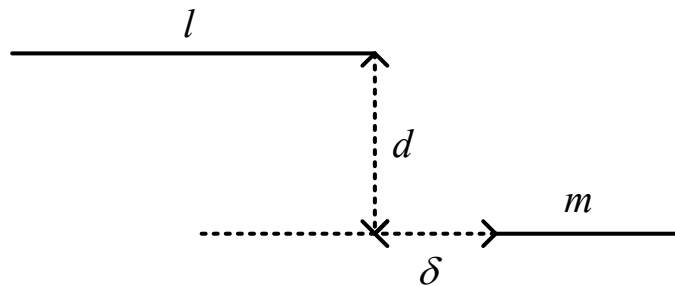


(a)

$$M_p(l) = 2l \left[\ln \left(\frac{l}{d} + \sqrt{1 + \frac{l^2}{d^2}} \right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right]$$



(c)

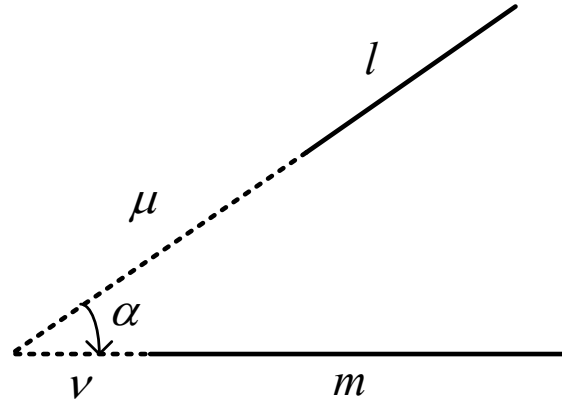


(b)

$$M_i(m, l) = \cos(\alpha) \cdot \left[l \cdot \ln \left(\frac{l + m + R}{l - m + R} \right) + m \cdot \ln \left(\frac{l + m + R}{m - l + R} \right) \right]$$

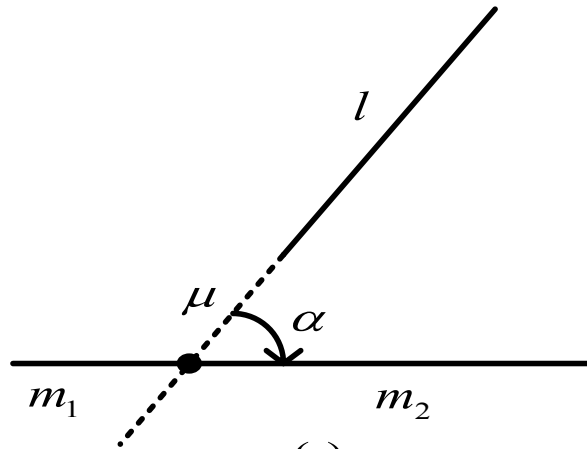
$$2M'_p = [M_p(l + m + \delta) + M(\delta)] - [M(l + \delta) + M(m + \delta)]$$

Modeling the Segment Box (Mutual inductance)



(d)

$$M'_i = [M_i(\mu + l, \nu + m) + M_i(\mu, \nu)] - [M_i(\mu + l, \nu) + M_i(\nu + m, \mu)]$$



(e)

$$M''_i = [M_i(m_1, \mu + l) + M_i(m_2, \mu + l)] - [M_i(m_1, \mu) + M_i(m_2, \mu)]$$

Modeling the Segment Box (Other components)

Line resistance: Oxide capacitance: Substrate conductance:

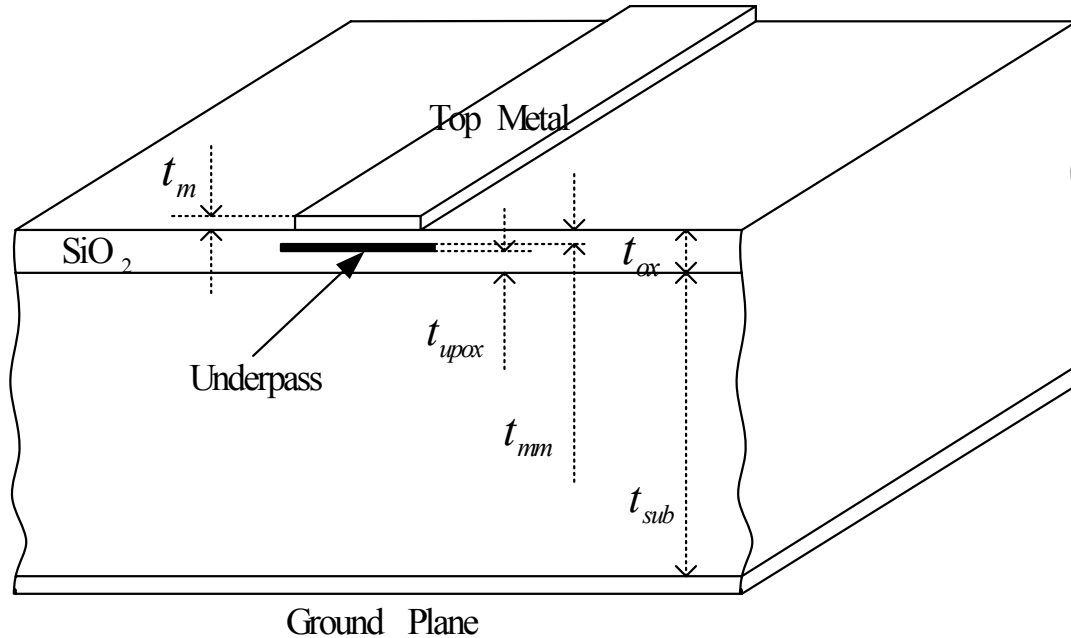
$$R_{line}(f) = \frac{\rho \cdot l}{w_{eff} \cdot t_m} \quad C_{ox}(f) = \frac{\epsilon_0 \epsilon_{eff}(f)}{2F(w, t_{ox})} \cdot l \quad G_{sub} = \frac{\sigma_{sub} [1 + (1 + 10 t_{sub}/w)^{-1/2}]}{2F(w, t_{sub})} \cdot l$$

Frequency-dependent dielectric permittivity is needed:

$$\epsilon_{eff}(f) = \epsilon_{ox} - \frac{\epsilon_{ox} - \epsilon'}{1 + (f/f_c)^2} \quad f_c = \frac{c^2 \epsilon_0 Z_0 \epsilon_{ox}^{1/2}}{2 t_{ox} \epsilon'^{1/2}} \quad Z_0 = \frac{120\pi \cdot F(w, t_{ox})}{\epsilon'^{1/2}}$$

$$F(w, t) = \begin{cases} \frac{1}{2\pi} \ln\left(\frac{8t}{w} + \frac{w}{4t}\right) & \text{for } t/w > 1 \\ \frac{1}{w/t + 2.42 - 0.44t/w + (1 - t/w)^6} & \text{for } t/w < 1 \end{cases}$$

Modeling the Overlap Components



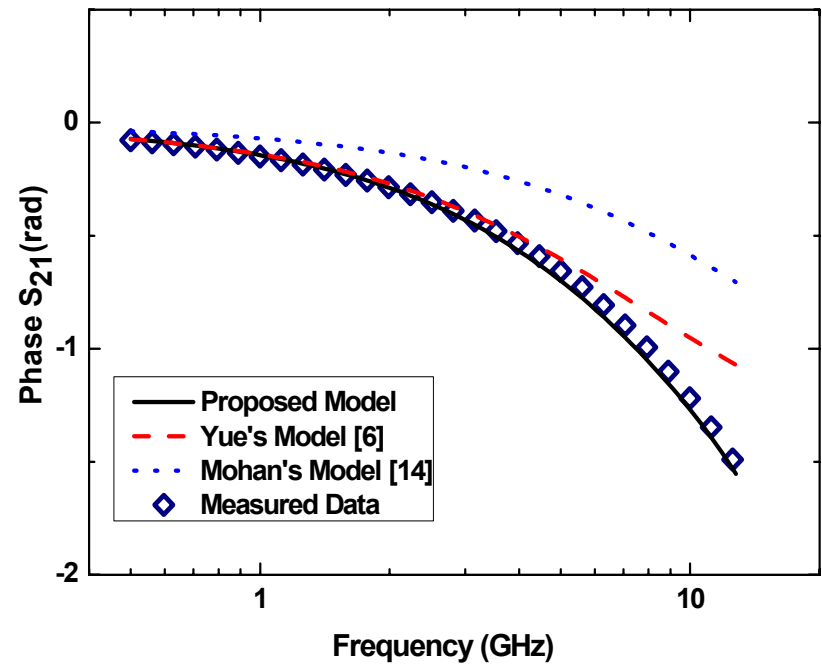
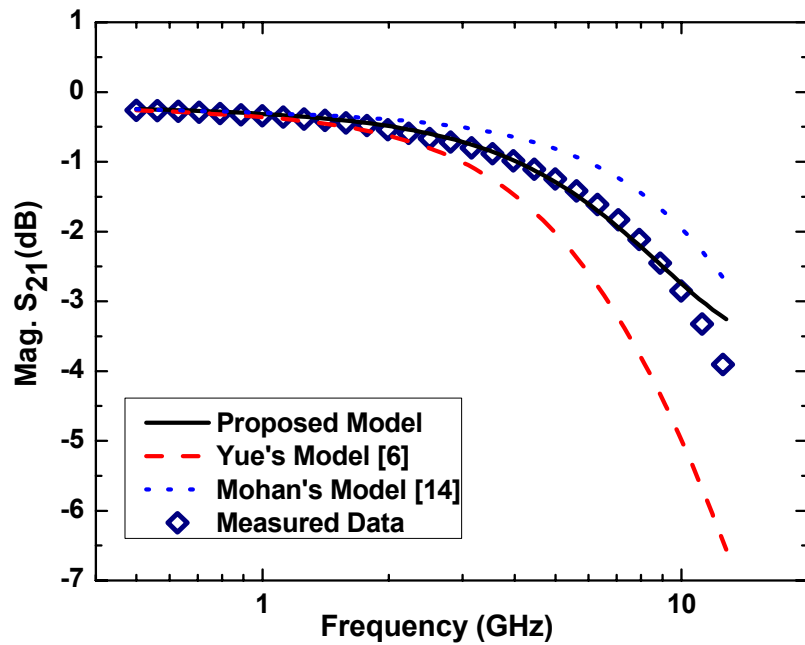
$$C_{ox_up} = \frac{\epsilon_0 \epsilon_{ox} (\text{underpass area})}{t_{upox}}$$

$$C_{sub_up} = \frac{\epsilon_0 \epsilon_{ox} (\text{underpass area})}{t_{sub}}$$

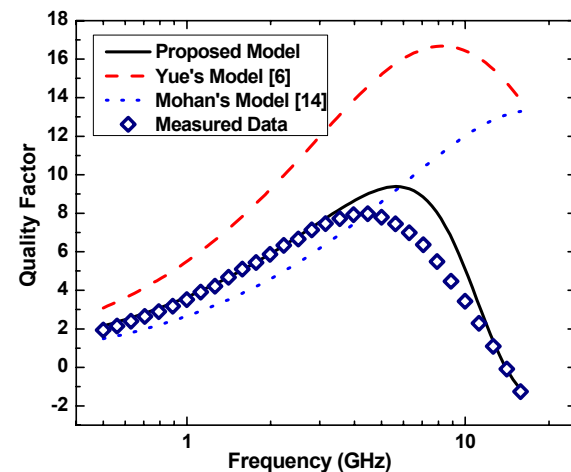
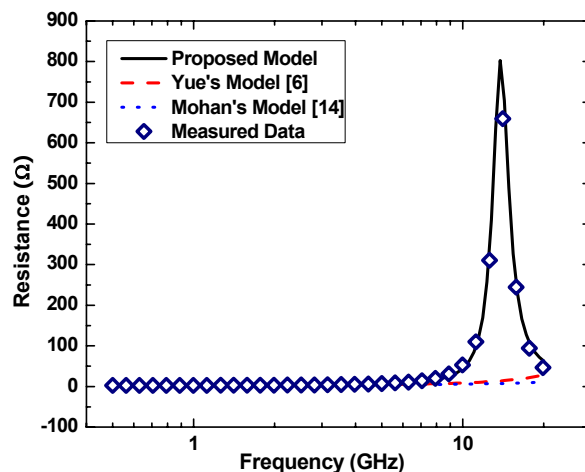
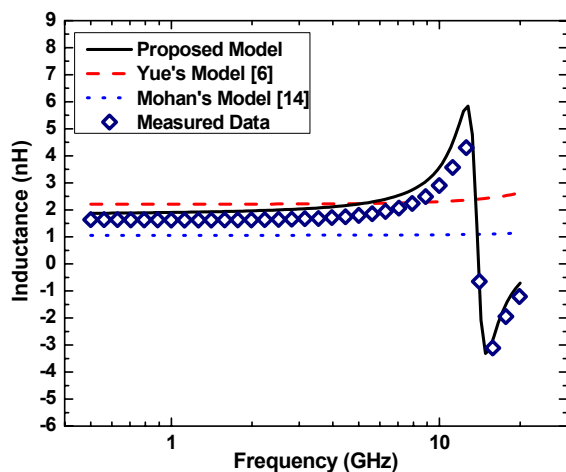
$$C_{mm} = \frac{\epsilon_0 \epsilon_{ox} (\text{overlap area})}{t_{mm}}$$

$$G_{sub_up} = \frac{\text{underpass area}}{\rho_{sub} \cdot t_{sub}}$$

Results of Single-Ended Inductor



Results of Single-Ended Inductor

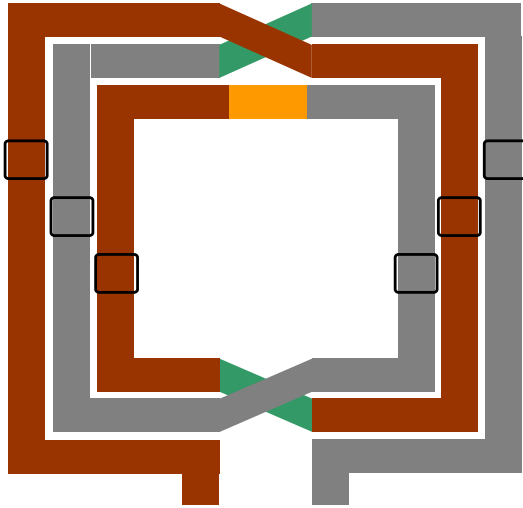


$$L = \frac{\text{Im}\left(\frac{1}{Y_{11}}\right)}{2 \cdot \pi \cdot f}$$

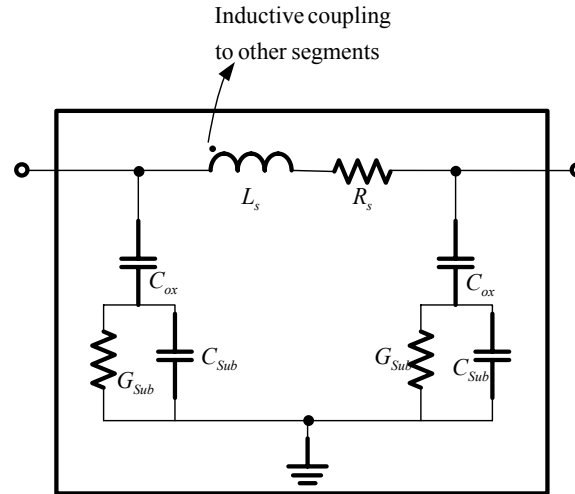
$$R = \text{Re}\left(\frac{1}{Y_{11}}\right)$$

$$Q = \frac{\text{Im}(Y_{11})}{\text{Re}(Y_{11})}$$

Model of Differential (Three-Terminal) Inductor



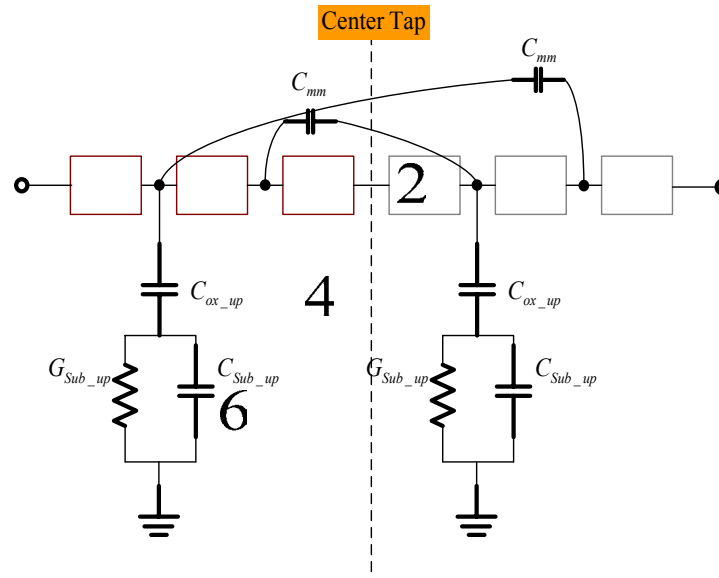
- Spiral 1
- Spiral 2
- Underpass
- Center Tap



Equivalent circuit for each line segment

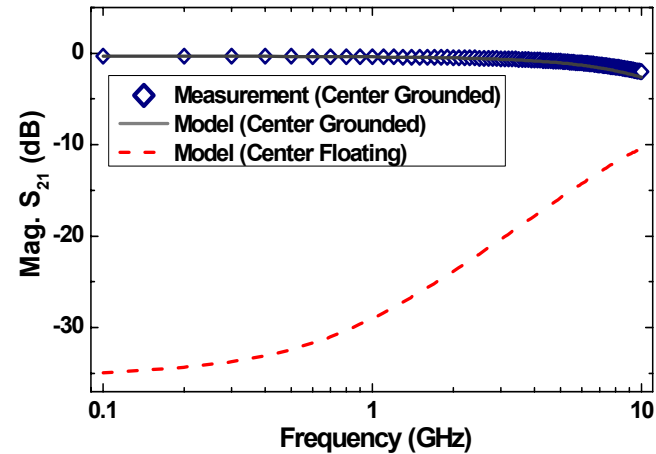
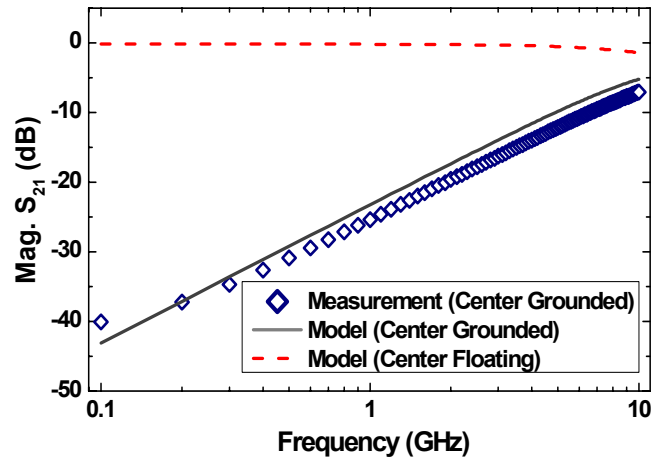
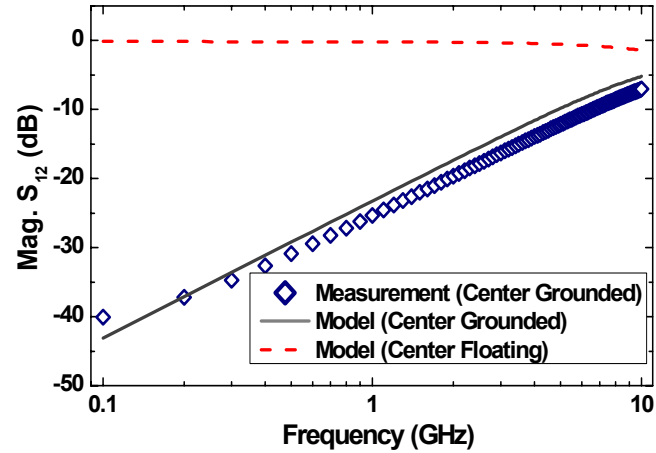
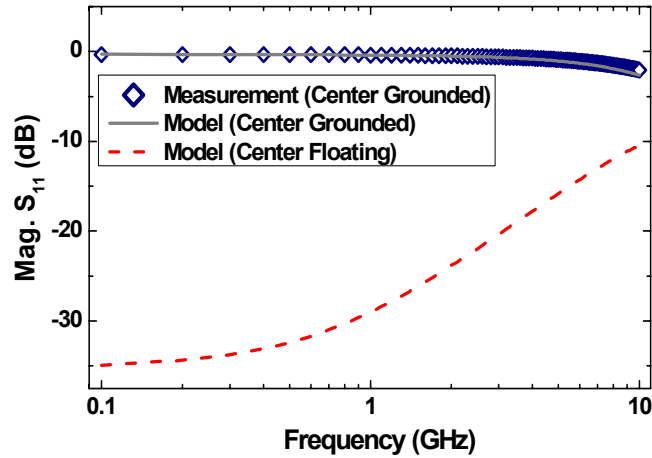
There are six line segments in the inductor, separated by the overlaps and center tap

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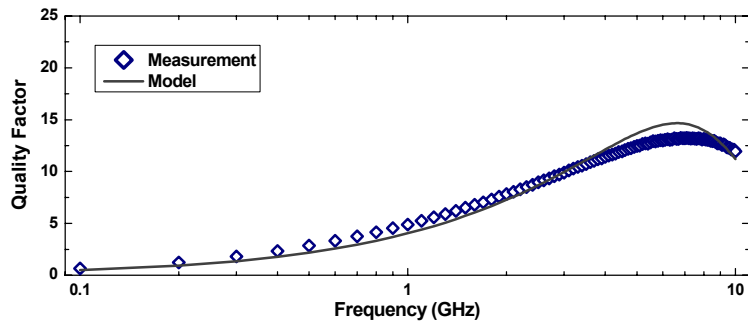
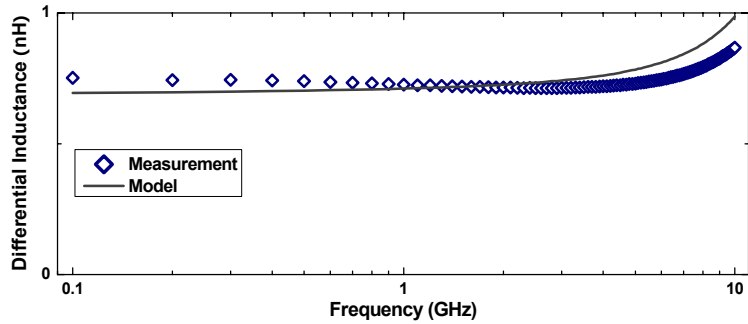
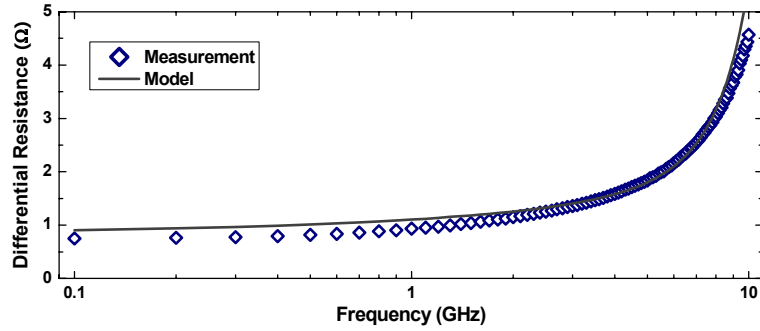


Complete equivalent circuit for the differential inductor

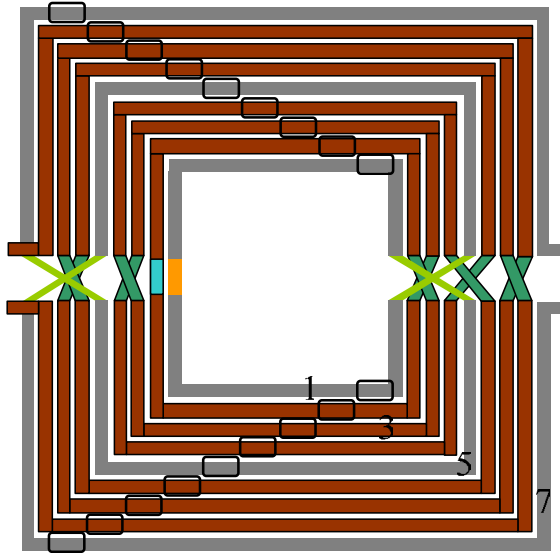
Results of Differential Inductor



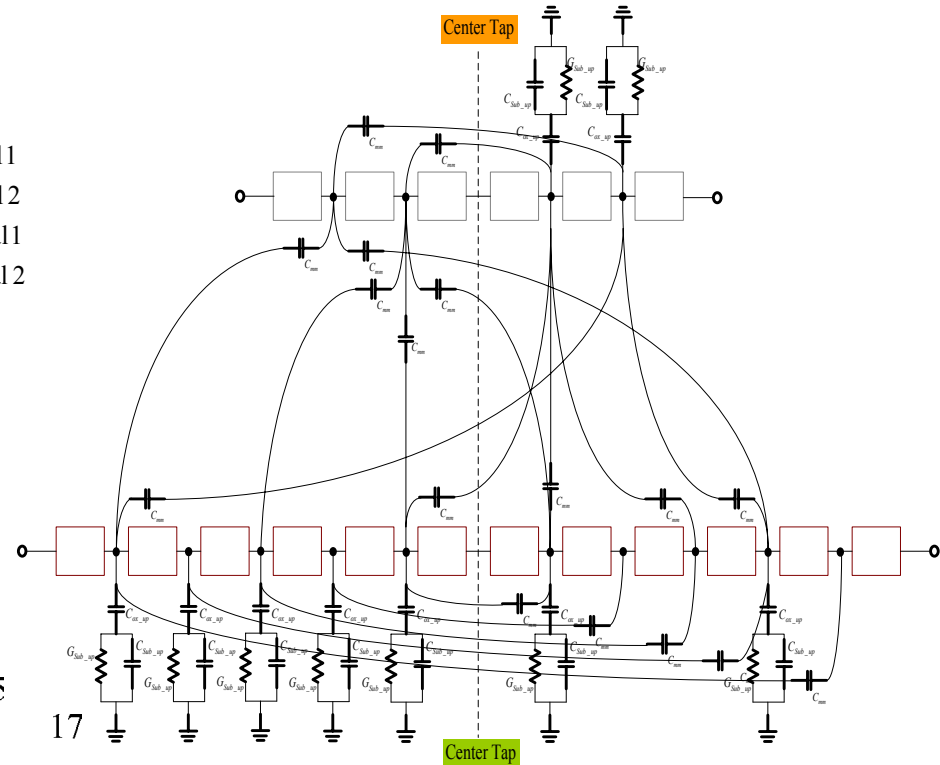
Results of Differential Inductor



Model of Transformer/Balun (Four Terminals)



- Spiral 1
- Spiral 2
- Underpass Spiral 1
- Underpass Spiral 2
- Center Tap Spiral 1
- Center Tap Spiral 2

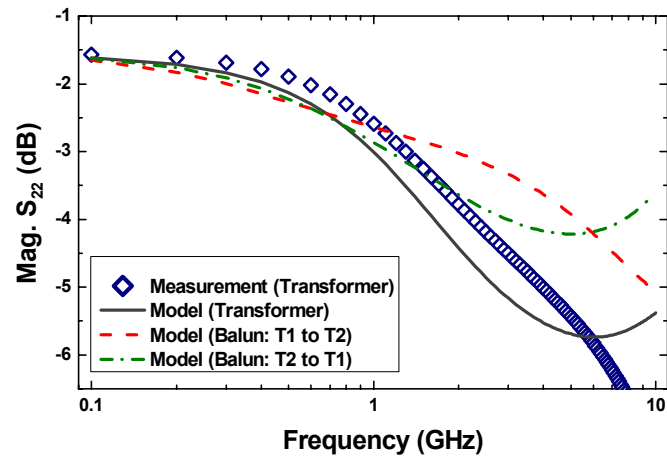
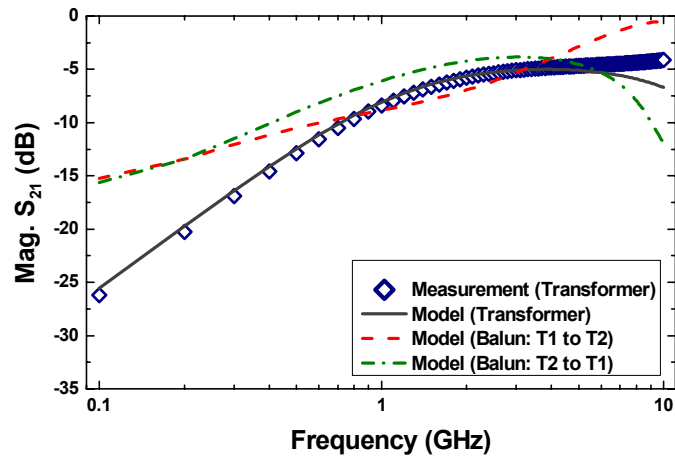
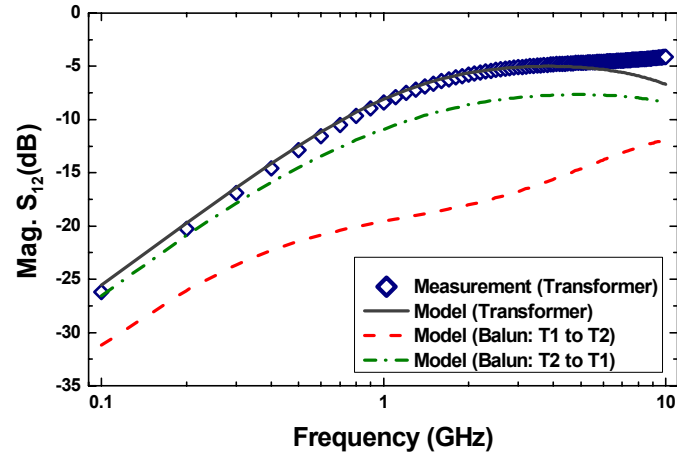
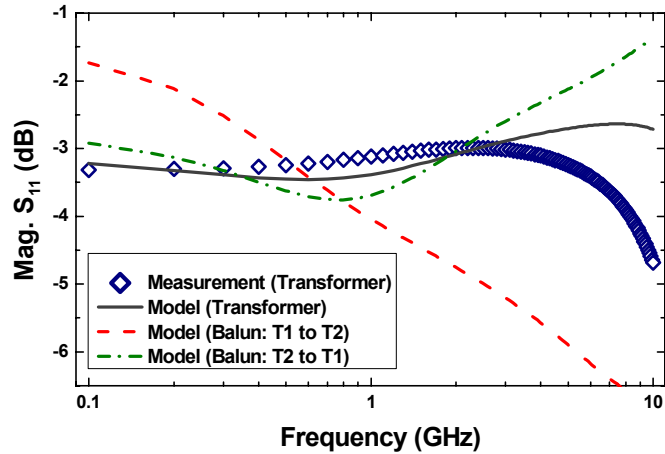


There are 18 line segments in the transformer, separated by the overlaps and center taps

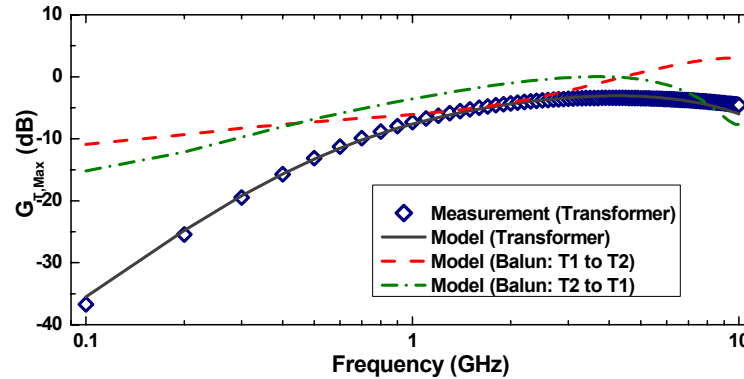
Complete equivalent circuit for the transformer/balun

The structure is called the balun when one of the center tap is grounded and the other is floating

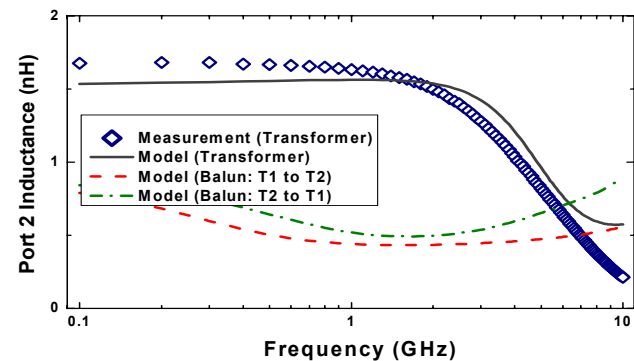
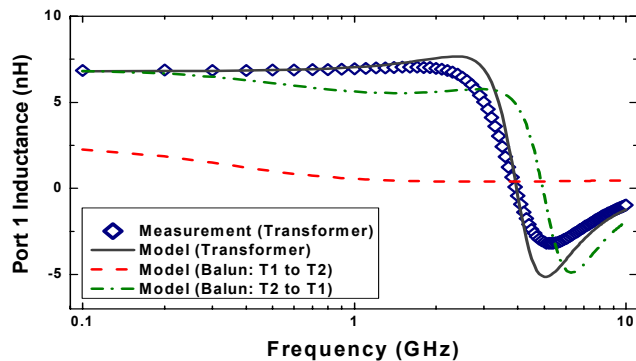
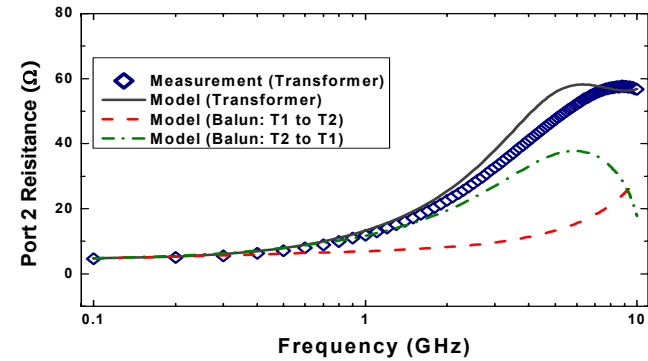
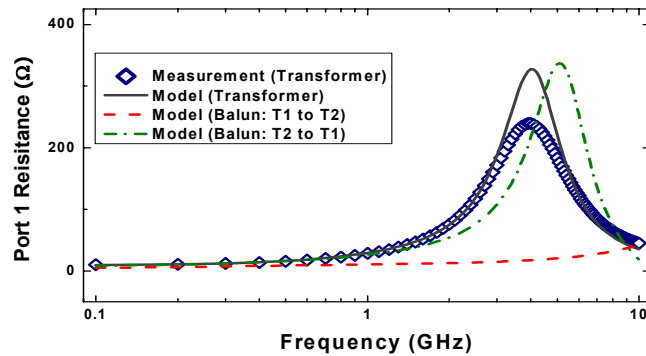
Results of Transformer/Balun



Results of Transformer/Balun



Port-matched transducer



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Conclusions

- Spiral inductors are often the components limiting the performance of RF ICs
- Concepts for design and modeling of spiral inductors have been reviewed
- An improved and scalable inductor model has been presented. The model is physics-based, scalable, applicable for different spiral patterns and geometries, and accounts for the parasitics associated with the overlaps and underpasses