An Improved MOS Transistor Model with an Integrated Mobility Model

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Outline

- MOS models
- Mobility models
- Incorporating mobility into basic equations
- New MOS model
- Some results
- Comparison with some experimental data
- Possible extensions of model

Introduction to Mobility Models

- Basic channel equation

$$I_d = \mu_n W Q_I(V_g, V)\frac{dV}{dx} \quad ; Q_I = \text{Inversion Layer Charge}$$

- Constant mobility

$$I_d = \frac{W}{L}\int \mu_n Q_I(V_g, V)\,dV \;\Rightarrow\; \mu_n \frac{W}{L} f(V_g, V_s, V_d)$$

- Problem is incorporating position (and potential) dependency of mobility in a first-order manner

Typical Approaches to Mobility

- Lateral (or drain) field effects

$$\mu = \frac{\mu_0}{1 + \frac{1}{E_C}\left|\frac{dV}{dx}\right|} \Rightarrow I_d = \frac{\mu_0 (W/L) f(V_g, V_s, V_d)}{\left[1 + (V_d - V_s)/E_C L\right]}$$

- Vertical (or gate) field effects

$$\Rightarrow I_d = \frac{\mu_0 (W/L) f(V_g, V_s, V_d)}{\left[1 + \theta_1 (V_g - V_T) + \theta_2 (V_g - V_T)^2\right]\left[1 + (V_d - V_s)/E_C L\right]}$$

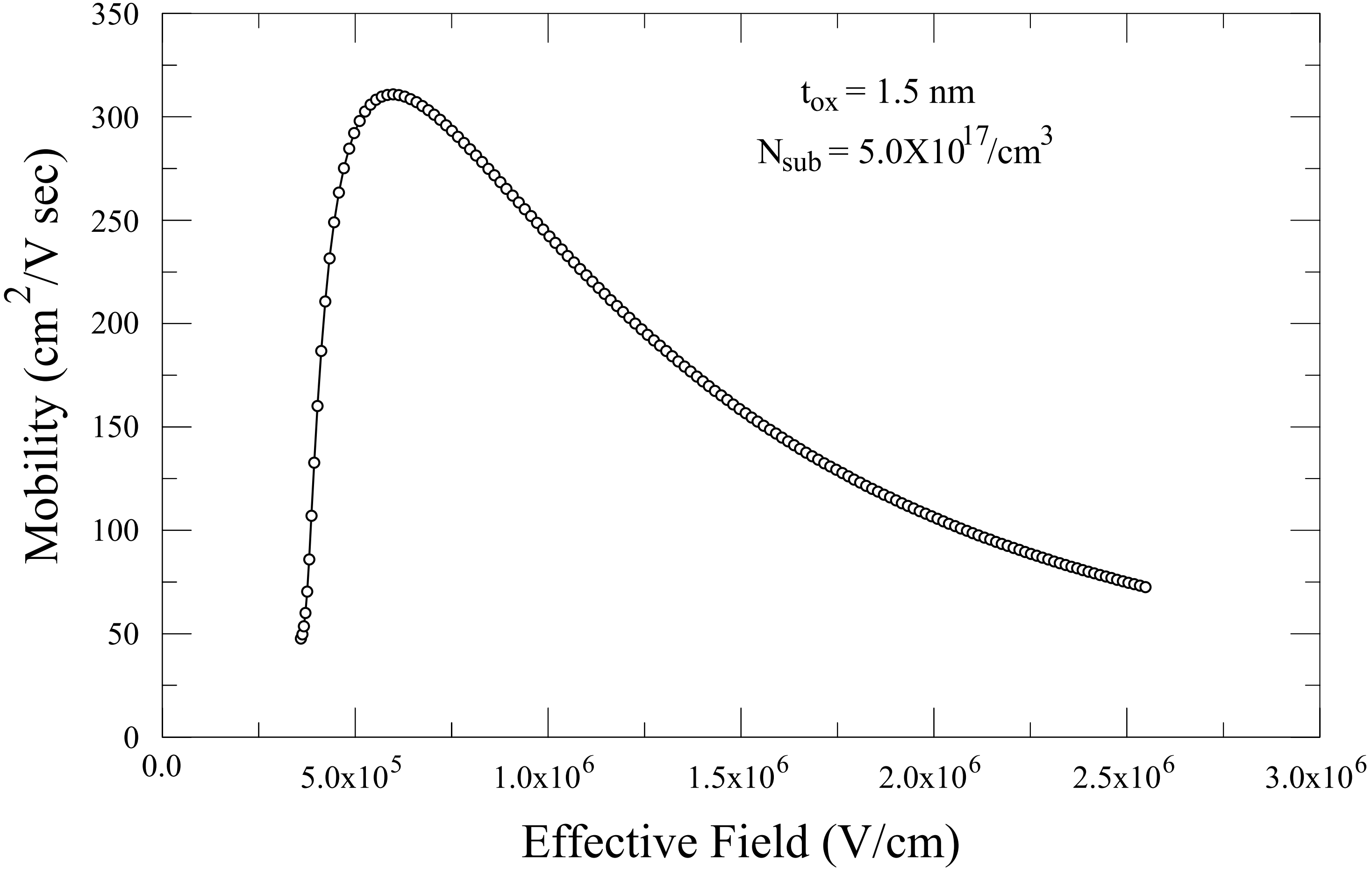
Limitations of Past Approaches

- Treats vertical field reduction as a second-order effect to be included after integrating differential equation
- Typically treats vertical field reductions and lateral field reductions as multiplicative effects
- As devices have been scaled to smaller dimensions, both vertical and lateral fields have increased making mobility reductions more important
- Improved understanding of surface mobility physics has not been integrated into MOS device models

Physics of vertical field mobility effects

- Surface mobility falls off at low vertical fields and high vertical fields
- Low field fall-off is due to bulk impurity scattering and surface charge scattering
- High field fall-off is due to surface roughness scattering where

$$\mu_{sr} = K_{sr} / E_{eff}^2 \;\; ; E_{eff} = \text{Effective surface electric field}$$



Typical variation of surface mobility with effective surface electric field

Modeling of high field mobility fall-off

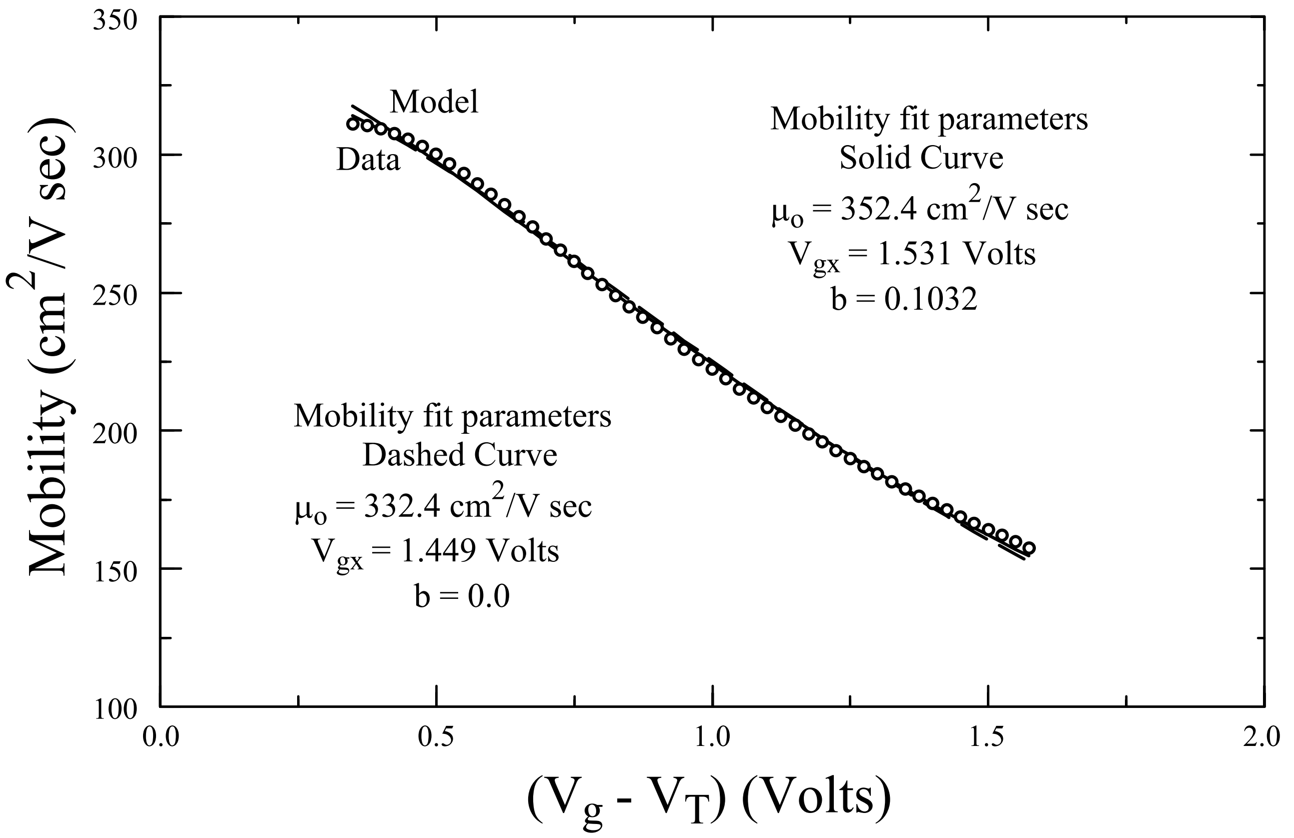
- Using

$$E_{eff} = (\eta Q_i + Q_d)/\varepsilon_{si} \;\; ; \; Q_d = \text{Depletion layer charge}$$

$$Q_i \simeq C_{ox}(V_g - V_T) = (\varepsilon_{ox}/t_{cox})(V_g - V_T)$$

- Results in

$$\frac{1}{\mu} = \frac{1}{\mu_o} + \frac{1}{\mu_{sr}} = \frac{1}{\mu_o}\left[1 + \left(\frac{V_g - V_T}{V_{gx}} + b\right)^2\right]$$



Fall-off in mobility at large effective surface fields. Data fitted to equation with b = 0 and with best-fit value of b

Combining vertical field fall-off with lateral field fall-off (v saturation)

- Effects are not multiplicative, but independent – vertical field is most important near source and lateral field is most important near drain

$$\frac{1}{\mu} = \frac{1}{\mu_o}\left[1 + \left(\frac{V_g - V_T - V}{V_{gx}} + b\right)^2 + \frac{1}{E_c}\left|\frac{dV}{dx}\right|\right]$$

$$\frac{I_d}{\mu_o} = \frac{[C_{ox}W(V_g - V_t - \alpha V) - I_d / \mu_o E_c]}{[1 + ((V_g - V_t - \alpha V)/V_{gx} + b)^2]}\;\frac{dV}{dx}$$

MOS model with integrated mobility

- Basic differential equation

$$\frac{I_d}{\mu_o} = \frac{[C_{ox}W(V_g - V_t - \alpha V) - I_d / \mu_o E_c]}{[1 + ((V_g - V_t - \alpha V)/V_{gx} + b)^2]} \frac{dV}{dx}$$

$$\alpha = 1 + \frac{0.5\sqrt{2\varepsilon_{si} q N_b}}{C_{ox}\sqrt{2\phi_F + V_s}} \quad \text{(first-order model of depletion layer charge)}$$

New MOS Model

$$I_d = \frac{I_o}{2}\frac{[\ln(1+y_s^2)-\ln(1+y_d^2)-2b[\tan^{-1}(y_s)-\tan^{-1}(y_d)]}{1+Z\,|\tan^{-1}(y_s)-\tan^{-1}(y_d)|}$$

$$I_o = \mu_o C_{ox}(W/\alpha L)V_{gx}^2,$$

$$y_s = b+(V_g - V_T - \alpha V_s)/V_{gx},$$

$$\mathrm{y}_d = b+(V_g - V_T - \alpha V_d)/V_{gx},$$

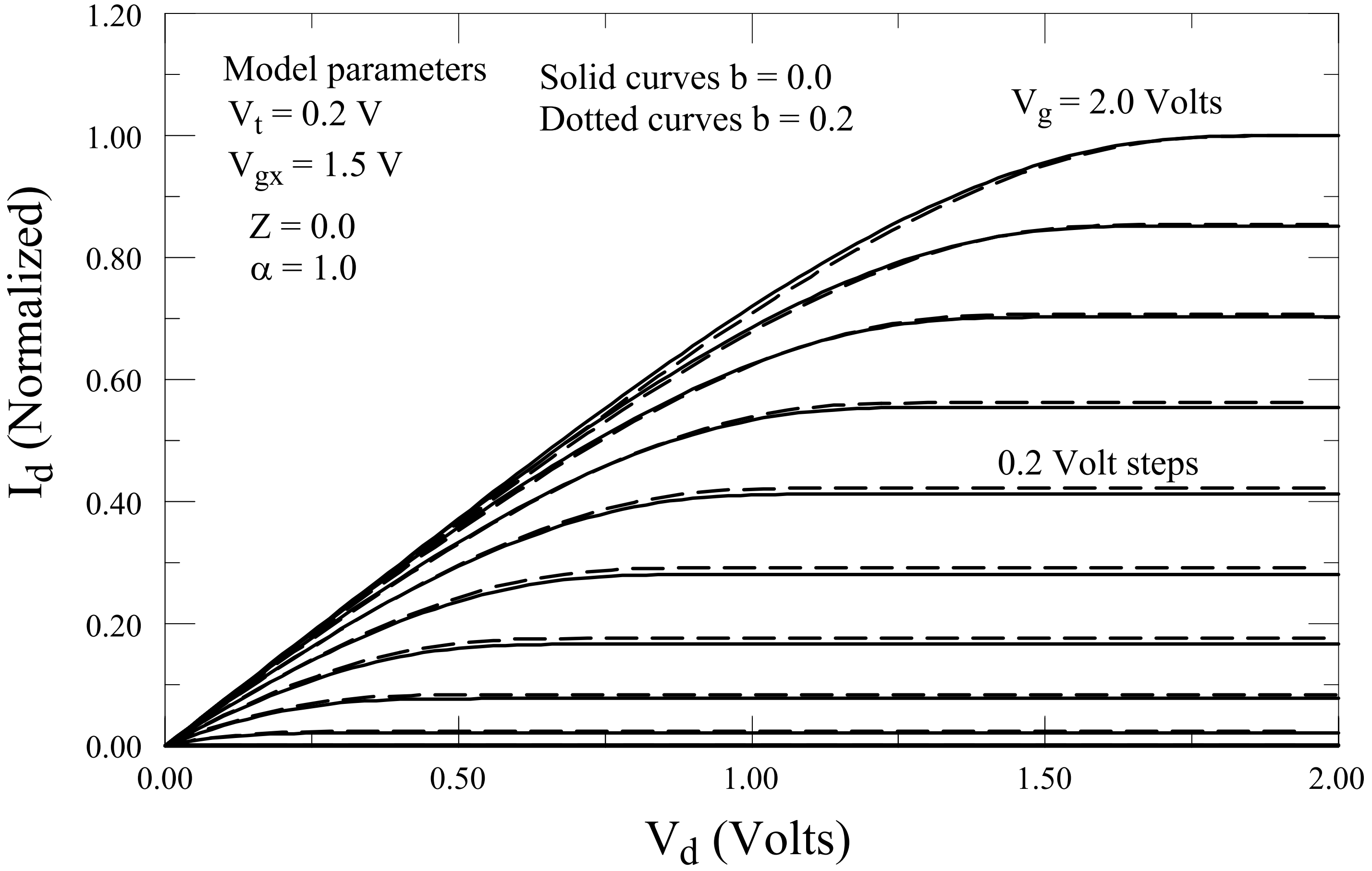
$$\mathrm{Z}{=}\mu_o V_{gx}/(\alpha v_{sat}L) = V_{gx}/V_{dx},$$

$$\text{with } V_{dx}{=}(\alpha v_{sat}/\mu_o)L = \alpha E_c L.$$

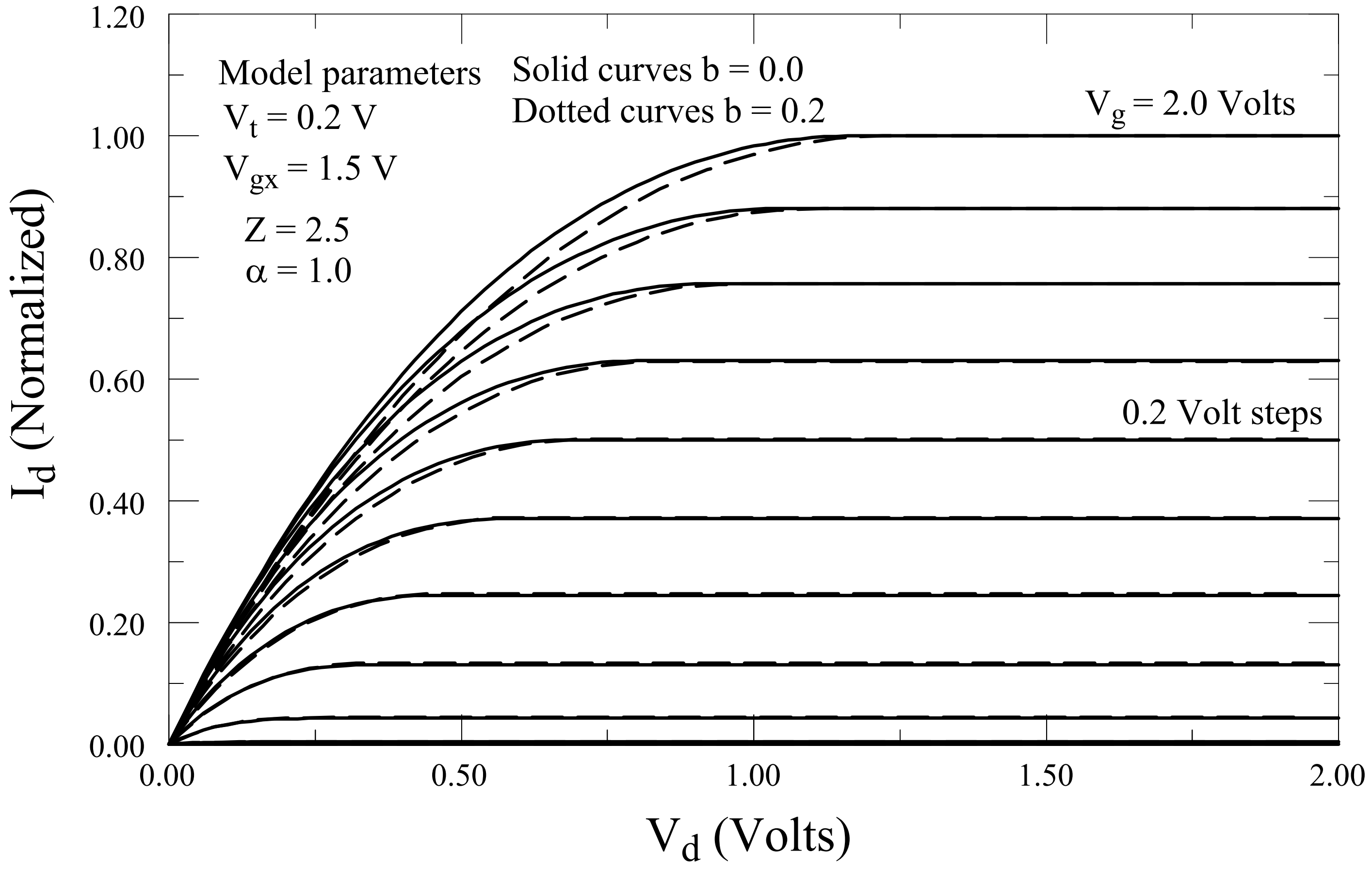
$$I_d = \frac{I_o}{2}\frac{[\ln(1+x_s^2)-\ln(1+x_d^2)]}{1+Z\,|\tan^{-1}(x_s)-\tan^{-1}(x_d)|} \quad \text{for } b=0$$

$$x_s = (V_g - V_T - \alpha V_s)/V_{gx},$$

$$x_d = (V_g - V_T - \alpha V_d)/V_{gx}.$$



Theoretical I-V characteristic for parameters for typical long channel MOS transistor with oxide thickness of about 1.3 nm



Theoretical I-V characteristic for parameters for typical short channel MOS transistor with oxide thickness of about 1.3 nm

Some Features of New Model

- Saturation point determined for b=0 by:

$$x_{dsat} = \frac{Z}{2} \frac{\ln(1+x_s^2) - \ln(1+x_{dsat}^2)}{1 + Z[\tan^{-1}(x_s) - \tan^{-1}(x_{dsat})]}$$

- Values and first derivatives continuous over all voltage values
- Continuous higher order derivatives require slight modifications (or smoothing functions) at zero drain-source voltage and at current saturation point – For details see IEEE Trans on ED, Dec 2005, pp. 2640-2647
- Requires additional factors to account for finite slope in current saturation region

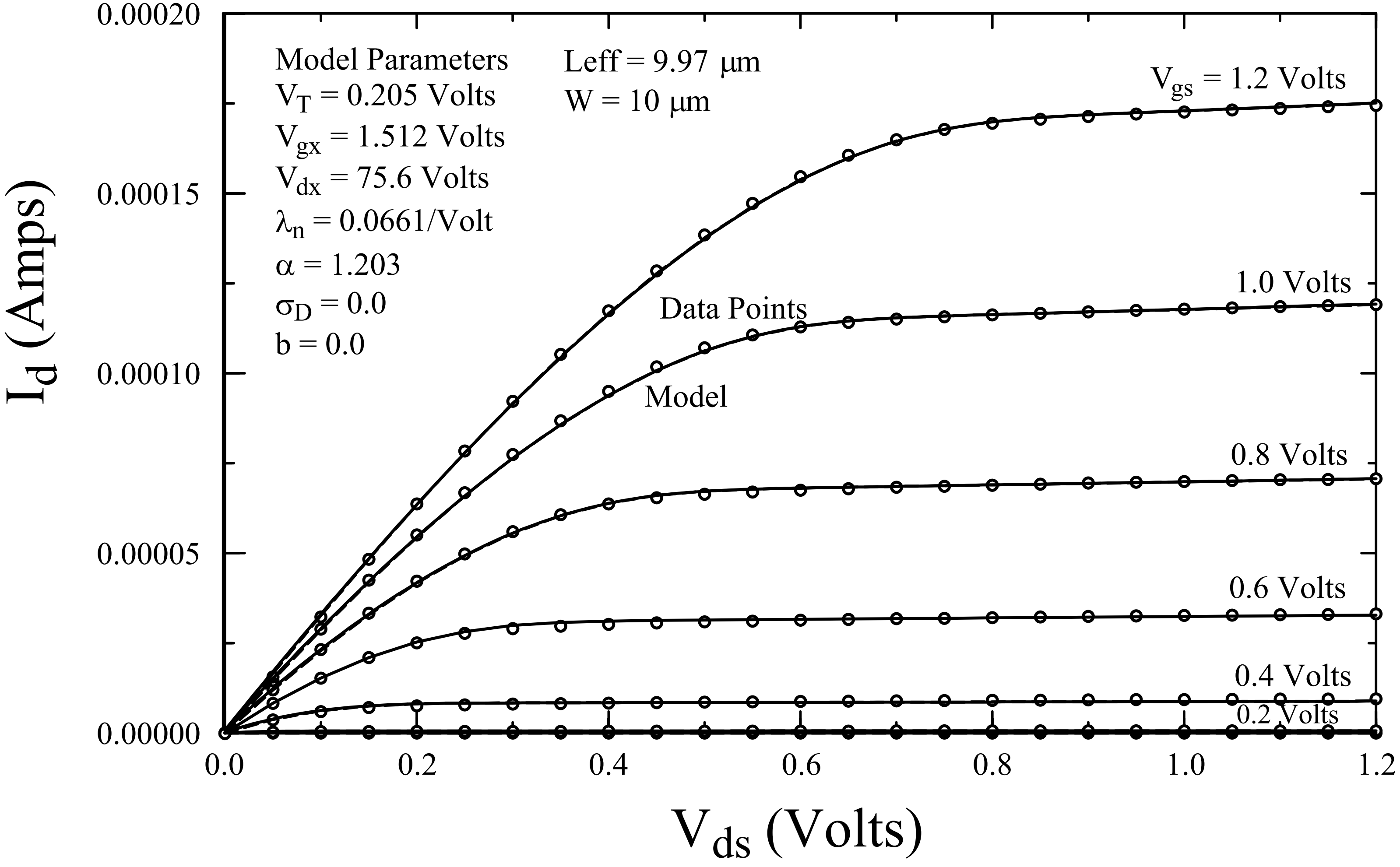
Saturation Region Additions

- At least two factors are needed to compare with experimental data
- Channel length modulation – modeled to first-order by factor:

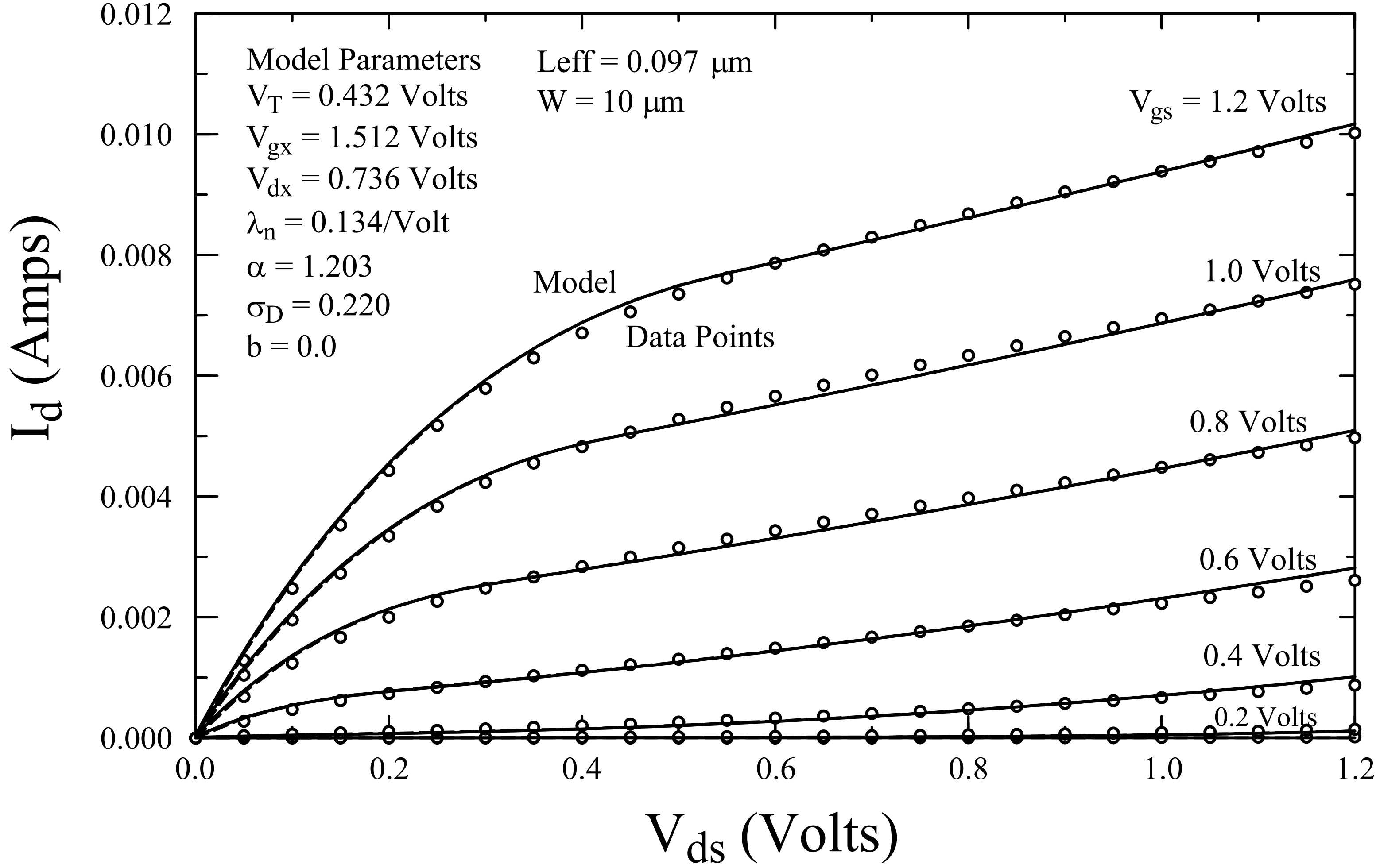
$$(1+\lambda_n V_{ds})$$

- Drain induced barrier lowering (DIBL) modeled by factor:

$$V_T \rightarrow V_T - \sigma_D V_{ds}$$



Model fit to experimental MOS I-V characteristic for a long channel MOS transistor with oxide thickness of about 1.5 nm



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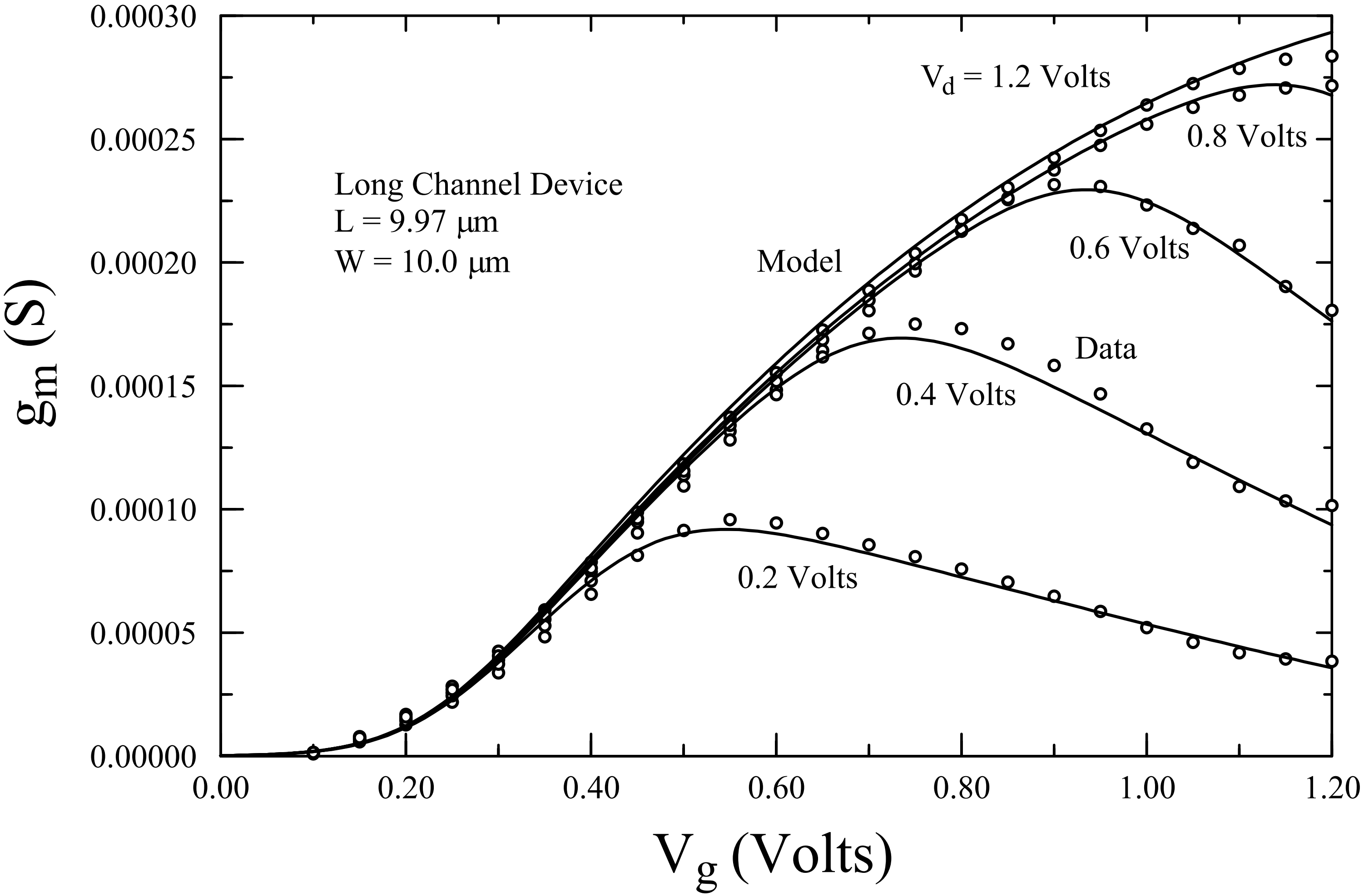
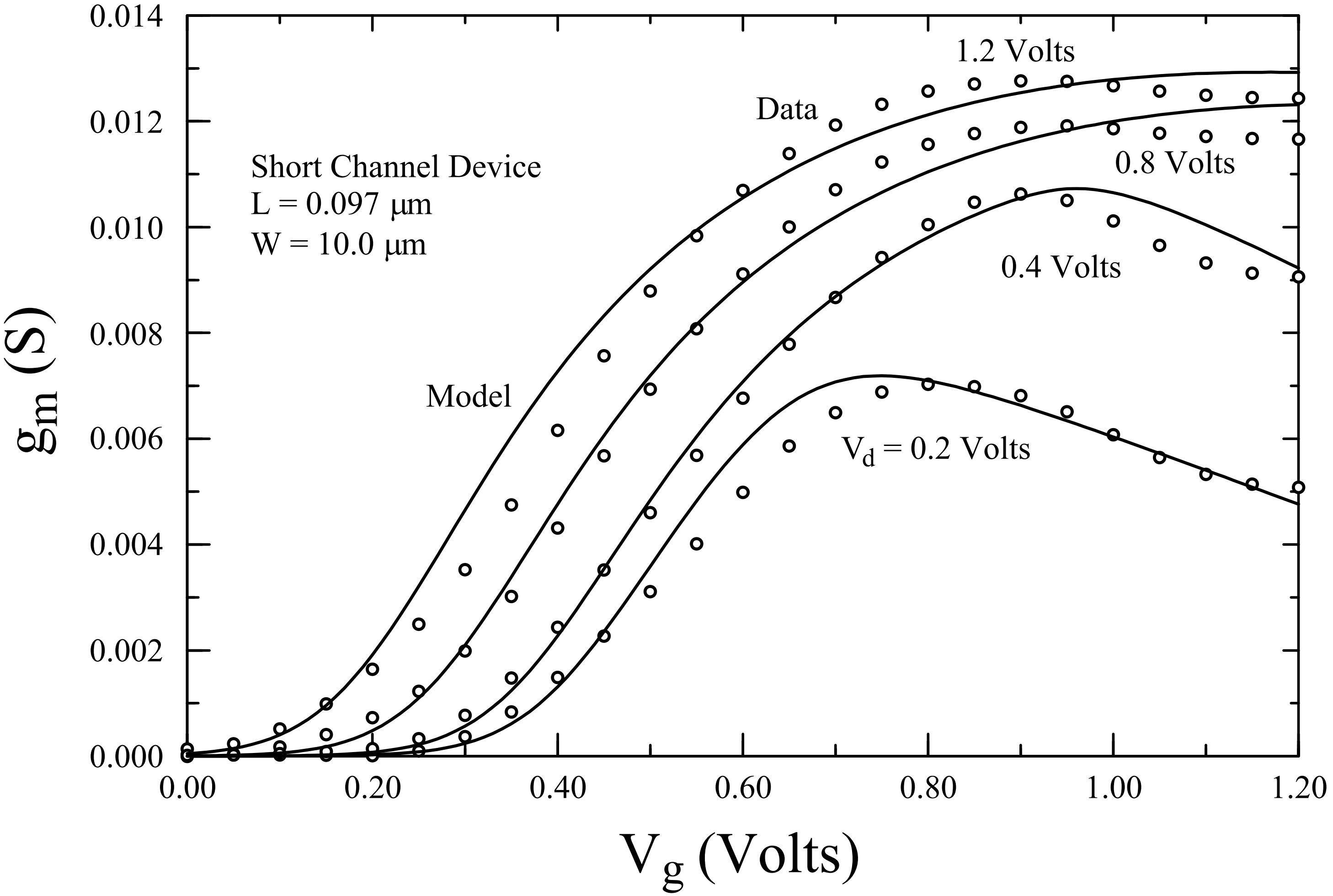
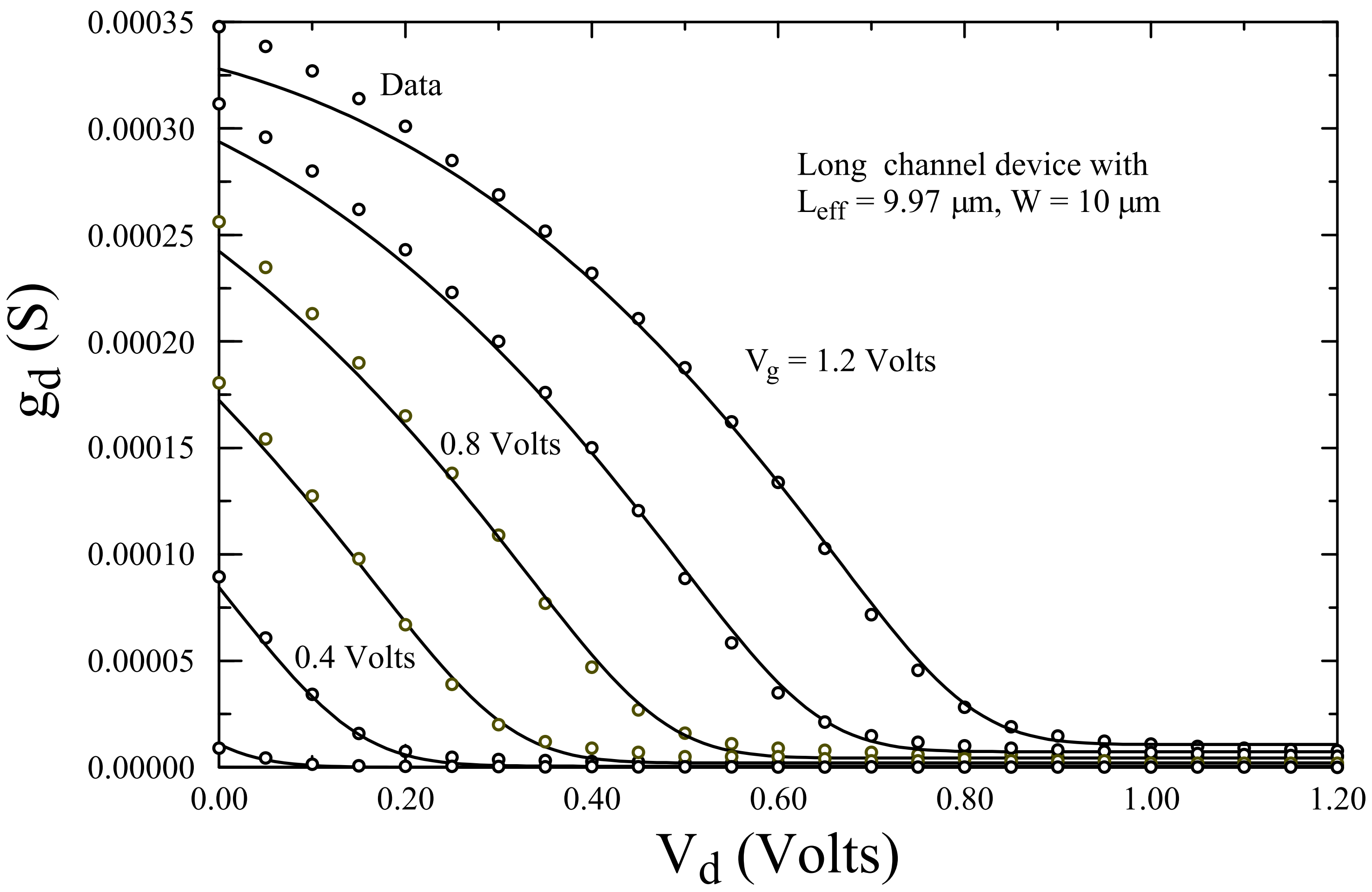


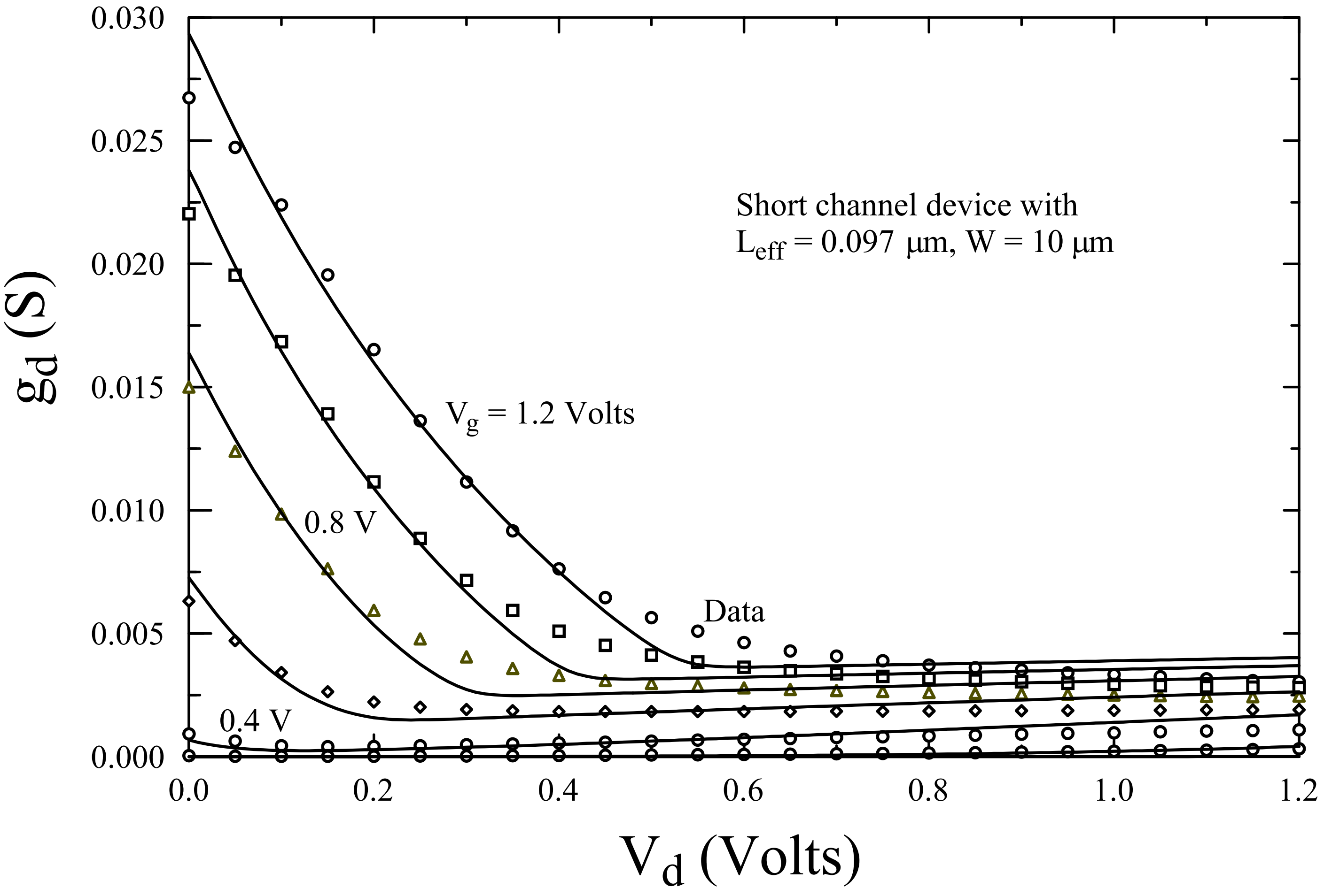
Comparison of experimental and theoretical transconductance for the long channel MOS transistor



Comparison of experimental and theoretical transconductance for the short channel MOS transistor



Comparison of experimental and theoretical drain conductance for the long channel MOS transistor



Comparison of experimental and theoretical drain conductance for the short channel MOS transistor

Further Extensions and Improvements

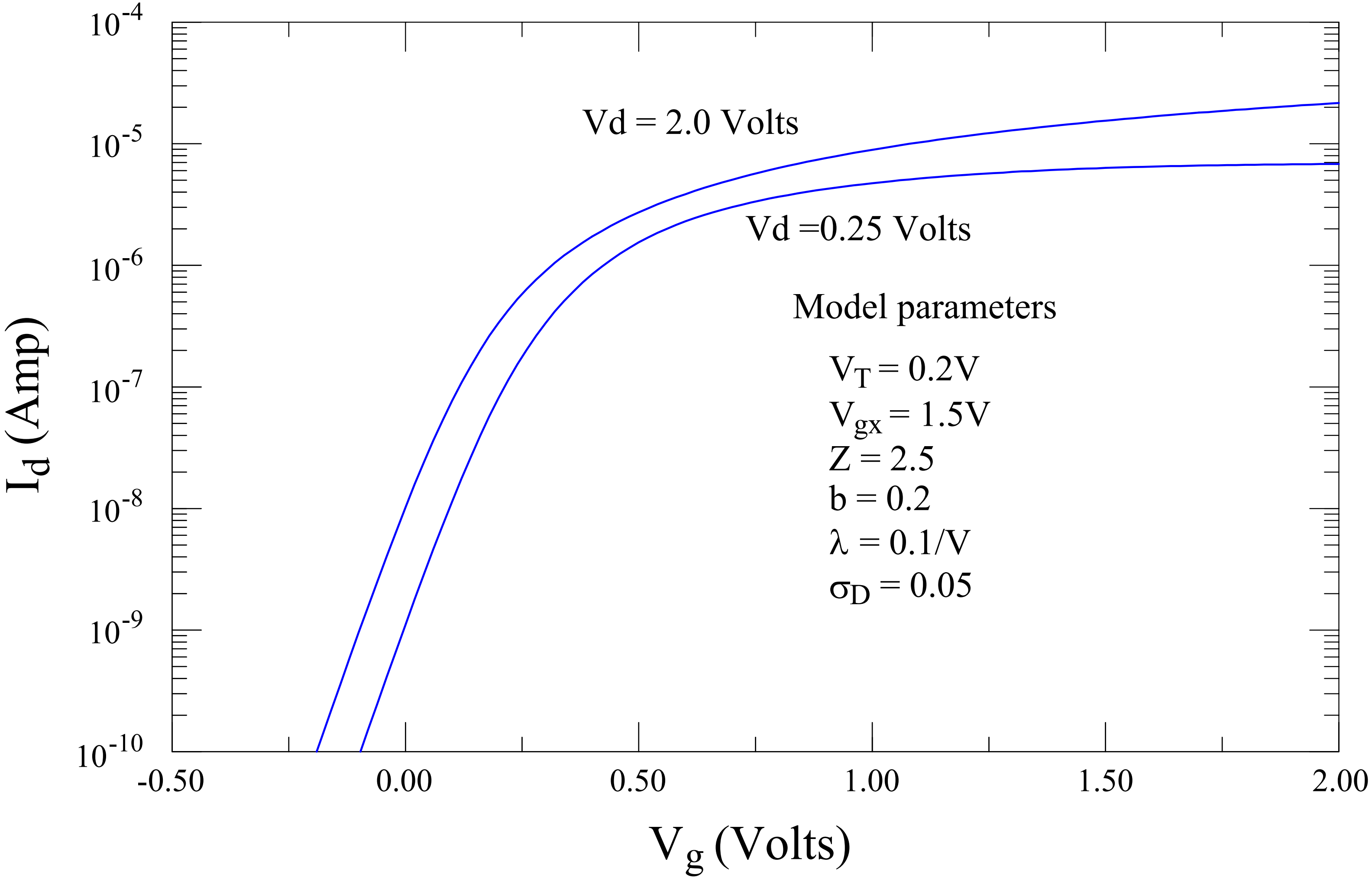
- Subthreshold region – Can be included by reformulation equations in terms of channel charge

$$y_s = b + (V_g - V_T - \alpha V_s)/V_{gx} \rightarrow b + q_s / Q_{gx},$$

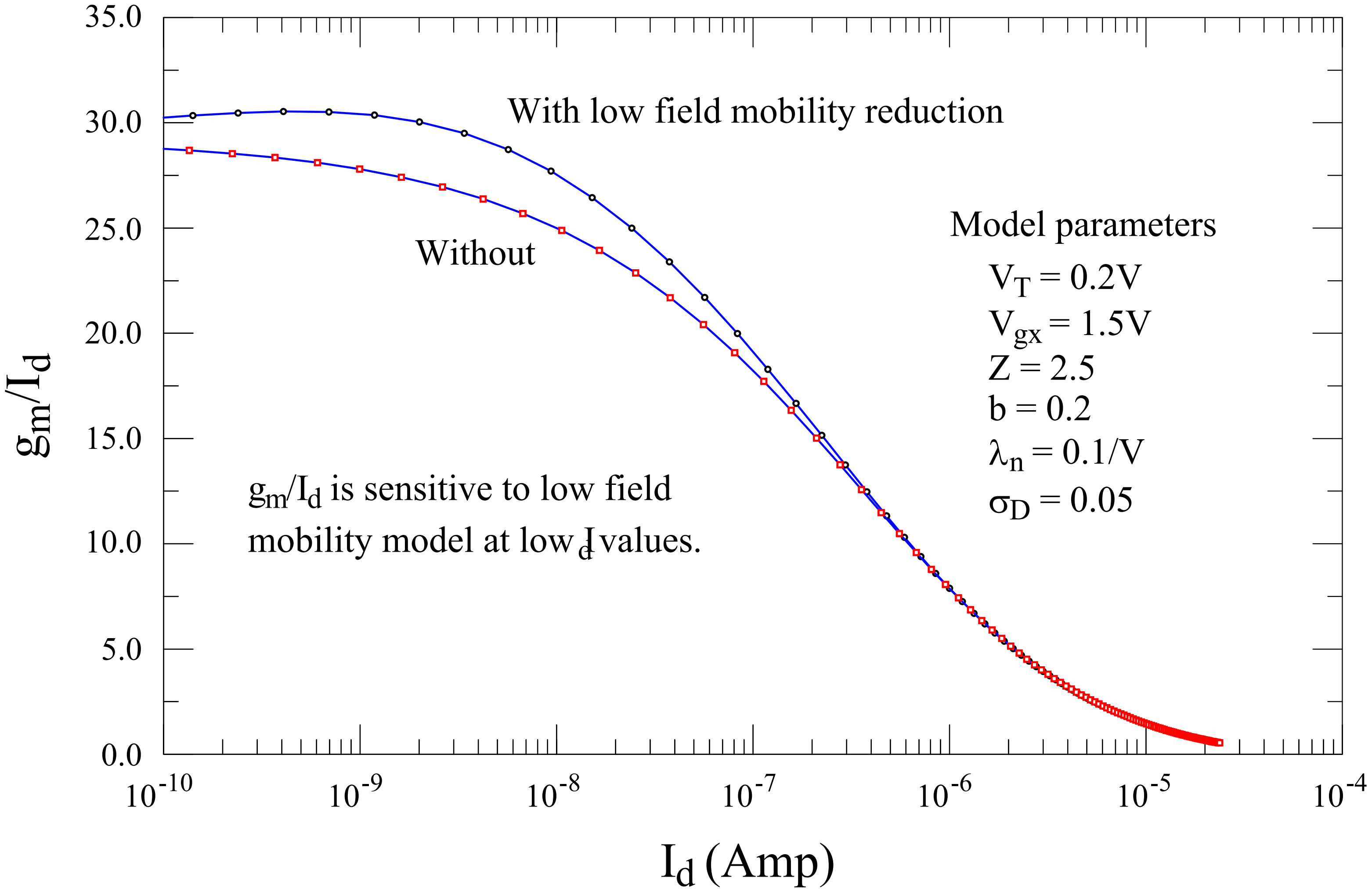
$$\mathrm{y}_d = b + (V_g - V_T - \alpha V_d)/V_{gx} \rightarrow b + q_d / Q_{gx},$$

$$Q_{gx} = C_{ox} V_{gx}.$$

- Model charge for an exponential dependence on voltage is subthreshold region and linear dependence above threshold with smooth transition – similar to previous charge control models



Example of model extension to subthreshold region



Low current transconductance properties with and without low field dependent mobility

Summary

- New MOS model developed that fully integrates a high field mobility model into the basic differential equation.
- Mobility model incorporates reductions due to both vertical gate field and lateral drain field.
- New model can provide an improved starting point for compact MOS models.
- Model can be extended to include subthreshold region and to include mobility fall-off at low surface effective fields.