Symbolic Charge-Based MOSFET Model

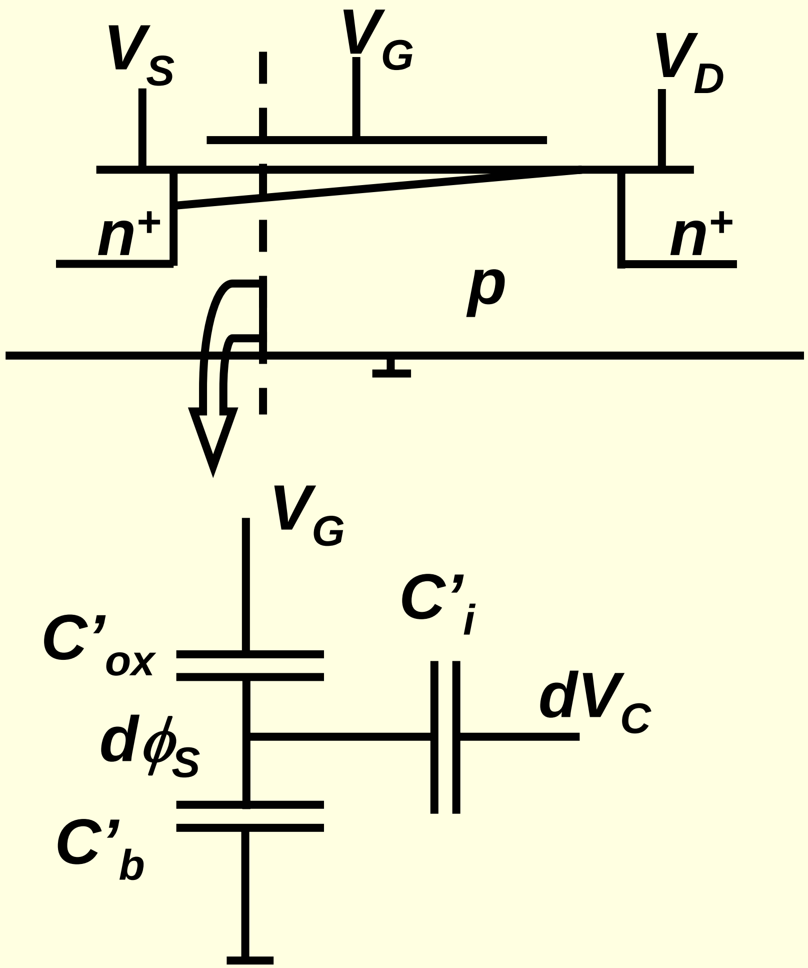
C. Galup-Montoro and M. C. Schneider

Federal University of Santa Catarina, Brazil

Outline

- **Charge-sheet I-V model**
- **Charge control model**
- **Saturation coefficient α**
- **Velocity saturation**
- **Virtual charge definition**
- **Stored charges and capacitive coefficients**
- **Conclusion**

Charge-sheet I-V model



$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

Basic approximations:

$$C_{ox}' + C_b' = nC_{ox}'$$

$$C_i' = -\frac{Q_I'}{\phi_t}$$

$$\Downarrow$$

$$dQ_I'\left(\frac{1}{nC_{ox}'} - \frac{\phi_t}{Q_I'}\right) = dV_C$$

Charge control model

$$I_D = -\mu W Q'_I \frac{dV_C}{dy} \qquad dV_C = \left(\frac{1}{nC'_{ox}} - \frac{\phi_t}{Q'_I} \right) dQ'_I$$

$$I_D = -\frac{\mu W}{nC'_{ox}} (Q'_I - nC'_{ox}\phi_t) \frac{dQ'_I}{dy} = -\frac{\mu W}{nC'_{ox}} Q'_{It} \frac{dQ'_{It}}{dy}$$

Drift diffusion

Saturation coefficient α

For constant mobility along the channel, the integration of $I_D = -\frac{\mu W}{nC'_{ox}} Q'_{It} \frac{dQ'_{It}}{dy}$ between source (*y*=*0*) and drain (*y*=*L*) yields

$$I_D = \frac{\mu W}{C'_{ox} L} \frac{Q'^2_F - Q'^2_R}{2n} = I_{D0}(1-\alpha^2)$$

where $$\alpha = \frac{Q'_R}{Q'_F} = \frac{Q'_{ID} - nC'_{ox}\phi_t}{Q'_{IS} - nC'_{ox}\phi_t}$$

In SI and assuming zero bulk charge the relations given below hold.

$$\alpha \cong \frac{Q'_{ID}}{Q'_{IS}} \cong \frac{V_P - V_D}{V_P - V_S} \cong \frac{V_{GD} - V_{T0}}{V_{GS} - V_{T0}}$$

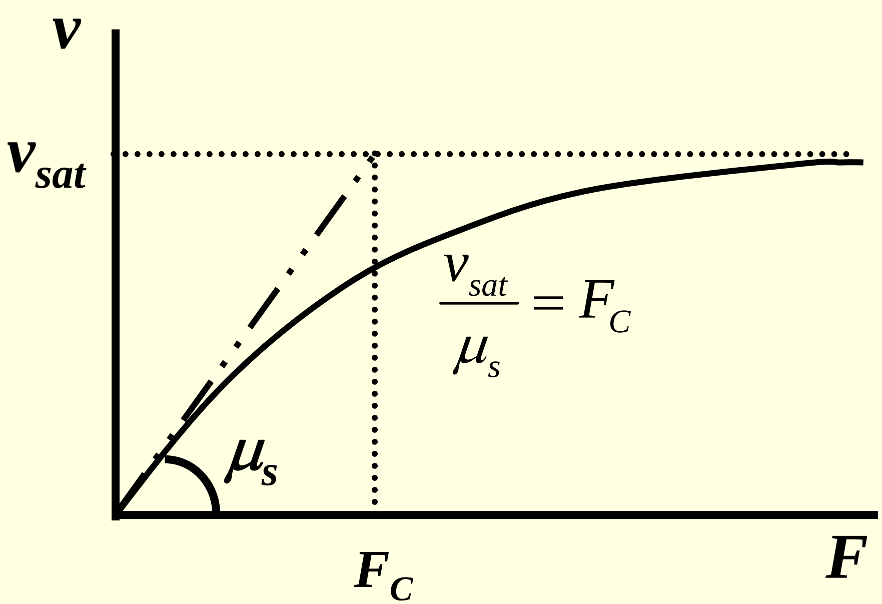
Velocity saturation -1

$$\mu \cong \frac{\mu_s}{1+\dfrac{\mu_s}{v_{sat}}\dfrac{d\phi_S}{dy}} = \frac{\mu_s}{1+\dfrac{\mu_s}{v_{sat}}\dfrac{dQ'_I}{nC'_{ox}dy}}$$

$$dQ'_I = nC'_{ox}d\phi_s$$

$$\frac{d\phi_s}{dy} = -F \quad \text{(longitudinal field)}$$

Allows analytical integration for I_D



Velocity saturation - 2

$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

$$\mu = \frac{\mu_s}{1 + \frac{\mu_s}{nC_{ox}' v_{sat}} \frac{dQ_I'}{dy}}$$

$$\frac{dQ_I'}{dy}\left(\frac{1}{nC_{ox}'} - \frac{\phi_t}{Q_I'}\right) = \frac{dV_C}{dy}$$

$$I_D = -\frac{\mu_s W}{nC_{ox}'} \frac{Q_I' - nC_{ox}'\phi_t}{1 + \frac{\mu_s}{v_{sat}} \frac{dQ_I'}{nC_{ox}' dy}} \frac{dQ_I'}{dy}$$

$$\Updownarrow$$

$$dy = -\frac{\mu_s W}{nC_{ox}' I_D}\left(Q_I' - nC_{ox}'\phi_t + \frac{I_D}{W v_{sat}}\right) dQ_I'$$

Virtual charge definition - 1

Virtual inversion charge density

$$Q'_V = Q'_I - nC'_{ox}\phi_t + \frac{I_D}{Wv_{sat}}$$

inversion +pinch off -saturation charge densities

Along the channel $dQ'_V = dQ'_I$

$$dy = -\frac{\mu_s W}{nC'_{ox} I_D}\left(Q'_I - nC'_{ox}\phi_t + \frac{I_D}{Wv_{sat}} \right) dQ'_I = -\frac{\mu_s W}{nC'_{ox} I_D} Q'_V dQ'_V$$

Virtual charge definition - 2

$$I_D = -\frac{\mu_s W}{nC'_{ox}} Q'_V \frac{dQ'_V}{dy} = -\mu_s W Q'_V \frac{d\phi_s}{dy}$$

The drift of the virtual charge produces the same current as the actual movement of the real charge, which includes drift, diffusion and velocity saturation

Virtual charge - 3

The integration of $I_D = -\frac{\mu_s W}{nC'_{ox}} Q'_V \frac{dQ'_V}{dy}$ from

source to drain results in

$$I_D = \frac{\mu_s W}{C'_{ox} L} \frac{Q'^2_{VS} - Q'^2_{VD}}{2n} = I_{D0}\left(1 - \alpha^2\right)$$

where

$$\alpha = \frac{Q'_{VD}}{Q'_{VS}} = \frac{Q'_{ID} - nC'_{ox}\phi_t + I_D / Wv_{sat}}{Q'_{IS} - nC'_{ox}\phi_t + I_D / Wv_{sat}}$$

Stored charges

The stored charge

$$Q_I = W\int_0^{L_e} Q_I' dy + W(L - L_e)Q_{IDsat}'$$

is calculated changing the integration variable from y to Q'_V

$$dy = -\frac{\mu_s W}{nC_{ox}' I_D} Q_V' dQ_V'$$

resulting in

$$Q_I = WL_e\left[\frac{2}{3}\frac{1+\alpha+\alpha^2}{1+\alpha}Q_{VS}' + nC_{ox}'\phi_t\right] - \frac{L_e I_D}{v_{sat}}$$

Capacitive coefficients - 1

$$C_{gs} = -\frac{\partial Q_G}{\partial V_S} = \frac{1}{n}\frac{\partial Q_I}{\partial V_S}$$

$$\frac{\partial Q_I}{\partial V_S} = \frac{\partial Q_I}{\partial Q'_{VS}}\frac{\partial Q'_{VS}}{\partial V_S} + \frac{\partial Q_I}{\partial Q'_{VD}}\frac{\partial Q'_{VD}}{\partial V_S} + \frac{L g_{ms}}{v_{sat}}$$

partial derivatives of the virtual charges have an additional term related to the saturation velocity

$$\frac{\partial Q'_{VS}}{\partial V_S} = \frac{\partial Q'_I}{\partial V_S} - \frac{g_{ms}}{W v_{sat}} \qquad \frac{\partial Q'_{VD}}{\partial V_S} = -\frac{g_{ms}}{W v_{sat}}$$

Capacitive coefficients - 2

$$C_{gs} = \frac{2}{3} W L_e C'_{ox} \frac{1+2\alpha}{(1+\alpha)^2} \frac{q'_{IS}}{1+q'_{IS}} + \frac{L_e g_{ms}}{3 n v_{sat}} \frac{(1-\alpha)^2}{(1+\alpha)^2}$$

$$C_{gd} = \frac{2}{3} W L_e C'_{ox} \frac{\alpha^2+2\alpha}{(1+\alpha)^2} \frac{q'_{ID}}{1+q'_{ID}} - \frac{L_e g_{md}}{3 n v_{sat}} \frac{(1-\alpha)^2}{(1+\alpha)^2}$$

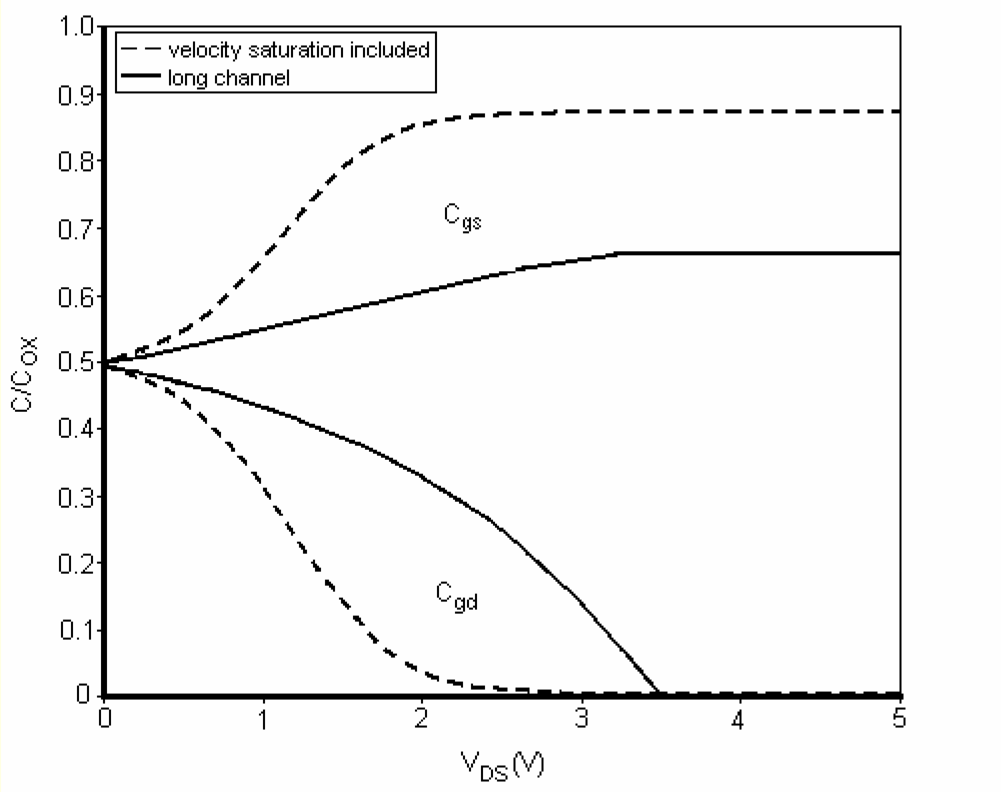
$$C_{gb} = C_{bg} = \frac{n-1}{n} \left(C_{ox} - C_{gso} - C_{gdo} - \frac{L_e g_{mg}}{3 v_{sat}} \frac{(1-\alpha)^2}{(1+\alpha)^2} \right)$$

$$C_{ds} = -\frac{4}{15} n C'_{ox} W \frac{L_e^2}{L} \frac{1+3\alpha+\alpha^2}{(1+\alpha)^3} \frac{q'_{IS}}{1+q'_{IS}} - \frac{1}{30} \frac{g_{ms} L_e^2}{v_{sat} L} \frac{(3\alpha+7)(1-\alpha)^2}{(1+\alpha)^3}$$

$$C_{sd} = -\frac{4}{15} n C'_{ox} W \frac{L_e^2}{L} \frac{\alpha+3\alpha^2+\alpha^3}{(1+\alpha)^3} \frac{q'_{ID}}{1+q'_{ID}} + \frac{1}{30} \frac{g_{md} L_e^2}{v_{sat} L} \frac{(3+7\alpha)(1-\alpha)^2}{(1+\alpha)^3}$$

$$C_{bs(d)} = (n-1) C_{gs(d)} \qquad C_{dg} = C_{gd} + (C_{sd} - C_{ds})/n$$

Simulation results



Normalized capacitances versus drain-source voltage voltage

Conclusion

Virtual inversion charge density

$$Q'_V = Q'_I + Q'_{pinch-off} - Q'_{saturation}$$

Drift (strong inversion) formalism with the virtual charge as variable can represent drift, diffusion and velocity saturation transport in the MOSFET