Accuracy of Surface-Potential-Based Long-Wide-Channel MOS Transistor Compact Model

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Outline

- Motivation
- Theoretical Formulas
  - One-current-term Double Integral
  - Two-current-term 4-component Formulas
- Accuracy of the Baseline Compact Models
- Summary

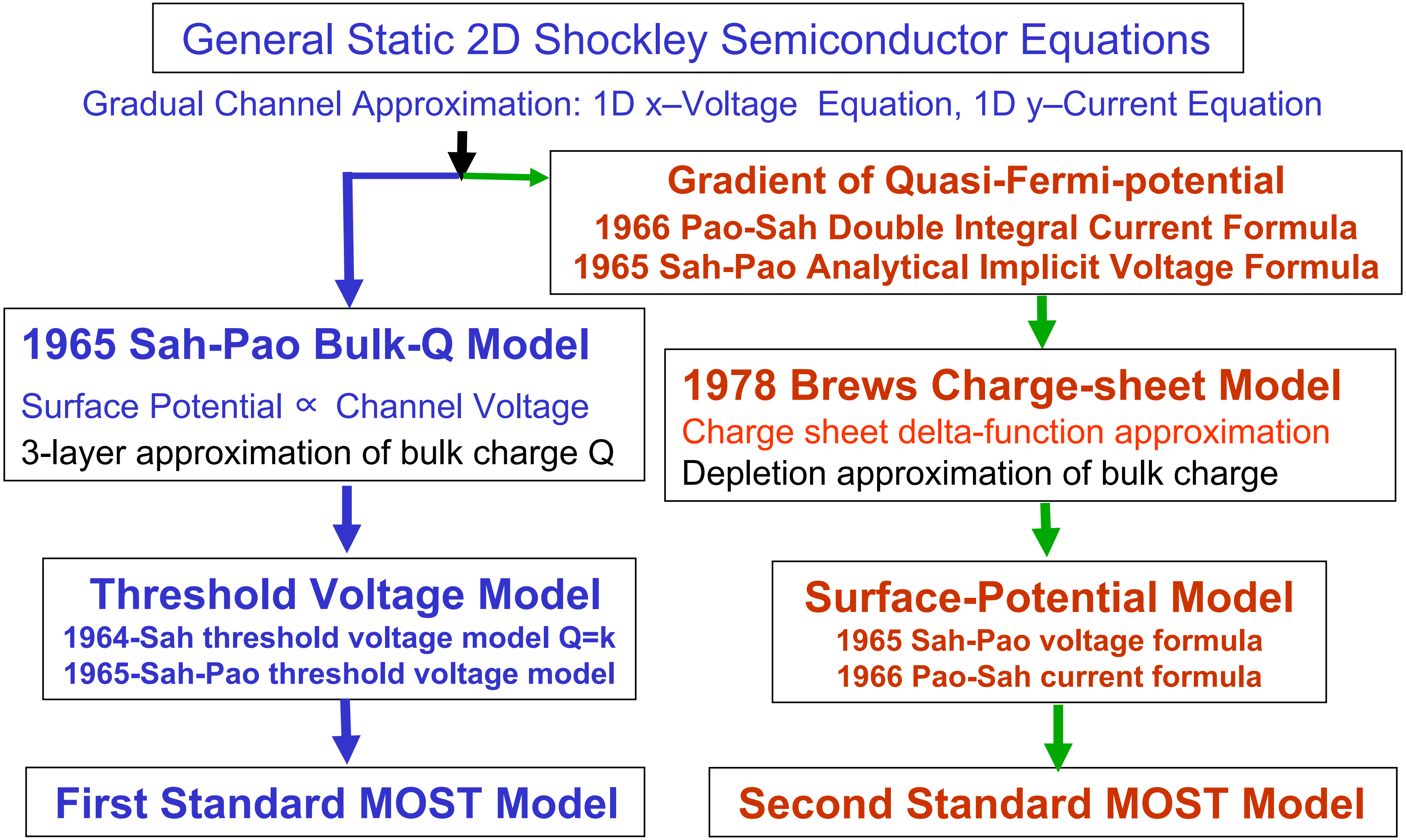
Motivation

You folks asked Sah last year

as his initiation test:

How accurate are the <u>approximate baseline long-and-wide-channel</u> MOS transistor models used by you to derive the compact models?

Historical Paths



General Static 2D Shockley Semiconductor Equations

Gradual Channel Approximation: 1D x–Voltage Equation, 1D y–Current Equation

1965 Sah-Pao Bulk-Q Model

Surface Potential ∝ Channel Voltage

3-layer approximation of bulk charge Q

Threshold Voltage Model

1964-Sah threshold voltage model Q=k

1965-Sah-Pao threshold voltage model

First Standard MOST Model

Gradient of Quasi-Fermi-potential

1966 Pao-Sah Double Integral Current Formula

1965 Sah-Pao Analytical Implicit Voltage Formula

1978 Brews Charge-sheet Model

Charge sheet delta-function approximation

Depletion approximation of bulk charge

Surface-Potential Model

1965 Sah-Pao voltage formula

1966 Pao-Sah current formula

Second Standard MOST Model

Theoretical Formulas

One-current-term Baseline Model

Gradient of Quasi-Fermi-Potential

1965 Sah-Pao Analytical Implicit Voltage Formula
1966 Pao-Sah Double Integral Current Formula

- n-channel MOS Transistor
- Constant Channel Impurity Concentration
- Constant Channel Electron Mobility
- Long-Wide Channel and Gate

One-term Double Integral

- MOSC Equation (2004-Sah Voltage Equation)
  - Self-Consistent Remote Charge-Neutrality Boundary Condition (1957-SNS, 1962-Sah, 1964-Sah, 1965-Sah-Pao, 2004-Sah-turkey)
- MOSR Equation (1966-Pao-Sah Current Equation)
  - Gradient of Quasi-Fermi-Potential
  - Subtraction of Unbounded Flatband Current.

$$I_D = \frac{Z}{L} q D_n n_i L_{Di} \int_{U_{N0}}^{U_{NL}} \exp(-U_N)\, dU_N \int_0^{U_S} \frac{[\exp(+U) - 1] dU}{\mathrm{Sign}U_S \times F_X(U, U_N, U_P)}$$

Two-term Drift+Diffusion Model

1996-Sah Space Charge Theory of MOST

(No Gradual Channel Approximation)

Four Components and Two 2D-terms

2D-terms cannot be reduced to two 1D terms

$$+I_S = (Z/L)\int_0^L\int_0^\infty J_{NY}(x,y,z)\,dxdy = -I_D - I_B$$

$$= (Z/L)\int_0^L\int_0^\infty \left(\underline{q\mu_n (N - N_B) E_Y} + \underline{qD_n \frac{\partial (N - N_B)}{\partial y}} \right) dxdy$$

Drift current term

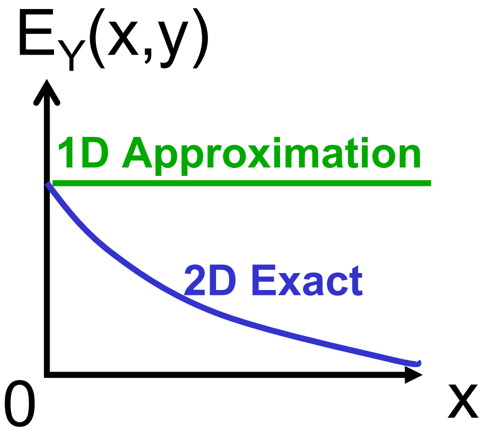
Diffusion current term

2D = NOT 1D numerically integrable

1D numerically integrable

x-independent E_Y Approximation

- Reject 1978-Brews charge sheet of inversion charge
- Assume x-dependence of $E_Y(x,y)$
- Give higher bulk-charge reduction of drift current. (the missing 2% I_D-inversion mystery solved here)



2D Drift current term

1D Drift current term

$$\mu_n \int_0^L \int_0^\infty (N - N_B) E_Y \, dx\,dy \propto \mu_n \int_{U_{SB}}^{U_{DB}} \frac{\partial U_S}{\partial U_{NP}} \int_0^{U_S} \frac{\exp(-U_N)\left[\exp(+U) - 1\right]}{\mathrm{Sign}U_S \times F_X(U, U_N, U_P)} \, dU\,dU_{NP}$$

4-Component Current Formula

Model 9604139 (1D integral model)

$$\mathrm{ID} = I_{\mathrm{NORM}} \times (\mathrm{139P1} + \mathrm{139P2} + \mathrm{139D1} + \mathrm{139D2})$$

☺Electron and hole carrier space-charge drift term

$$\mathrm{139P1} = \mu_n [(U_{GB} - U_{FB} - U_{S0})^2 - (U_{GB} - U_{FB} - U_{SL})^2] / (2 U_{II}^{1/2})$$

☹Ionized impurity space-charge or bulk-space-charge drift term

$$139\mathrm{P}2 = -\mu_n \int_{U_{SB}}^{U_{DB}} \frac{\partial U_S}{\partial U_{NP}} \int_0^{U_S} \frac{\exp(U_P)[1 - \exp(-U)]}{\mathrm{Sign}U_S \times F_X(U, U_N, U_P)} dU dU_{NP}$$

☺Electron and hole carrier space-charge diffusion term

$$\mathrm{139D1} = D_n (U_{SL} - U_{S0}) / U_{II}^{1/2}$$

☹Ionized impurity space-charge or bulk-space-charge diffusion term

$$139\mathrm{D}2 = D_n \int_0^{U_{SL}} \frac{\exp(U_{PL})[1 - \exp(-U)] dU}{\mathrm{Sign}U_{SL} \times F_X(U, U_{NL}, U_{PL})} - \int_0^{U_{S0}} \frac{\exp(U_{P0})[1 - \exp(-U)] dU}{\mathrm{Sign}U_{S0} \times F_X(U, U_{N0}, U_{P0})}$$

Bulk Charge Approximations

Bulk Charge = Excess Hole Charge (over flatband P_B)

$$-Q_P = -q\int_0^{\infty}(P-P_B)dx = qn_iL_{Di}\int_0^{U_S}\frac{\exp(+U_P)[1-\exp(-U)]}{\text{Sign}U_S\times F_X(U,U_N,U_P)}dU \equiv -Q_{PB}$$

Electron terms ↓

$$-Q_{PB} = qn_iL_{Di}\text{Sign}U_S\int_0^{U_S}\frac{d\{\exp(+U_P)[U-1+\exp(-U)]\}}{\sqrt{\exp(+U_P)[U-1+\exp(-U)]+\exp(-U_N)[\exp(+U)-1-U]}} \qquad B=9$$

$$\cong qn_iL_{Di}\text{Sign}U_S\times 2\sqrt{\exp(+U_P)\times[U_S-1+\exp(-U_S)]} \qquad B=3$$

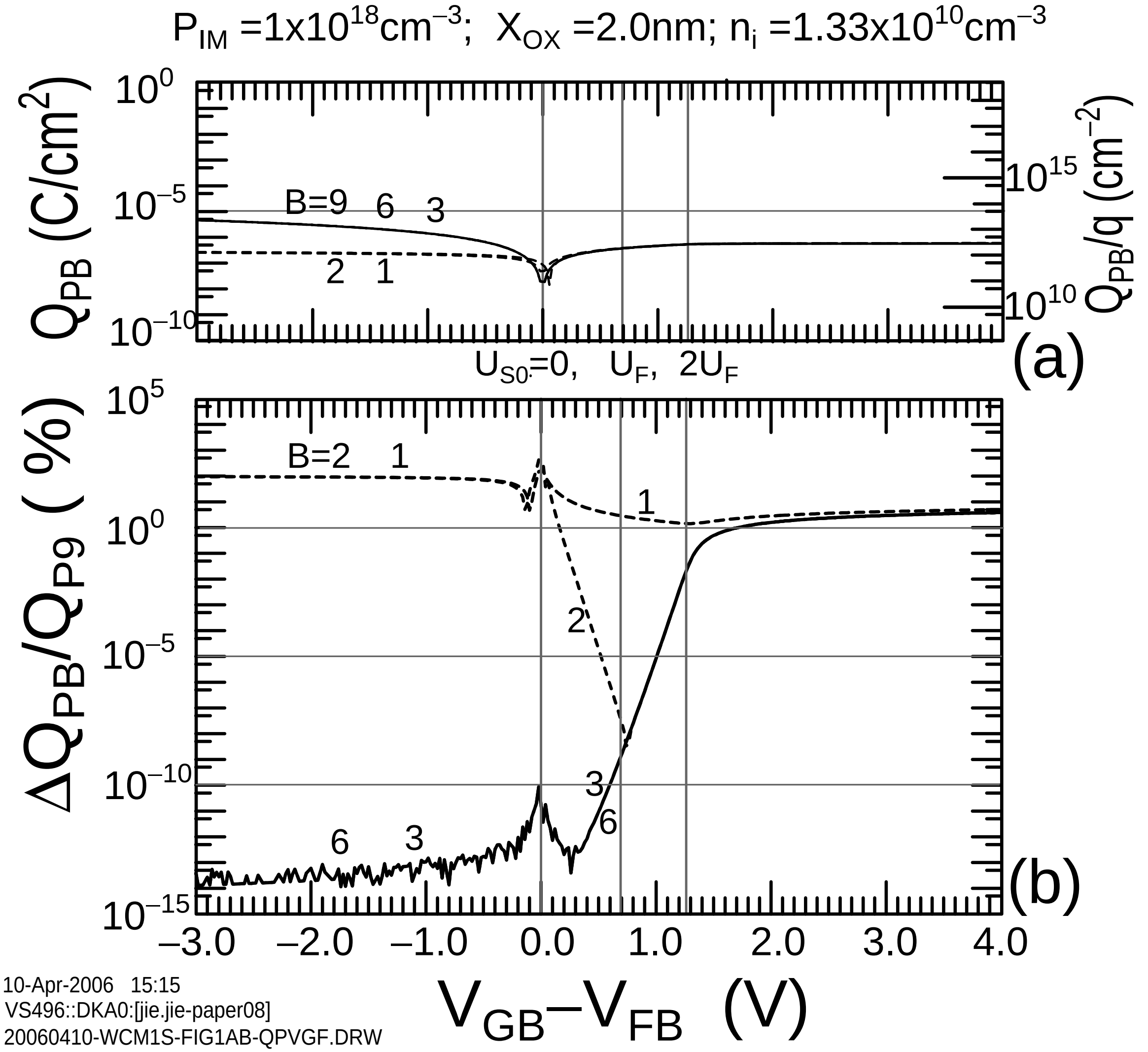
$$\cong qn_iL_{Di}\text{Sign}U_S\times 2\sqrt{\exp(+U_P)\times|U_S-1|} \qquad B=2$$

$$\cong qn_iL_{Di}\text{Sign}U_S\times 2\sqrt{\exp(+U_P)\times|U_S|} \qquad B=1$$

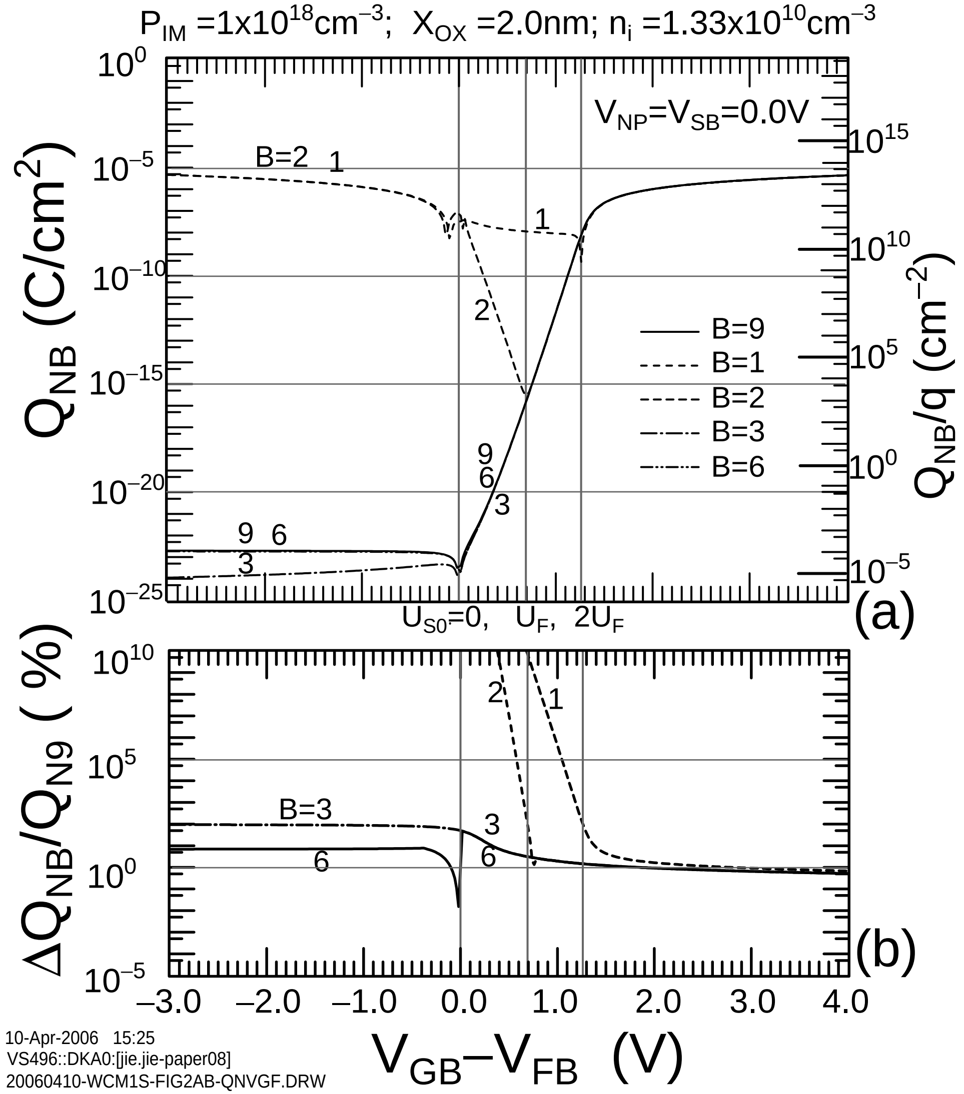
≅ see text (B=6 better than B=3 in accumulation) B=6

Baseline: B=9 (exact); Compact: B=1, 2, 3, 6

Induced Hole Charge Density



Induced Electron Charge Density



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20060410-WCM1S-FIG2AB-QNVGF.DRW

4-component Compact Models

- Compact model 9604131 (B=1)
- Compact model 9604132 (B=2)

  ≡1978 Brews charge-sheet model

  U_{S0}(Accu, FB, Subth, TH, Inv) ≡ (<–2, 0, U_F, $2U_F$, >$2U_F$ +2)
  With reference to the long-wide-channel 4-component formula:
  %dev (Accu, FB, Subth, TH, Inv) = (300, 100, 4, 2, 1)%

- Compact model 9604133 (B=3)

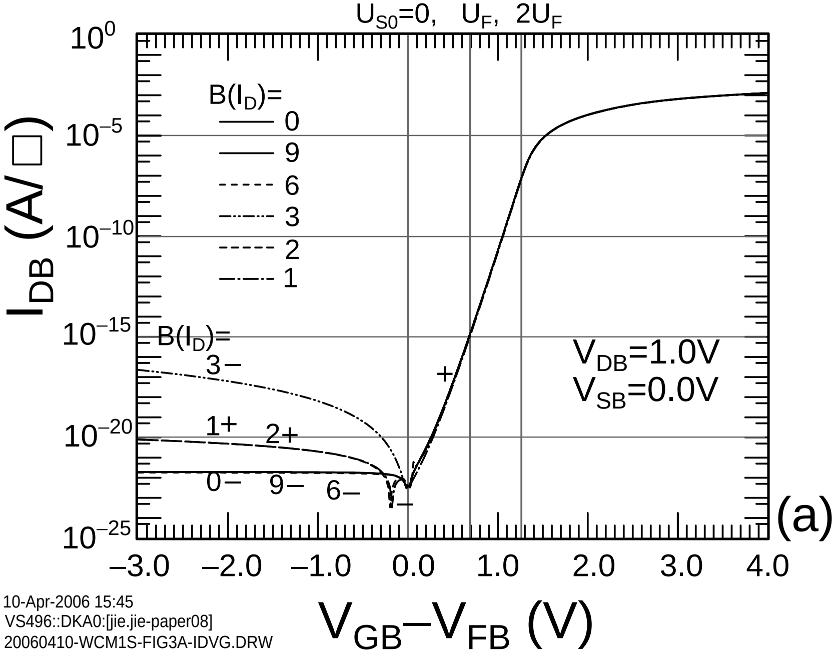
  Good FB to Inv ranges, but 500% off in accumulation

- Compact model 9604136 (B=6)

  Good FB to Inv ranges, only 10% off in accumulation

Transfer Characteristics I_D-V_G

P_{IM}=1x10^{18}cm^{-3}; X_{OX}=2.0nm; μ_n=400cm^2/Vs



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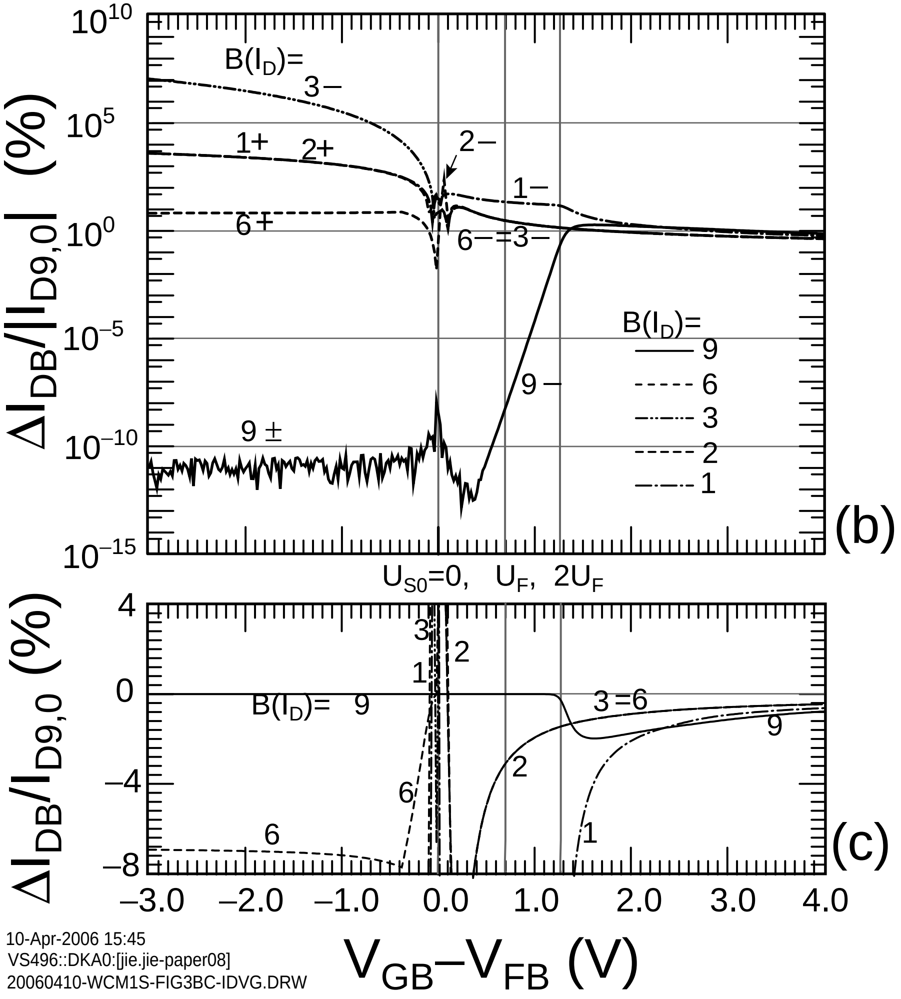

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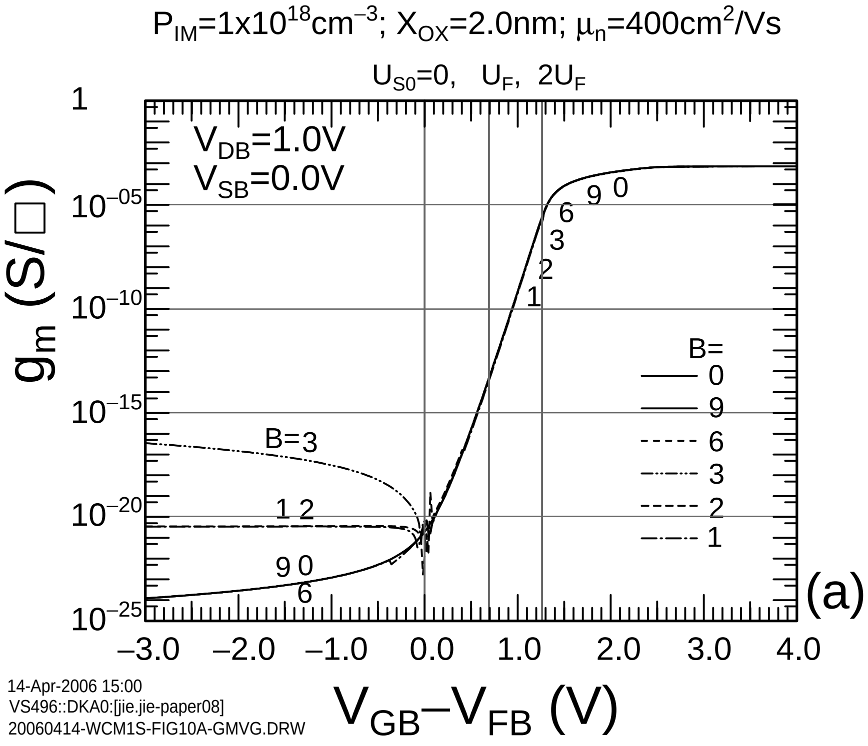
20060410-WCM1S-FIG3BC-IDVG.DRW

Transfer Characteristics g_m-V_G



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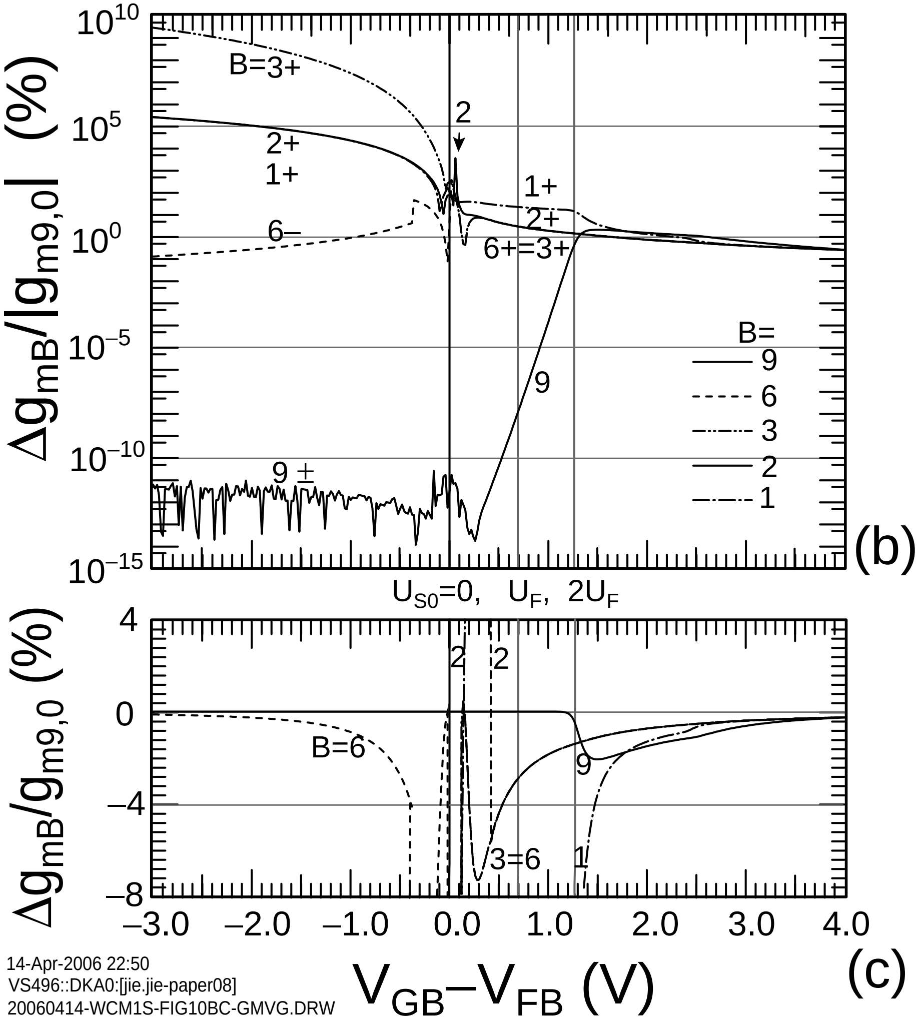
20060414-WCM1S-FIG10A-GMVG.DRW



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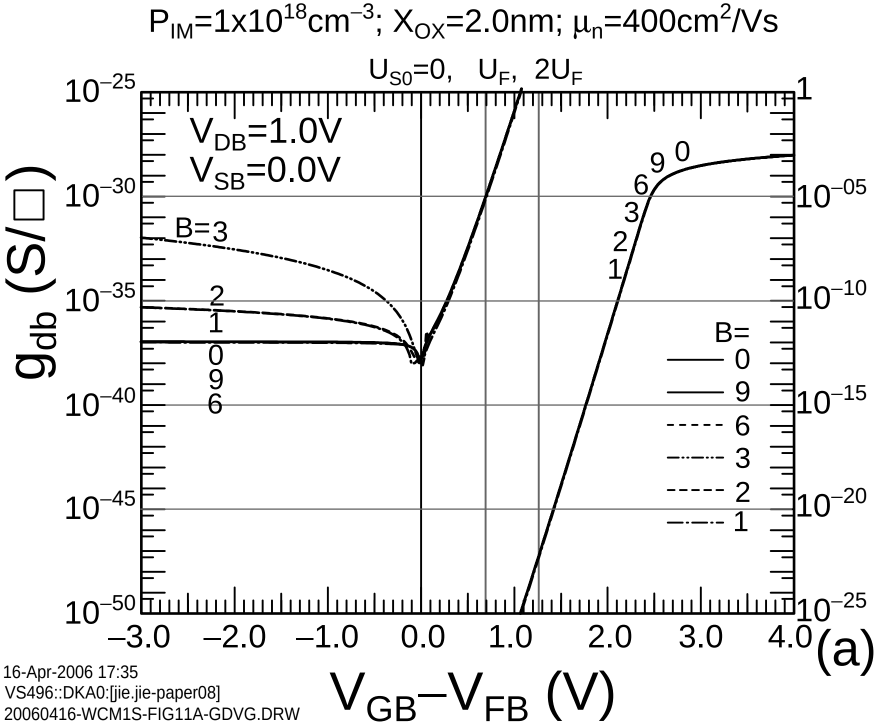
20060414-WCM1S-FIG10BC-GMVG.DRW

Transfer Characteristics g_d-V_G



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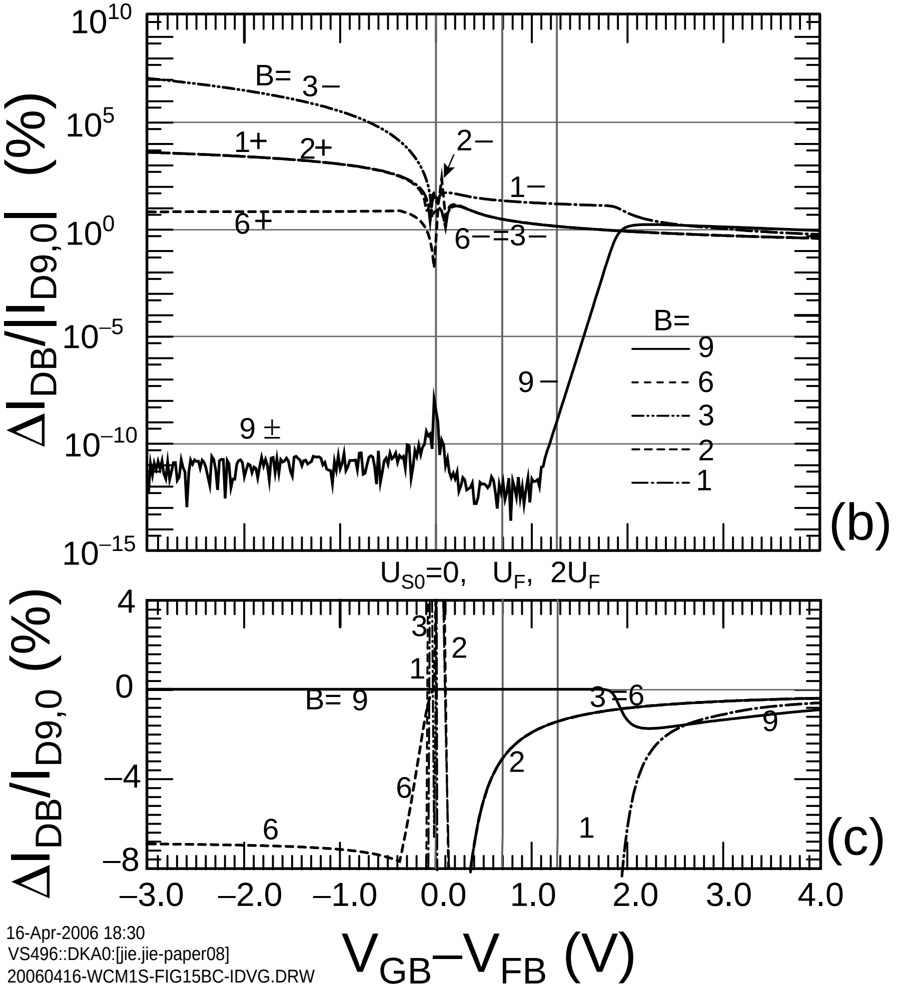
20060416-WCM1S-FIG11A-GDVG.DRW



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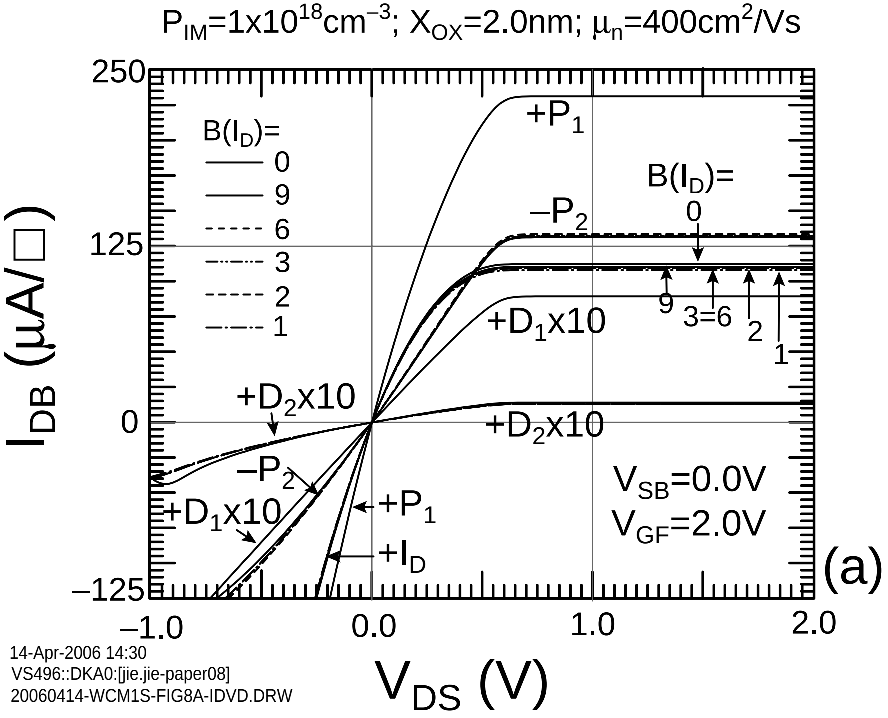
20060416-WCM1S-FIG15BC-IDVG.DRW

Output Characteristics I_D-V_D



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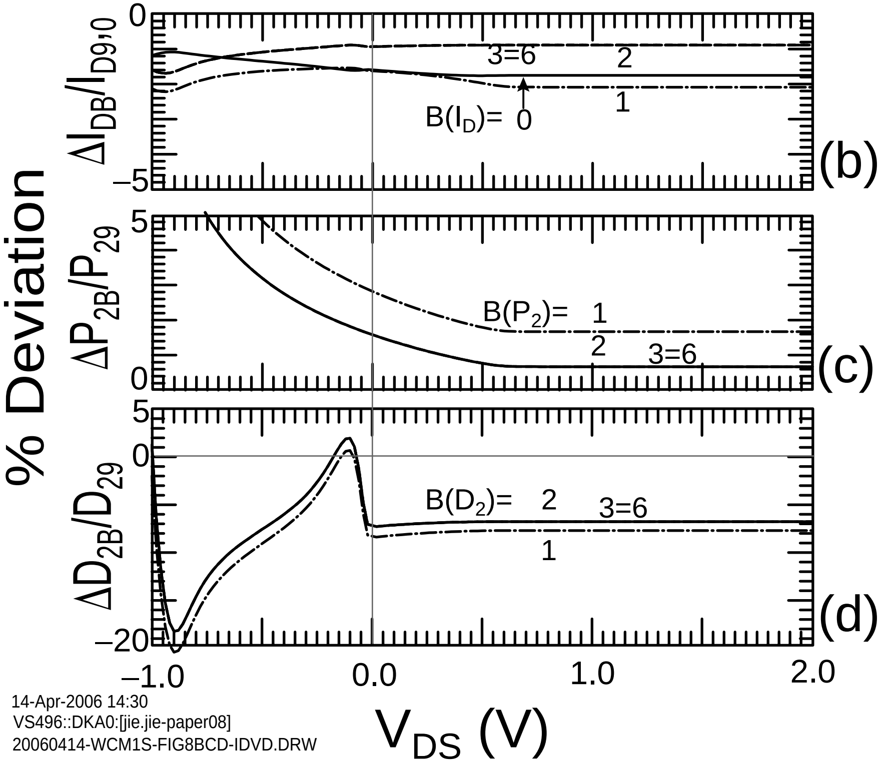

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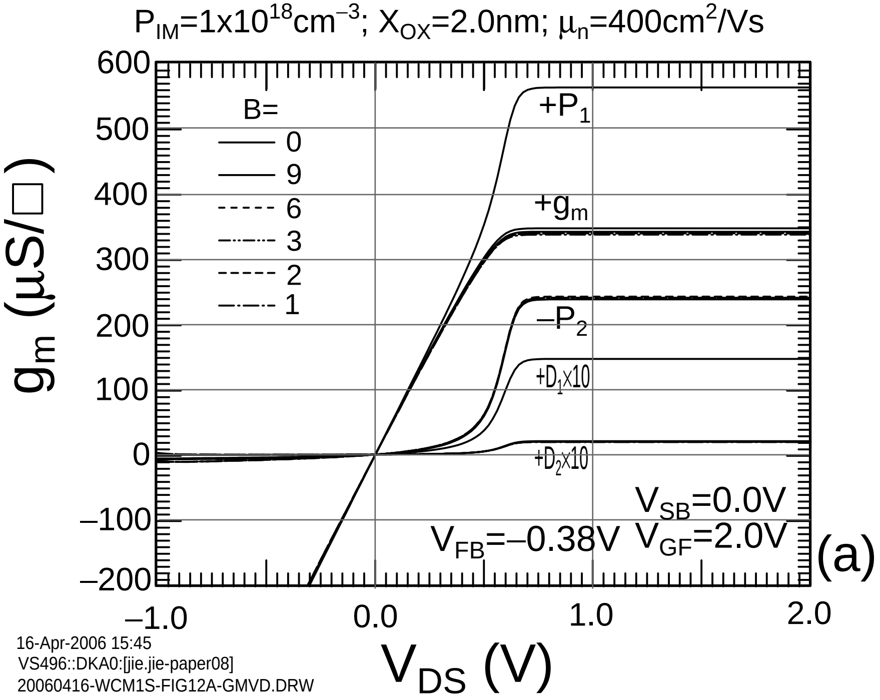
20060414-WCM1S-FIG8BCD-IDVD.DRW

Output Characteristics g_m-V_D



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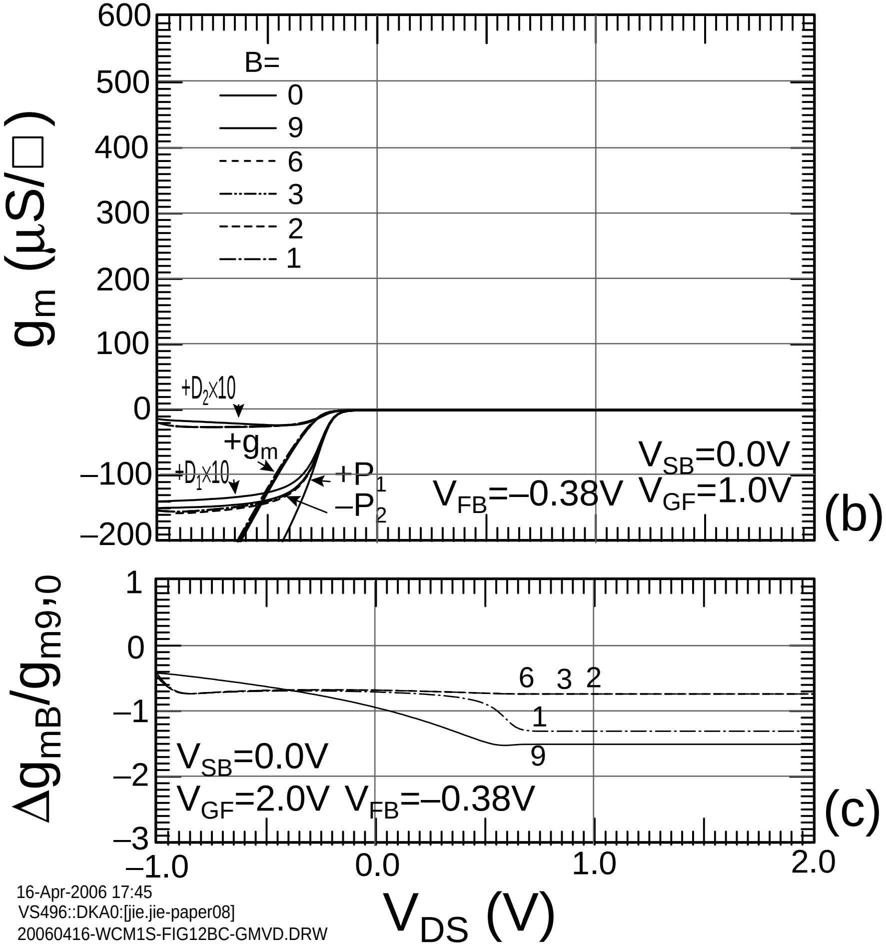

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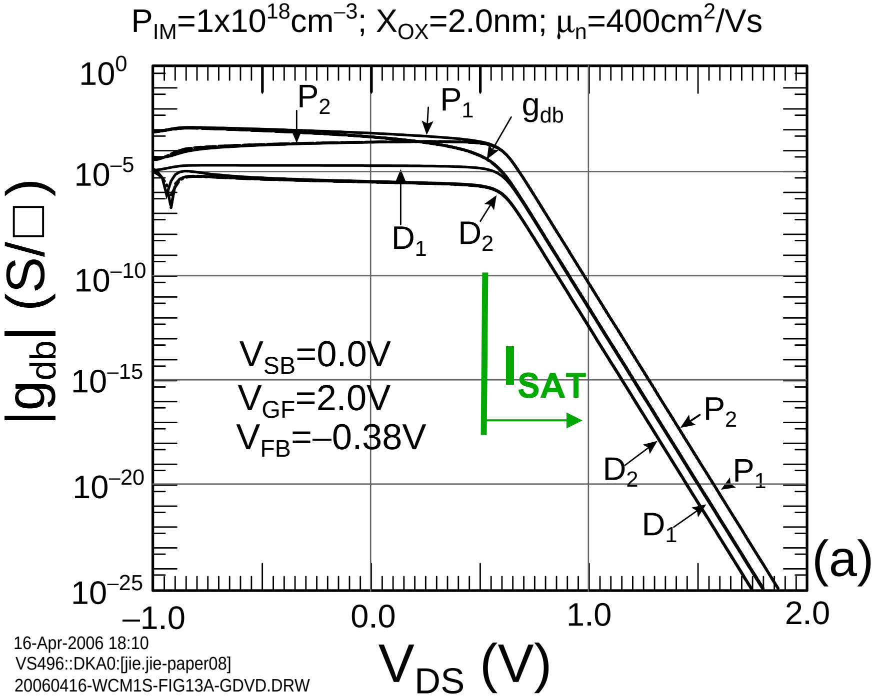
Output Characteristics g_d-V_D



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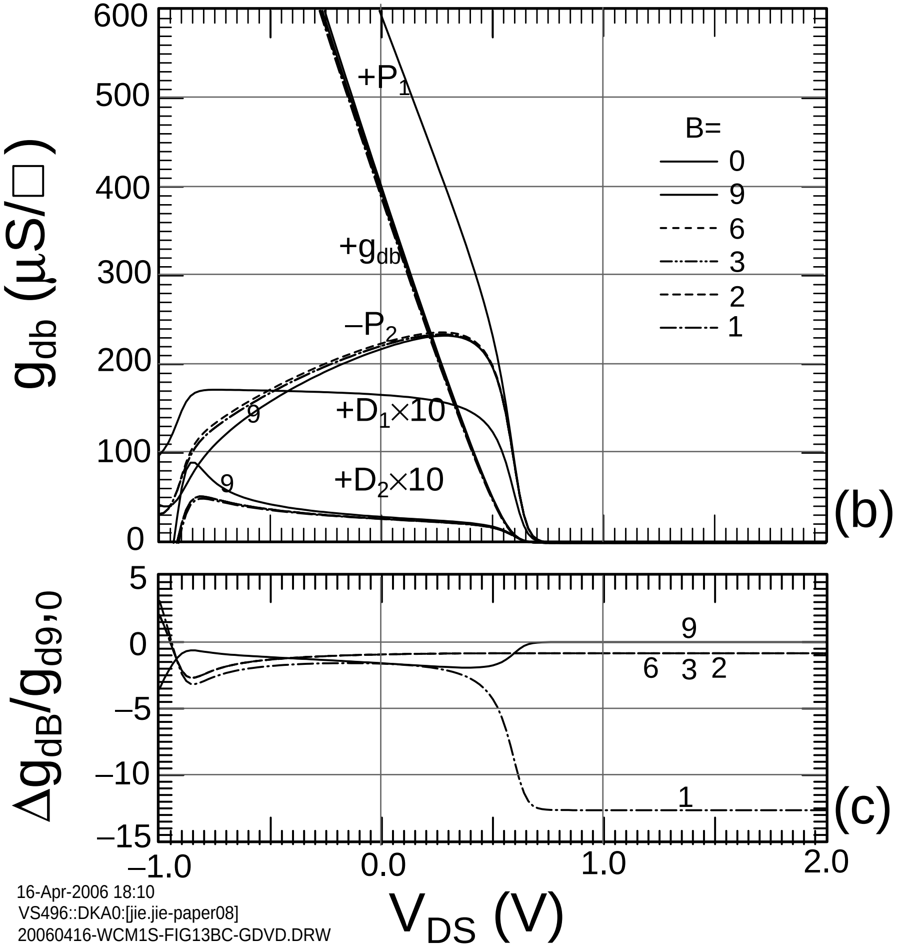

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Summary

One-term Double Integral Drift-Diffusion Formula

Long-wide-channel 4-Component Formula

x-independent E_Y approximation

Excellent in subthreshold and accumulation; < 2% in strong inversion.

Mysterious missing physics on the 2% deviation now solved!

Long-wide-channel Surface-Potential Compact Models

Bulk charge approximations B= 1, 2, 3, 6

Strong inversion:	B=1: ~5%;	B=2, 3, 6: ~2%
Subthreshold:	B=1: >16%;	B=2, 3, 6: ~4%
Accumulation:	B=1, 2, 3: >100%;	B=6: ~10%