

A Compact model to Predict QM Levels and Inversion Layer Centroid with Parabolic Potential Well Approximation

Jin He , Mansun*, and Chenming Hu

EECS, University of California, Berkeley, CA 94720, USA

*Department of Electrical & Electronic Engineering, Hong Kong
University of Science & Technology



jinhe@eecs.berkeley.edu

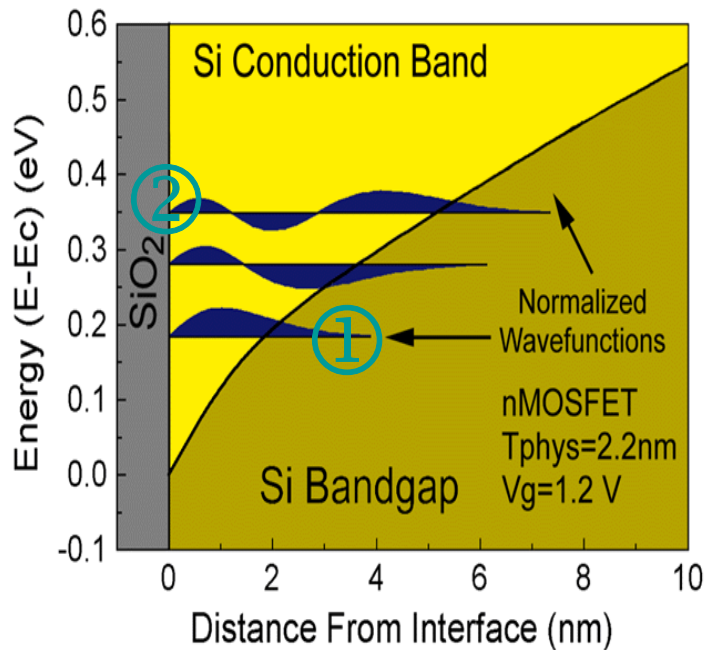
OUTLINE

- ◆ **INTRODUCTION: QME important**
- ◆ **QME Modeling approach reviewing**
- ◆ **NEW QME model development**
- ◆ **MODEL results and verification**
- ◆ **SUMMARY**

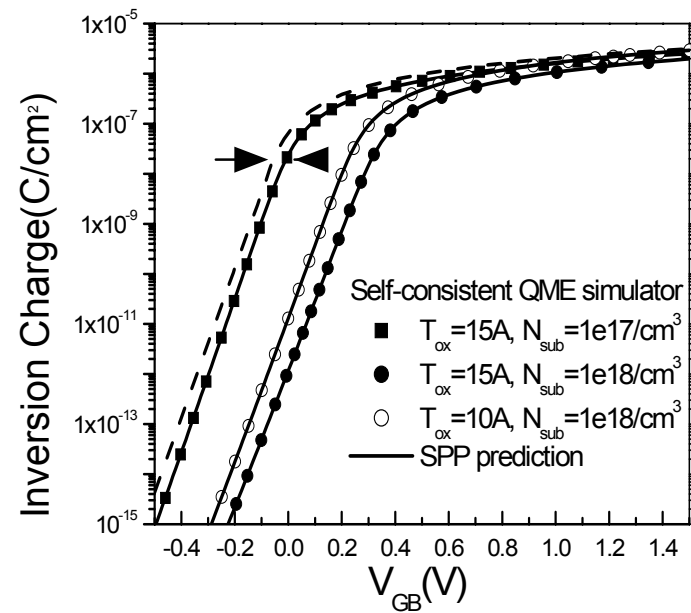
INTRODCUTION

- ◆ **QME phenomena:** coming from Ultra-thin Tox and High doping-Modern CMOS use
- ◆ **QME significant:** Energy level quanization and QME capacitance.
- ◆ **QME results:** VT shift; gate capacitance degradation, mobility degradation, slope degradation.

INTRODUCTION



- ◆ QME phenomena



- ◆ QME results

QME MODELING

Numerical self-consistent solvers

- ❖ Physics picture is complete, Result is high accurate,
- ❖ cannot be available for circuit simulation level

Simple empirical approach

- ❖ Demonstrate correct physical trend.
- ❖ Can fit most data from processing
- ❖ Need more parameters

Physics based analytical approach

- ❖ Triangular well approximation: too simple, too large error
- ❖ Parabolic potential approximation: more accurate, difficult ?

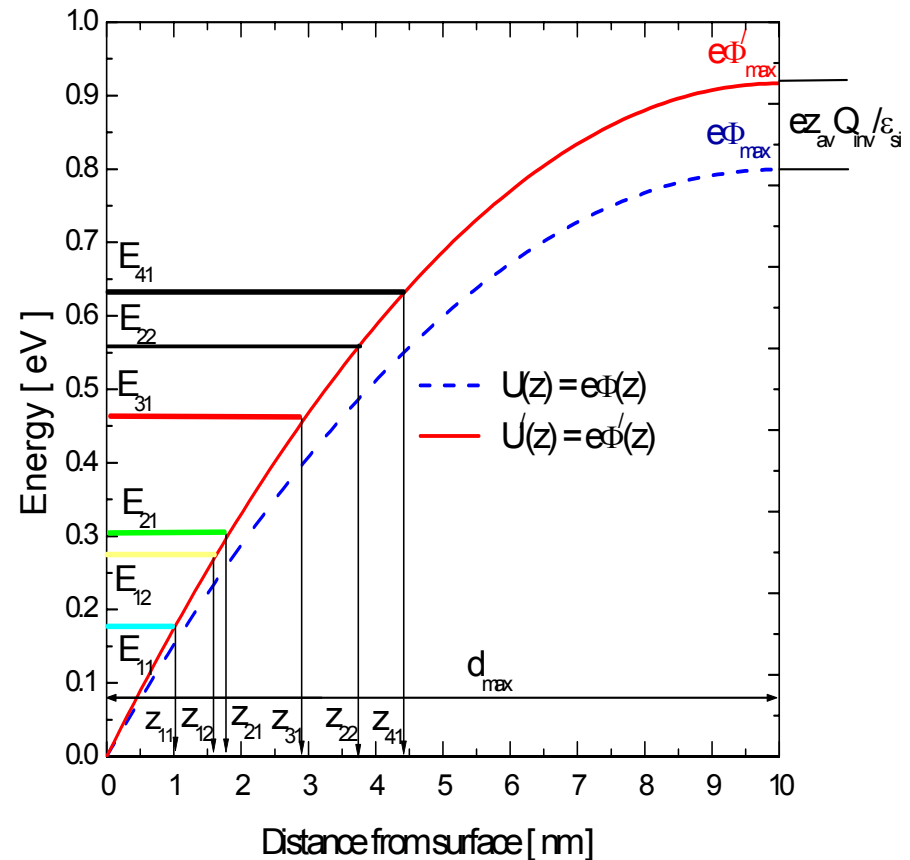
NEW MODEL DEVELOPMENT

- ◆ Effective field definition

$$E_{eff} = (Q_B + \eta Q_{inv}) / \epsilon_{si}$$

- ◆ *New inversion centroid definition*

$$z_I = z_0 \left| U'(z_0) = E_{ij} \right.$$



NEW MODEL DEVELOPMENT

- ◆ Poisson equation

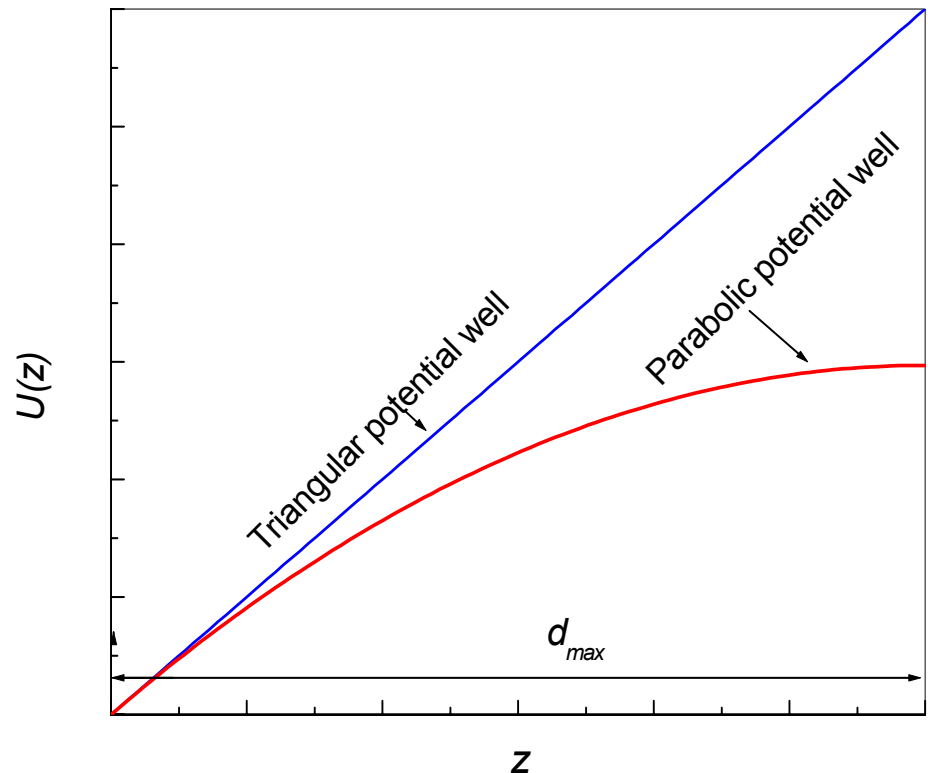
$$\frac{d^2 \phi(z)}{dz^2} = \frac{eN_a}{\epsilon_{si}}$$

- ◆ Parabolic potential

$$\phi'(z) = -\frac{E_{eff}}{2d_{max}}(2zd_{max} - z^2)$$

- ◆ Parabolic energy

$$U'(z) = -\frac{eE_{eff}}{2d_{max}}(z^2 - 2zd_{max})$$



NEW MODEL DEVELOPMENT

- ◆ Corrected max potential due to QME

$$\phi'_{\max} = \frac{E_{\text{eff}} d_{\max}}{2} = \frac{eN_a d_{\max}^2}{2\epsilon_{si}} + \frac{\eta Q_{in} d_{\max}}{2\epsilon_{si}} = \phi_{\max} + \frac{\eta Q_{in} d_{\max}}{2\epsilon_{si}}$$

- ◆ *Analytical expression of inversion centroid*

$$z_I = d_{\max} \left[1 - \sqrt{1 - \frac{2E_{ij}}{eE_{\text{eff}} d_{\max}}} \right] = d_{\max} \left[1 - \sqrt{1 - \frac{E_{ij}}{e\phi'_{\max}}} \right]$$

- ◆ *Analytical expression of the weight factor*

$$\eta = \frac{2z}{d_{\max}}$$

NEW MODEL DEVELOPMENT

- ◆ General form of the QM wave function

$$\psi_{ij}(z) = \frac{A}{\sqrt{k_{ij}(z)}} \sin\left(\int k_{ij}(z) dz + \frac{\pi}{4}\right)$$

- ◆ *General QM wave vector equation*

$$k_{ij} = \frac{P}{h} = \sqrt{\frac{2m_z(E_{ij} - U'(z))}{h^2}}$$

- ◆ *Concise WKB-path integral method*

$$\int_0^{z_1} \frac{P}{h} dz = n\pi - \frac{\pi}{4}$$

COMPLETE QM-ENERGY EQUATION

Integral of the inversion centroid results in

$$d_{\max} \sqrt{\frac{2d_{\max} E_{ij}}{eE_{\text{eff}}}} + \left(d_{\max}^2 - \frac{2d_{\max} E_{ij}}{eE_{\text{eff}}} \right) \left[\ln \sqrt{d_{\max}^2 - \frac{2d_{\max} E_{ij}}{eE_{\text{eff}}}} - \ln \left(d_{\max} + \sqrt{\frac{2d_{\max} E_{ij}}{qE_{\text{eff}}}} \right) \right] = 2\pi\hbar \left(n - \frac{1}{4} \right) \sqrt{\frac{d_{\max}}{m_z e E_{\text{eff}}}}$$

The use of the Taylor expand on ln function

$$\left[\ln \sqrt{d_{\max}^2 - \frac{2d_{\max} E_{ij}}{eE_{\text{eff}}}} - \ln \left(d_{\max} + \sqrt{\frac{2d_{\max} E_{ij}}{eE_{\text{eff}}}} \right) \right] \approx -\sqrt{\frac{2E_{ij}}{eE_{\text{eff}} d_{\max}}}$$

Analytical QM-based energy level expression

$$E_{ij} = \left(\frac{e^2 E_{\text{eff}}^2 \hbar^2}{2m_z} \right)^{\frac{1}{3}} \left[\pi \left(n - \frac{1}{4} \right) \right]^{\frac{2}{3}}$$

COMPACT MODELING

QM-based analytical energy levels

$$E_{ij} = \left(\frac{e^2 E_{eff}^2 \hbar^2}{2m_z} \right)^{\frac{1}{3}} \left[\pi \left(n - \frac{1}{4} \right) \right]^{\frac{2}{3}}$$

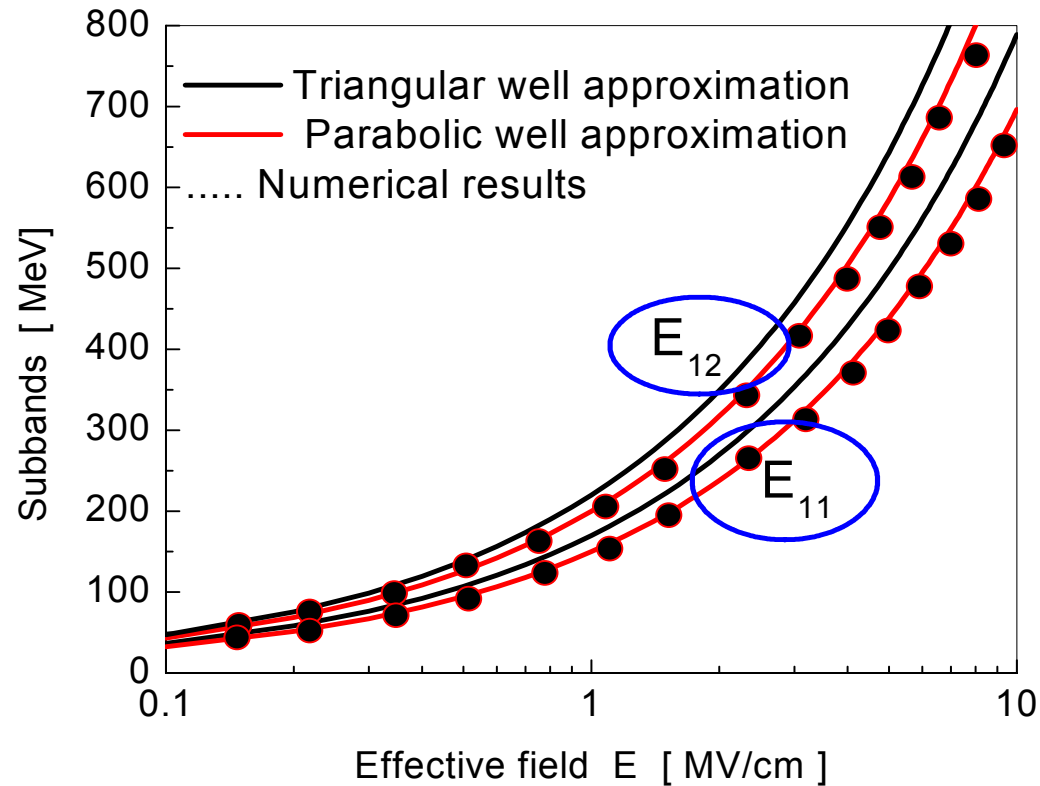
QM-based analytical inversion charge centroid

$$z_I = d_{\max} \left[1 - \sqrt{1 - \frac{[\pi \hbar (2n - 1/2)]^{2/3}}{(em_z E_{eff})^{1/3} d_{\max}}} \right] = d_{\max} \left[1 - \sqrt{1 - \frac{E_{ij}}{e\phi'_{\max}}} \right] \approx KE_{eff}^{-1/3}$$

QM-based analytical weight factor of E-field

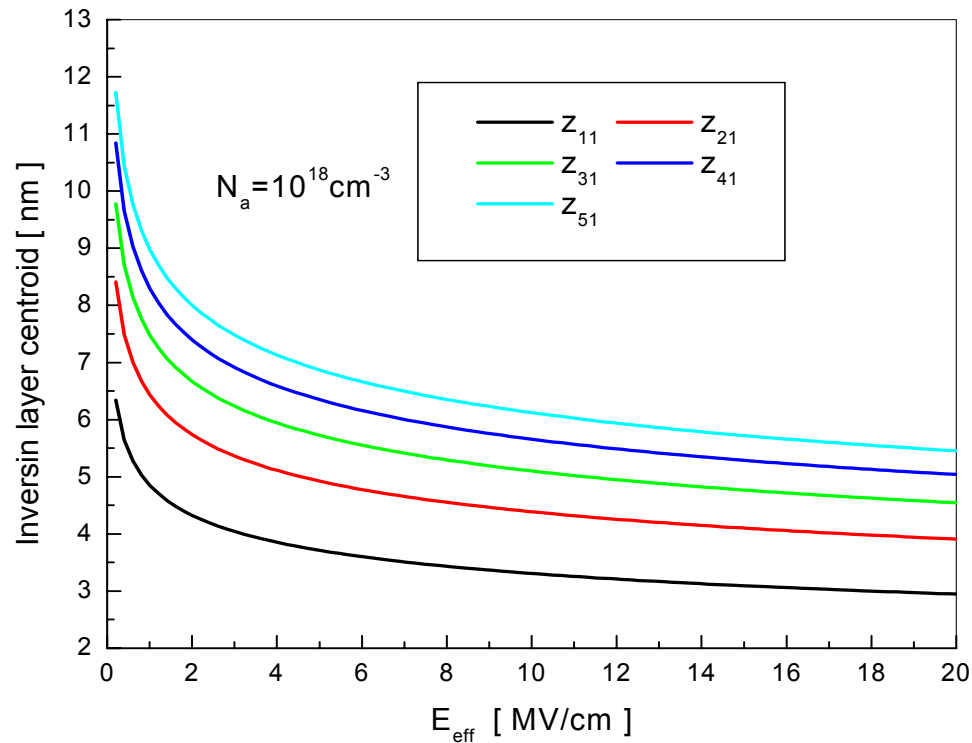
$$\eta = 2 \left[1 - \sqrt{1 - \frac{[\pi \hbar (2n - 1/2)]^{2/3}}{(em_z E_{eff})^{1/3} d_{\max}}} \right] = 2 \left[1 - \sqrt{1 - \frac{E_{ij}}{e\phi'_{\max}}} \right]$$

RESULT and DISCUSSION



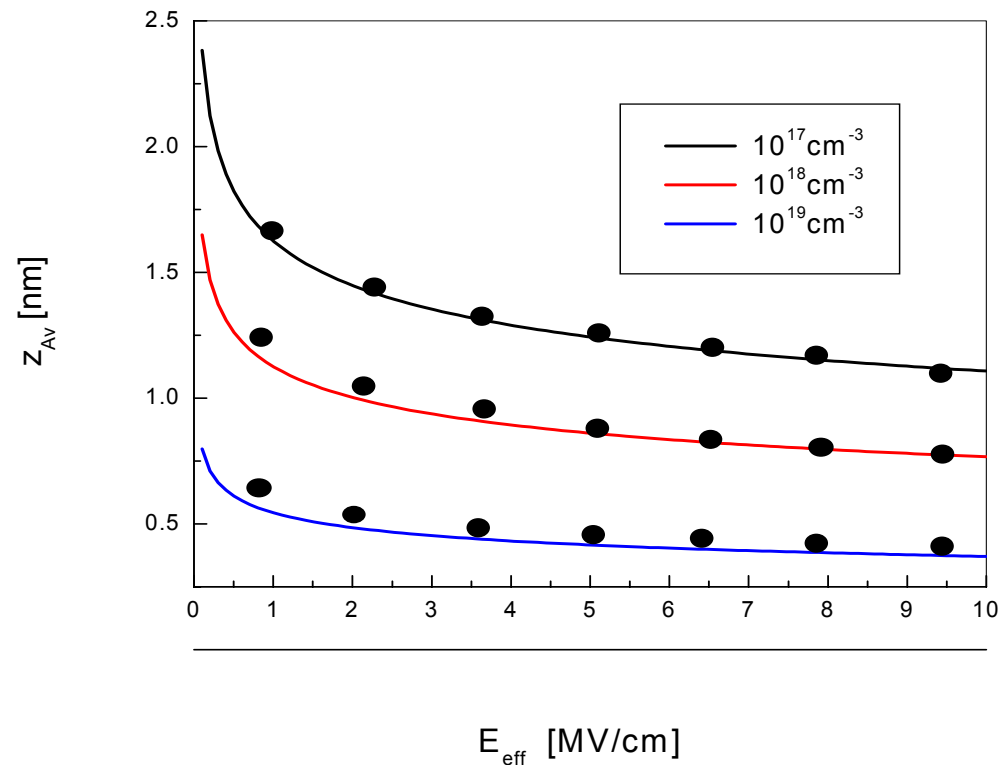
QM energy levels of different valley versus E-field

RESULT and DISCUSSION



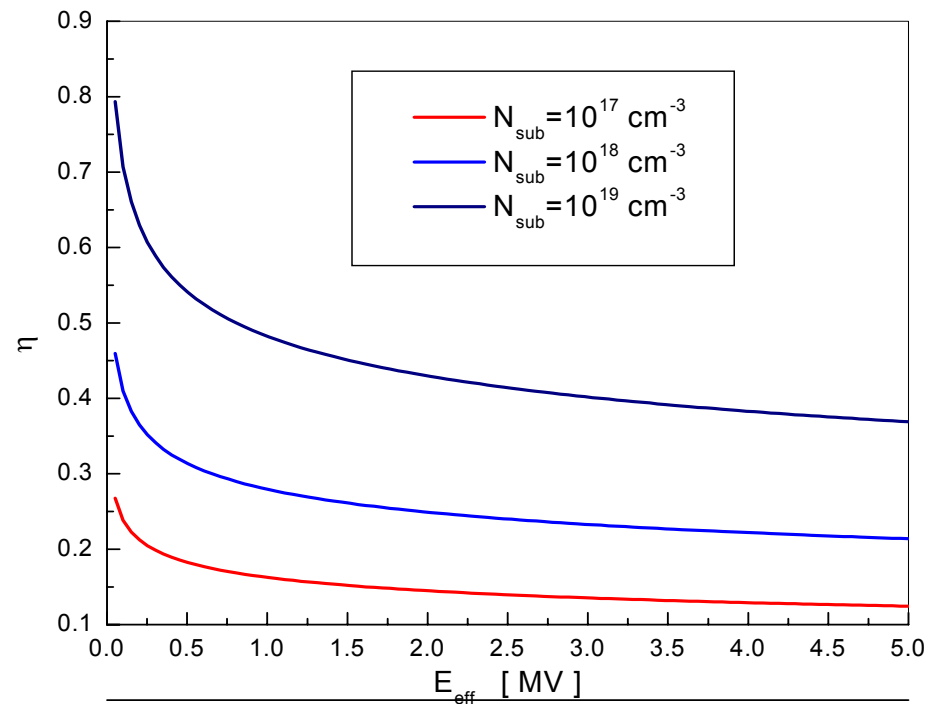
Centriod of different energy level versus E-field

RESULT and DISCUSSION



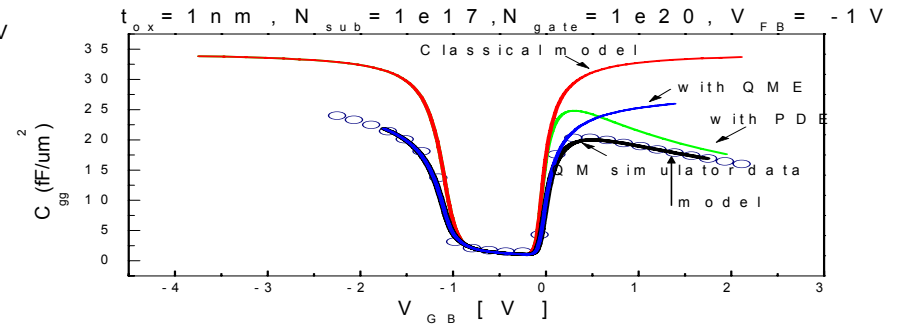
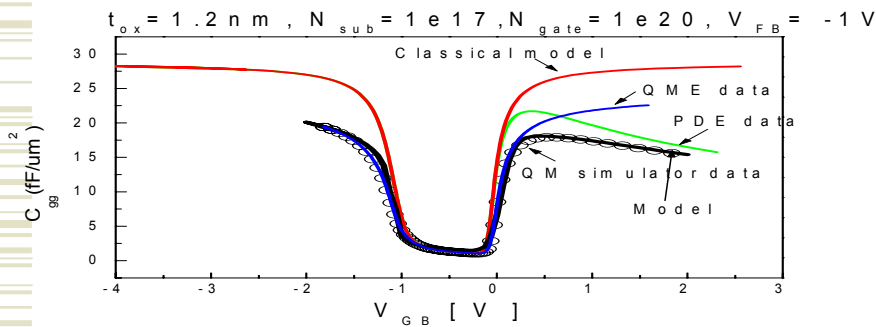
Centriod of different substrate doping versus E-field

RESULT and DISCUSSION



Weight factor of different doping versus E-field

NEW MODEL APPLICATION in BSIM5



Comparison between analytical and numerical of C-V

SUMMARY

- ◆ Physical based analytical model for QME of MOSFET is developed based on the parabolic potential approximation
- ◆ Analytical equations of sub-band energy level, inversion charge centroid and weight factor of E-field are derived
- ◆ The comparison of the QM levels and average centroid between the analytical result and numerical analysis is demonstrated and agreement of both is found
- ◆ New model application has been demonstrated in BSIM5

- ◆ **Acknowledgement:**

This work was supported by SRC Contract #2003-NJ-1149: *BSIM Next Generation Model Research and Development: Surface-Potential-Plus* and partially supported by SRC Contract #2002-NJ-1001: Compact Modeling of Double Gate MOSFET.