Optimized Threshold Voltage MOS Transistor Compact Model from the 4-Component Theory

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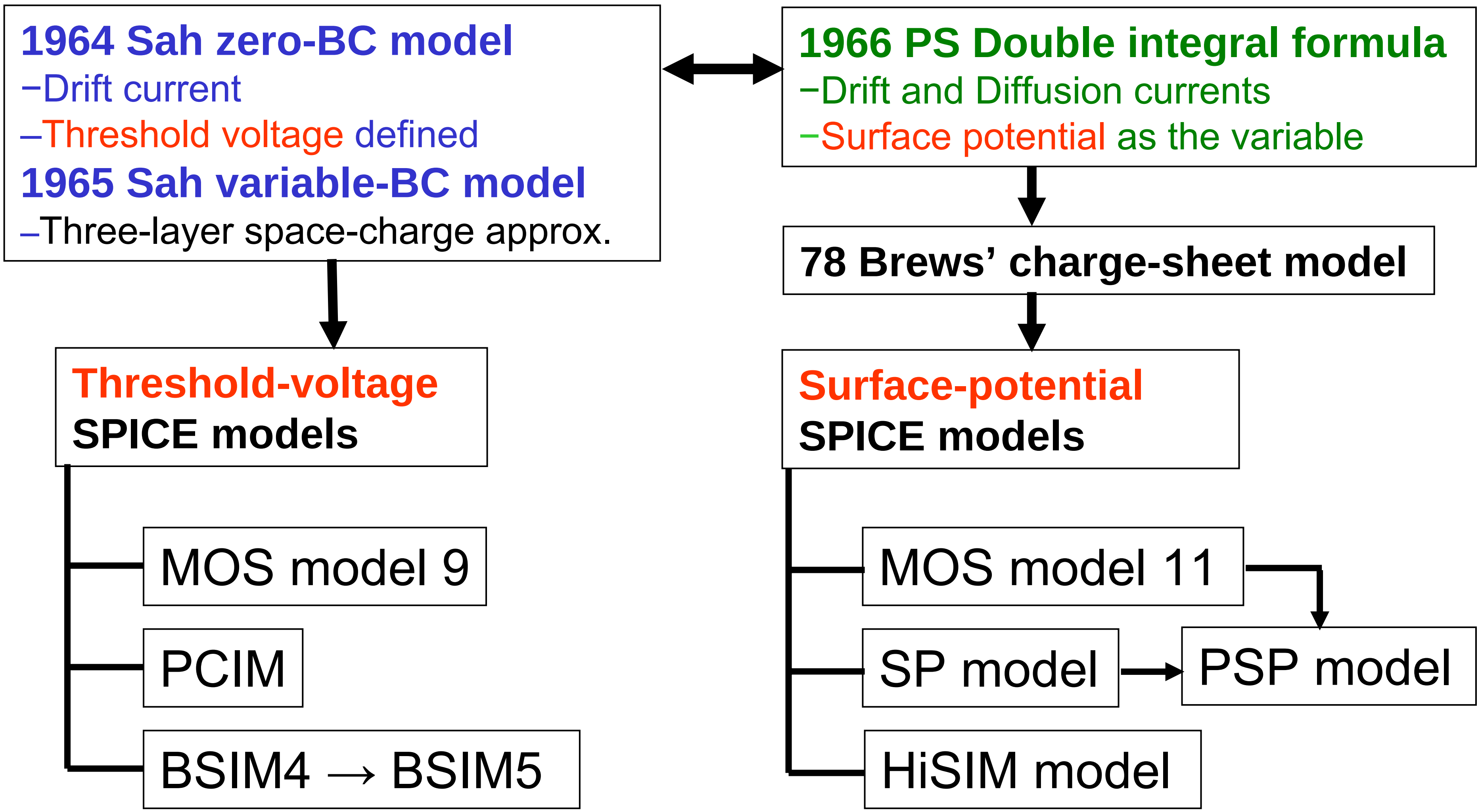
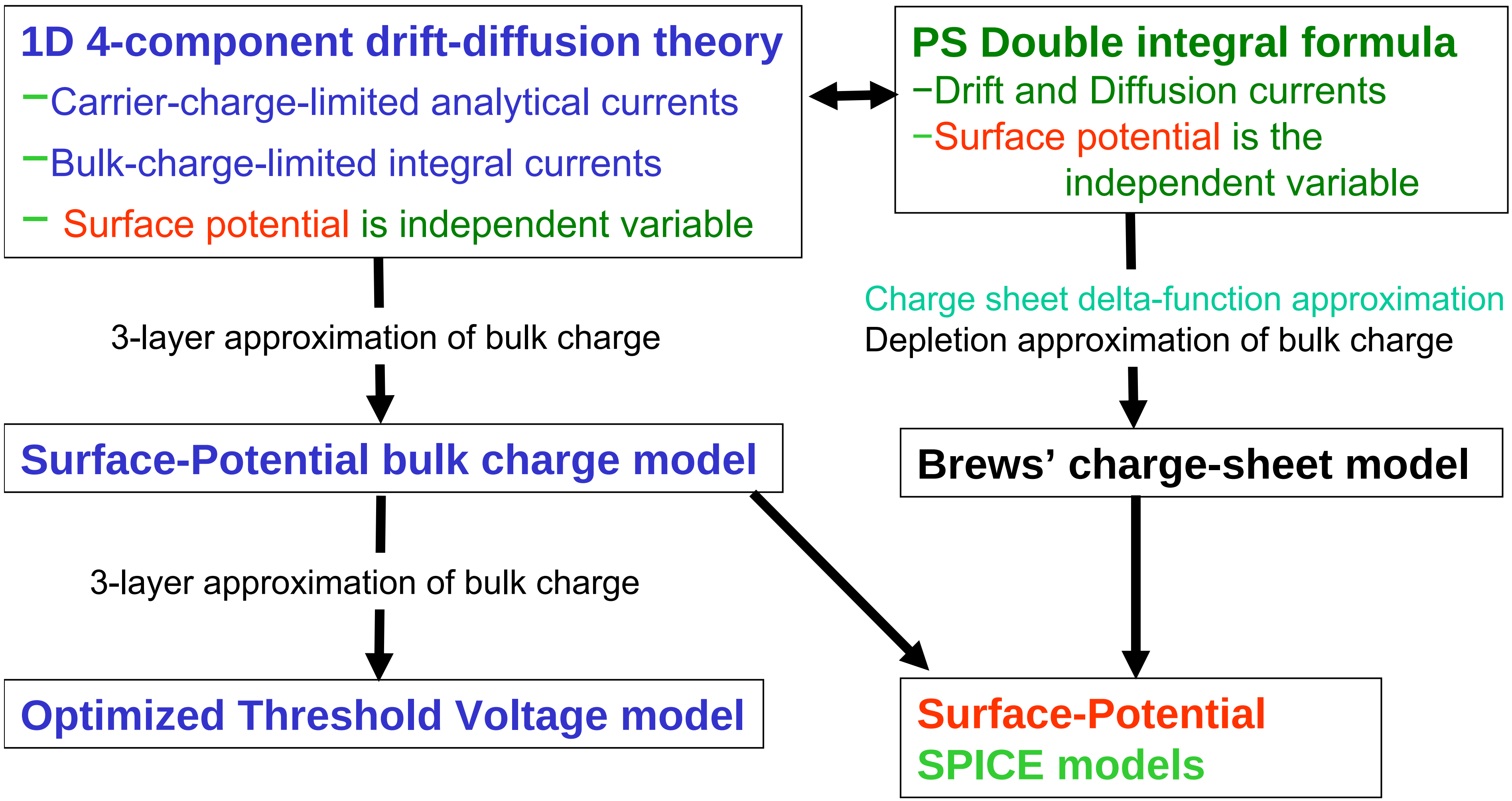
Outline

- 1D 4-Component Theory
- Two possible models
  - Surface Potential (SP) Bulk Charge (BC) Model
  - Optimized Threshold Voltage (TV) Model
- Summary

Connection to Advanced Models



Connection to Advanced Models



1D 4-Component Theory

This started from 1996 Sah Space Charge Theory

Compared with 1966 Pao-Sah Double Integral Formula

- n-channel MOS Transistor
- Constant Channel Impurity Concentration
  - Spatial variation excluded.
- Constant Electron Mobility
  - Mobility variation excluded.
- Long-Wide Channel and Gate
  - Short channel and narrow gate effects excluded.

Drift+Diffusion Model

$$-I_D = (Z/L)\int_0^L \int_0^\infty J_{NY}(x,y,z)dxdy$$

$$= (Z/L)\int_0^L \int_0^\infty \left(q\mu_n (N - N_B) E_Y + qD_n \frac{\partial (N - N_B)}{\partial y} \right) dxdy$$

$$= q\mu_n (Z/L) \left(\int_0^L \int_0^\infty (N - N_B) E_Y dxdy + \frac{kT}{q} \int_0^L \int_0^\infty \frac{\partial (N - N_B)}{\partial y} dxdy \right)$$

Drift Component
Baseline N_B subtracted OK.

Diffusion Component

Integration of Poisson Equation

Poisson Equation

$$\varepsilon_S\left(\frac{\partial E_X}{\partial x}+\frac{\partial E_Y}{\partial y}\right)=q\left[P(x,y)-P_B\right]-q\left[N(x,y)-N_B\right]$$

Gauss Law $\varepsilon_S E_X(x=0)=C_O\left[V_{GX}-V_{FB}-V(0,y)\right]$

nMOST **Bulk Charge**

$$Q_P(0,y)\equiv\int_0^\infty q\left[P_B-P(x,y)\right]dx$$

$$\cong\sqrt{2qP_{AA}\varepsilon_S\{V(0,y)-(kT/q)+(kT/q)\exp[-qV(0,y)/kT]\}}$$

$$\cong\sqrt{2qP_{AA}\varepsilon_S\{V(0,y)-(kT/q)\}}$$

4-Component-Current Formula

$$\frac{I_D L}{\mu_n Z} = \frac{C_O}{2}\left[V_{GX} - V_{FB} - V(0,0)\right]^2$$

Parabolic **Carrier-Space-Charge-Limited** Drift current — Source bias

$$- \frac{C_O}{2}\left[V_{GX} - V_{FB} - V(0,L)\right]^2$$

Drain bias

$$- \int_0^L \int_0^\infty q\,(P_B - P)(-E_Y)\,dxdy$$

Bulk-Charge-Limited Drift depressment

$$+ \frac{kT}{q}\left[V(0,L) - V(0,0)\right]C_O$$

Carrier-Space-Charge-Limited Diffusion

$$+ \frac{kT}{q}\left[Q_P(0,L) - Q_P(0,0)\right]$$

Bulk-Charge Diffusion enhancement

$$+ \frac{1}{2}\varepsilon_S \int_0^L \int_0^\infty \frac{\partial}{\partial y}\left(E_Y^2 - E_X^2\right)dxdy$$

2D Drift enhancement

$$+ \frac{kT}{q}\varepsilon_S \int_0^\infty \left(\left.\frac{\partial E_Y}{\partial y}\right|_{y=L} - \left.\frac{\partial E_Y}{\partial y}\right|_{y=0} \right) dx$$

2D Diffusion dilution

Bulk Charge Approximations

Surface-Potential **Bulk Charge** Approximation

$$Q_P(0,y) \cong \sqrt{2qP_{AA}\varepsilon_S(kT/q)\{U(0,y)-1+\exp[-U(0,y)]\}}$$

Threshold-Voltage 3-Layer **Bulk Charge** Approximation

$$Q_P(0,y) = \sqrt{2qP_{AA}\varepsilon_S}\left\{\left[2V_F - 3\frac{kT}{q} + V_{NP}(0,y)\right]^{+1/2} + 2\frac{kT}{q}\left[2V_F - \frac{kT}{q} + V_{NP}(0,y)\right]^{-1/2} + e^{-1}\frac{kT}{q}\left[2V_F + \frac{kT}{q} + V_{NP}(0,y)\right]^{-1/2}\right\}$$

Threshold-Voltage **Bulk Charge** with Optimization V_θ

$$V(0,y) \equiv V_\theta(y) + 2V_F + (kT/q) + V_{NP}(0,y)$$

Surface Potential Bulk Charge Model

$$\frac{I_D L}{\mu_n Z} \equiv I_{CM} \equiv I_{CM-P1} + I_{CM-P2} + I_{CM-D1} + I_{CM-D2}$$

$$= \frac{C_O}{2}\left\{\left[V_{GX} - V_{FB} - V(0,y)\right]^2 - \left[V_{GX} - V_{FB} - V(0,L)\right]^2\right\}$$

$$- \sqrt{2qP_{AA}\varepsilon_S}\,\frac{2}{3}\left\{\left[V(0,L) - kT/q\right]^{3/2} - \left[V(0,0) - kT/q\right]^{3/2}\right\}$$

$$+ C_O\,\tfrac{kT}{q}\left[V(0,L) - V(0,0)\right]$$

$$+ \sqrt{2qP_{AA}\varepsilon_S}\,\tfrac{kT}{q}\left\{\left[V(0,L) - kT/q\right]^{1/2} - \left[V(0,0) - kT/q\right]^{1/2}\right\}$$

Optimized Threshold Voltage Model

Note $V(0,y) = V_{NP}(0,y) + 2V_F + (kT/q) + V_\theta(y)$

$$\frac{I_D L}{\mu_n Z} \equiv I_{CM} \equiv I_{CM-P1} + I_{CM-P2} + I_{CM-D1} + I_{CM-D2}$$

$$= \frac{C_O}{2}\left[\left(V_{GX} - V_{FB} - V_{SX} - 2V_F - P_1\right)^2 - \left(V_{GX} - V_{FB} - V_{DX} - 2V_F - P_1\right)^2\right]$$

$$- \sqrt{2qP_{AA}\varepsilon_S}\,\frac{2}{3}\left[\left(V_{DX} + 2V_F + P_2\right)^{3/2} - \left(V_{SX} + 2V_F + P_2\right)^{3/2}\right]$$

$$+ C_O \tfrac{kT}{q}\left(V_{DS} + D_1\right)$$

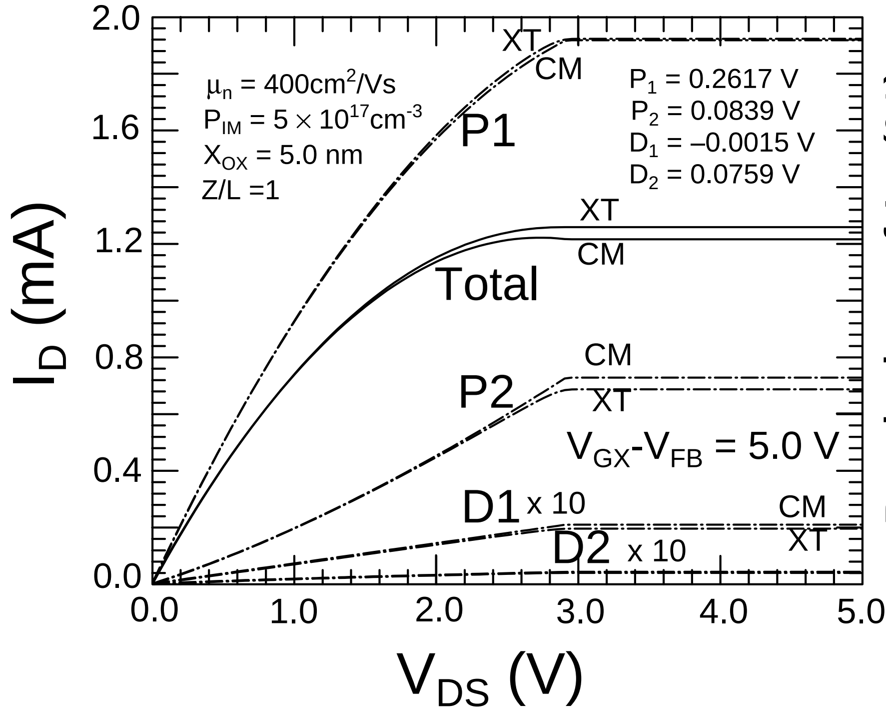
$$+ \sqrt{2qP_{AA}\varepsilon_S}\,\tfrac{kT}{q}\left[\left(V_{DX} + 2V_F + D_2\right)^{1/2} - \left(V_{SX} + 2V_F + D_2\right)^{1/2}\right]$$

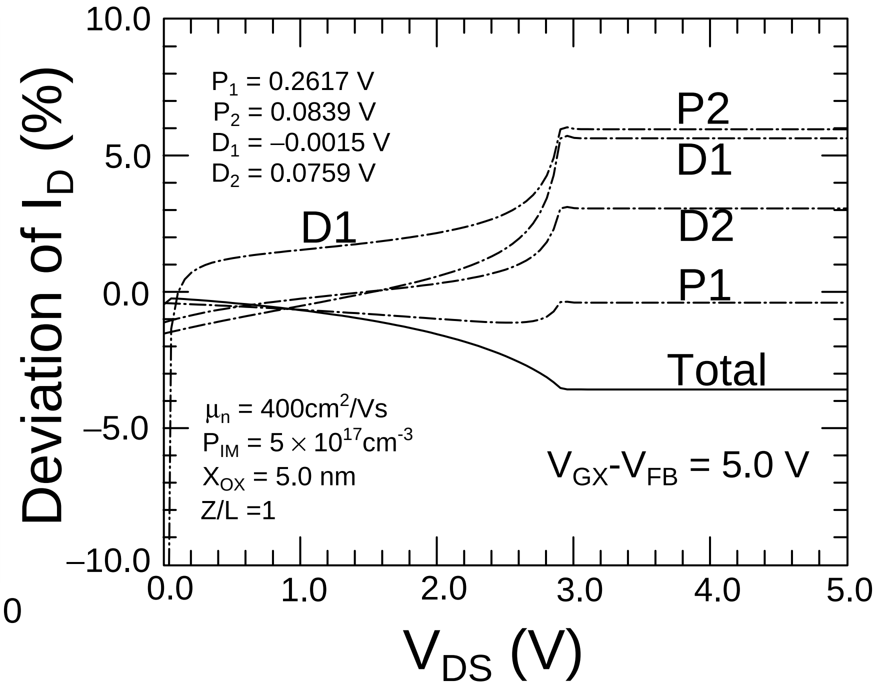
Four Optimization Parameters

- P_1 in **Carrier-Space-charge**-limited Drift
- P_2 in **Bulk-Space-charge** Drift **depressment**
- D_1 in **Carrier-Space-charge**-limited Diffusion
- D_2 in **Bulk-Space-charge** Diffusion **enhancement**
- Gate voltage dependence of the four parameters due to their dependences on surface potential.

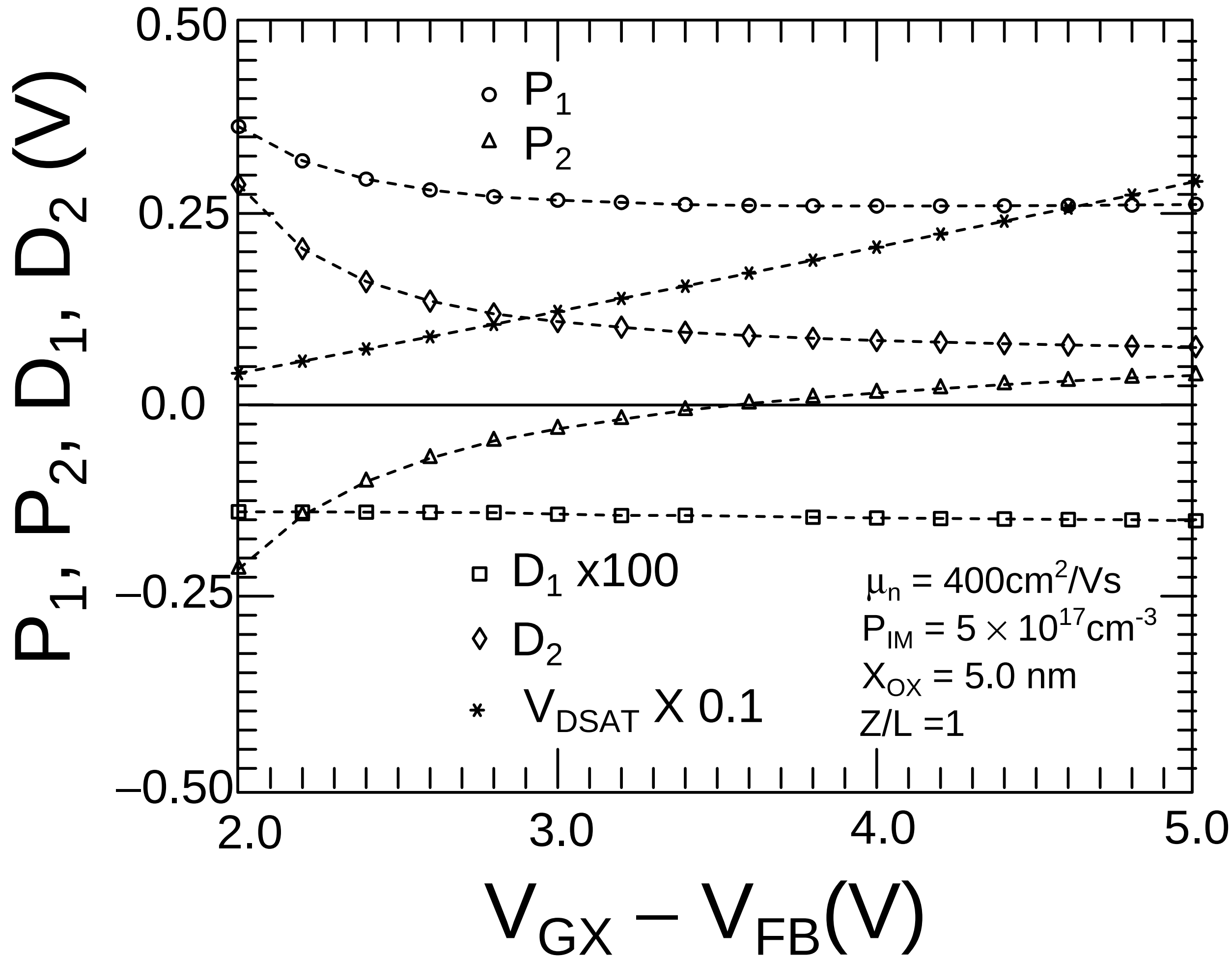
Optimized Model $I_D V_D$ Curve

The four parameters were extracted by Least-Square-Fit of the four components of Optimized model (CM) to those of 4-component theory (XT) in the drain voltage range $(0, V_{DSAT})$ at a gate voltage of $V_{GB}-V_{FB}=5.0V$.





V_{GB} Dependence of P_1, P_2, D_1, D_2



Summary

1996 Sah 1D 4-Component Formula with 2D Terms

Surface-Potential Bulk-Charge Model

No charge-sheet approximation but mathematical result or formula identical to Brews' charge-sheet model

(In press, IEEE-TED.)

Optimized Threshold Voltage Model

Applied to the inversion range

This poster presentation.