

# Analytic Solution for the Drain Current of Undoped Symmetric DG MOSFET

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# Motivation:

- Undoped DG MOSFETs are very attractive for scaling devices because they permit the lessening of SCE by means of an ultra thin body.
- Surface potential-based description seems to be the way to attain model robustness for reliable circuit simulation.
- Surface potential descriptions require that the channel potential be expressed in terms of the terminal voltages, ideally by exact explicit analytic expressions.
- Highly accurate and physics based compact models which are computationally efficient are required.

## Previous work on DG MOSFET:

- In 2000, Taur [1] obtained a system of two equations to describe the potential.
- In 2004, He [2] developed a charge-based drain current model.
- In 2004, Taur [3] derived a drain current model.
- In 2005, Sallese [4] obtained an approximate expressions for the current.

[1] Y. Taur, . IEEE Electron Device Lett., 21, 245–247, 2000.

[2] J . He et al, WCM, NSTI-Nanotech, Boston, 2, 124-127, 2004.

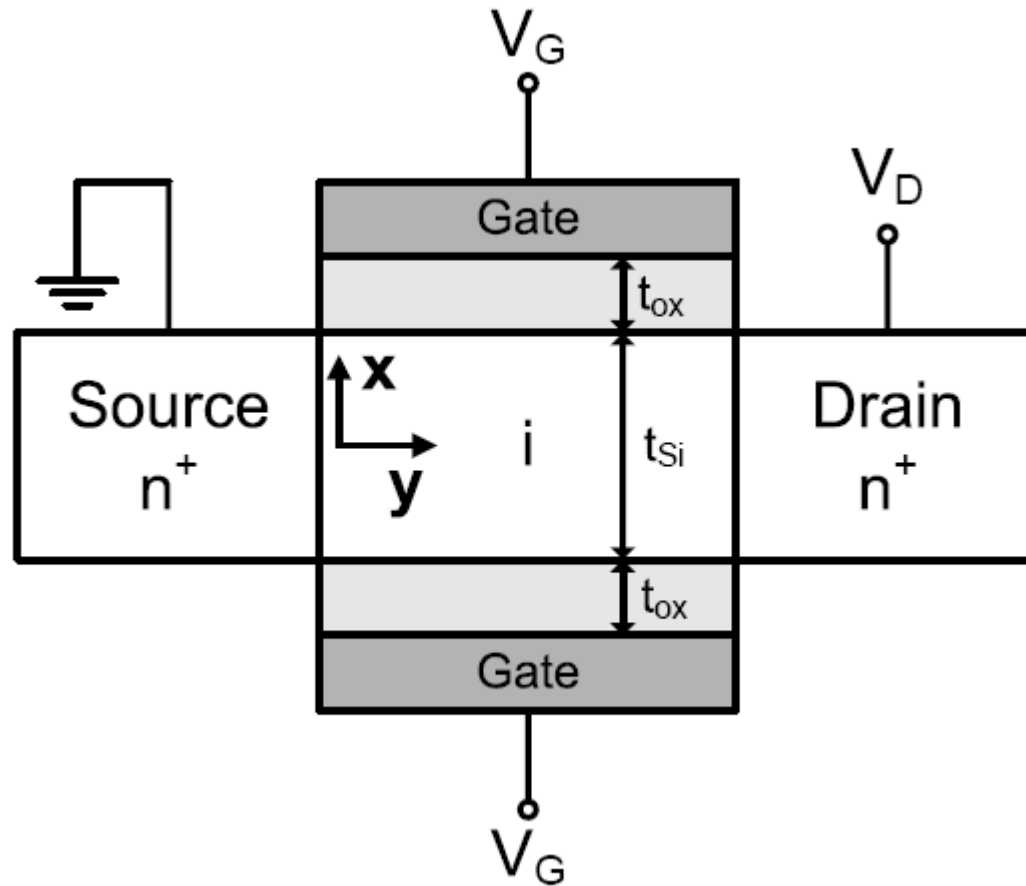
[3] Y. Taur et al, IEEE Electron Device Lett, 25, 107-109, 2004.

[4] J.M. Sallese et al, Solid-St. Electron. 49, 485–489, 2005.

# Contents:

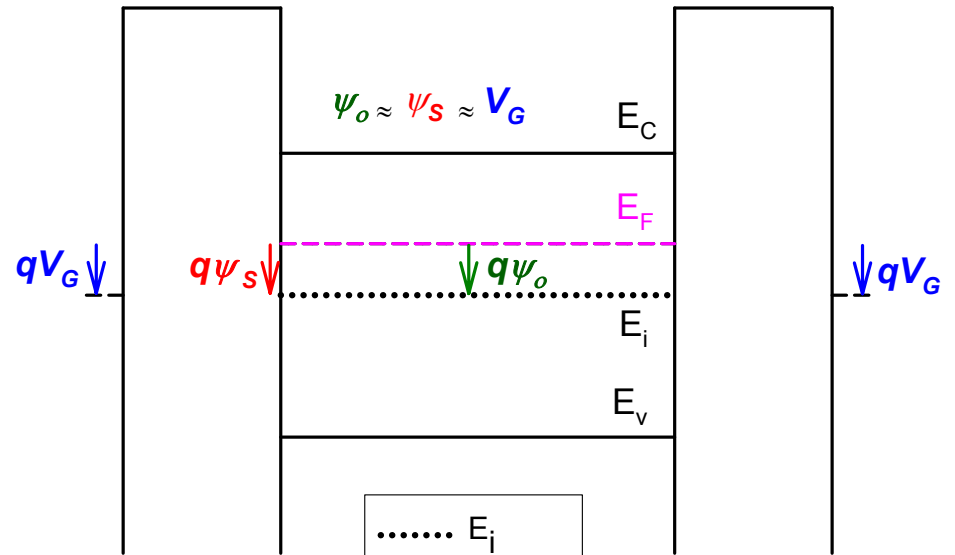
- Part I: A potential-based, analytic, continuous, and fully consistent physical description of the drain current of the undoped body symmetric double gate MOSFET.
- Part II: Considerations about the threshold voltage of undoped DG MOSFETs.

# Symmetric DG n-MOSFET:

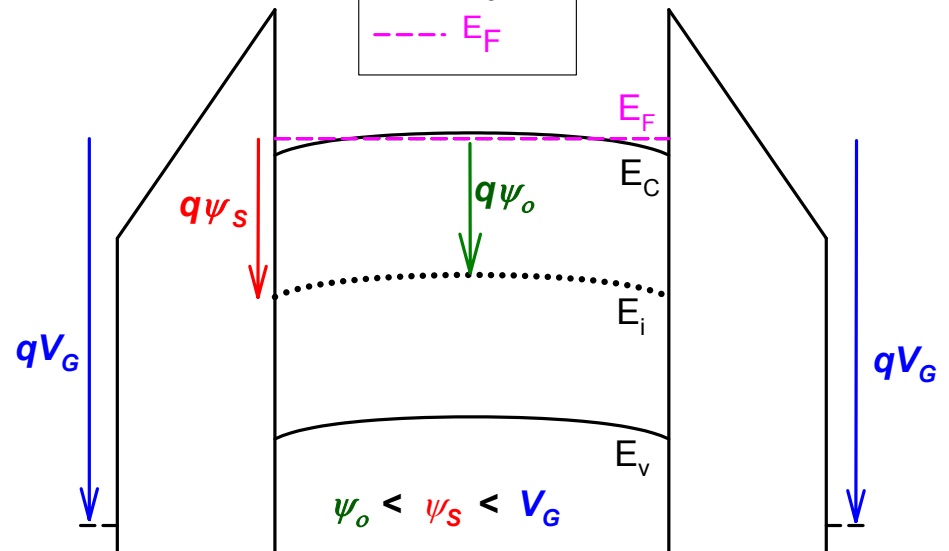


Both gates have same: work function, oxide thickness and applied bias

Below threshold



Above threshold





# Assumptions:

- Quasi-Fermi level is constant across thickness (x direction)
- Current flows only along channel length (y direction)
- The energy levels are referenced to the electron quasi-Fermi level of the n+ source.
- Our formulation does not account for short-channel effects, carrier confinement energy quantization, interface roughness, ballistic-type transport, and mobility degradation.

# Pao-Sah's type formulation:

Under the assumption that the mobility is independent of position in the channel:

$$I_D = 2 \mu \frac{W}{L} \int_0^{V_{DS}} \int_{\psi_o}^{\psi_s} \frac{q n}{F} d\psi dV$$

where: 
$$Q_I \equiv -2q \int_0^{t_{Si}/2} (n - n_i) dx = -2q \int_{\psi_o}^{\psi_s} \frac{n - n_i}{F} d\psi$$

substituting: 
$$I_D = 2 \mu \frac{W}{L} \epsilon_s \int_0^{V_{DS}} \int_{\psi_o}^{\psi_s} \left( \frac{1}{2F} \frac{d\alpha}{dV} - \frac{\partial F}{\partial V} \right) d\psi dV$$

# Pierret and Shields's type model:

In 1983, Pierret and Shields [7] transform the Pao-Sah's double integral formulation, for bulk MOSFET, into a single integral without introducing any additional approximation.

In 1992, we obtained [8] a single integral current equation for a SOI MOSFET by using the previous ideas.

The charge coupling factor,  $\alpha$ , was used:

$$\alpha \equiv F_s^2 - G^2(\psi_s, V)$$

where  $G(\psi, V)$  is the Kingston function.

Now, we will use the same procedure for a DG MOSFET.

[7] R.F. Pierret, J.A. Shields, *Solid-St. Electron.* 26, 143-147, 1983.

[8] A. Ortiz-Conde et al, *Solid-St. Electron.* 35, 1291-1298, 1992.

# Pierret and Shields's type model:

Electric field: 
$$F = -\sqrt{\frac{2kT n_i}{\epsilon_s} e^{\beta(\psi-V)} + \alpha}$$

where: 
$$\alpha \equiv F_s^2 - G^2(\psi_s, V) = -\frac{2kT n_i}{\epsilon_s} e^{\beta(\psi_o-V)}$$

is the charge coupling factor.

Taking the partial derivative: 
$$\frac{\partial F}{\partial V} = \frac{1}{2F} \frac{d\alpha}{dV} - \frac{1}{F} \frac{q n_i}{\epsilon_s} e^{\beta(\psi-V)}$$

recalling that: 
$$n = n_i e^{\beta(\psi-V)}$$

we obtain: 
$$\frac{q n}{F} = \epsilon_s \left( \frac{1}{2F} \frac{d\alpha}{dV} - \frac{\partial F}{\partial V} \right)$$

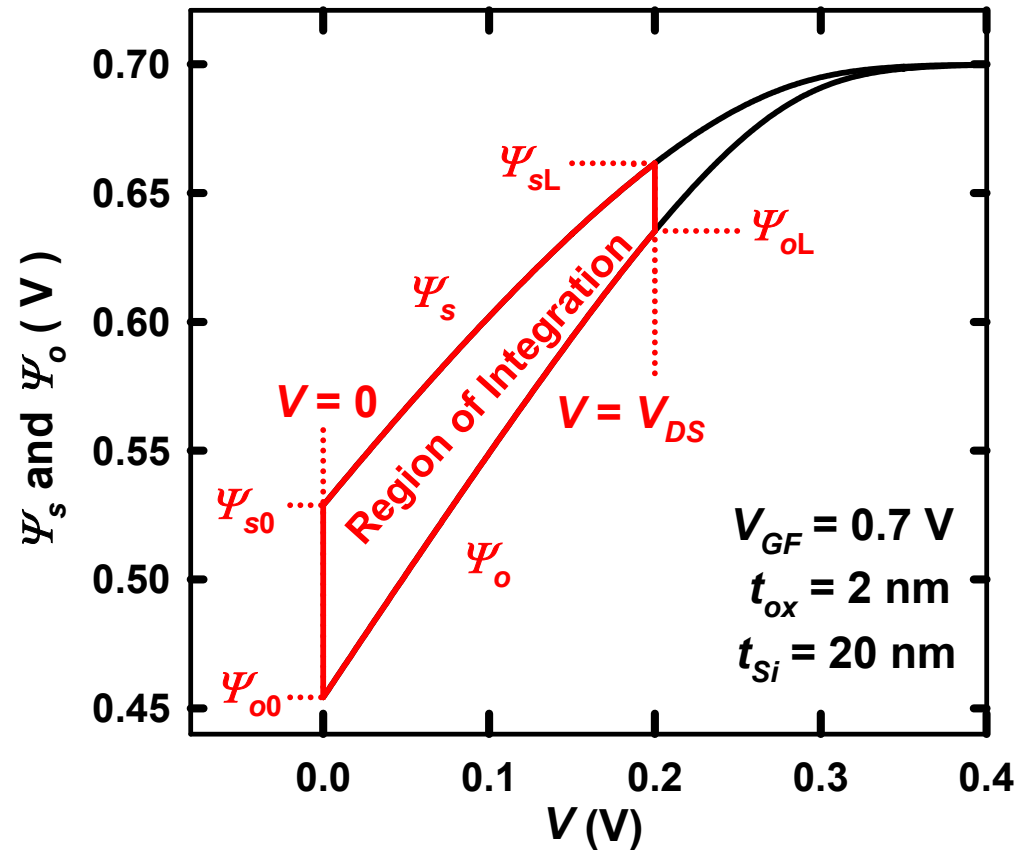
Substituting into the double integral:

$$I_D = 2 \mu \frac{W}{L} \epsilon_s \int_0^{V_{DS}} \int_{\psi_o}^{\psi_s} \left( \frac{1}{2F} \frac{d\alpha}{dV} - \frac{\partial F}{\partial V} \right) d\psi dV$$

# Pierret and Shields's type model:

The region of integration is shown in the figure.

Breaking the integral into different parts and then solving all of them, we obtain an analytic solution without introducing any additional approximation.



## Drain current equation: [9]

$$I_D = \mu \frac{W}{L} \left\{ \begin{array}{l} \boxed{2C_o \left[ V_{GF} (\psi_{SL} - \psi_{S0}) - \frac{1}{2} (\psi_{SL}^2 - \psi_{S0}^2) \right]} \text{ first term} \\ \boxed{+ 4 \frac{kT}{q} C_o (\psi_{SL} - \psi_{S0})} \text{ second term} \\ \boxed{+ t_{Si} kT n_i \left[ e^{\beta(\psi_{oL} - V_{DS})} - e^{\beta\psi_{o0}} \right]} \text{ Third term} \end{array} \right\}$$

where  $V_{GF}$  is the difference between the gate-to-source voltage and the flat-band voltage. This equation is equivalent to that of Taur [10], although the derivation was different.

This equivalence is established, after some algebraic manipulations, by noting that the beta,  $\beta_T$ , introduced in [10] is related to our charge coupling factor  $\alpha$  by:

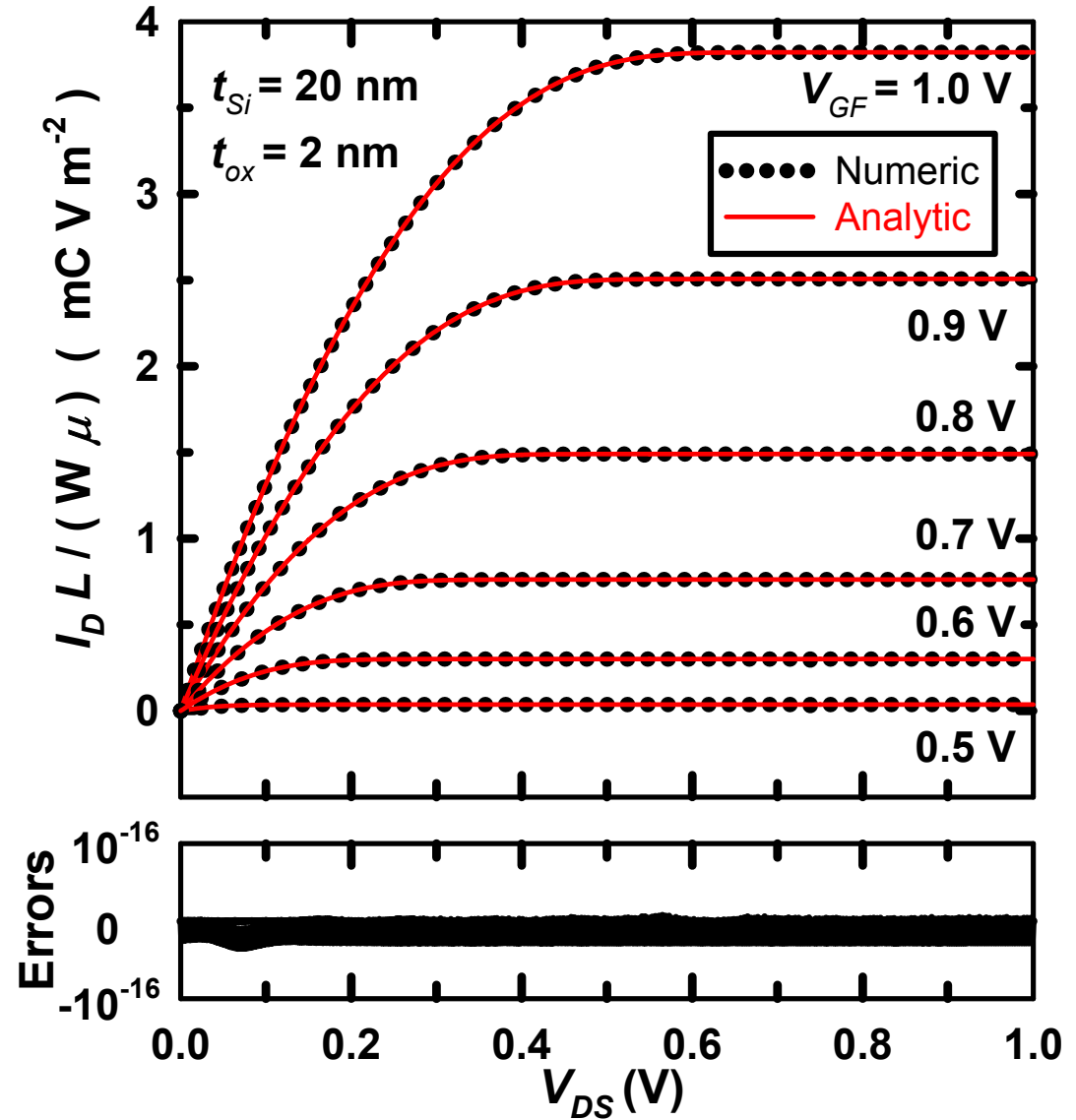
$$\beta_T \equiv \frac{-\alpha}{\left(4 \frac{kT}{q t_{Si}}\right)^2}$$

[9] A. Ortiz-Conde et al, "Rigorous analytic solution for the drain current of undoped symmetric dual-gate MOSFETs", Solid-State Electronics, 49, 640-647, 2005.

[10] Y. Taur et al, "A Continuous, Analytic Drain-Current Model for DG MOSFETs", IEEE Electron Device Lett, 25, 107-109, 2004.

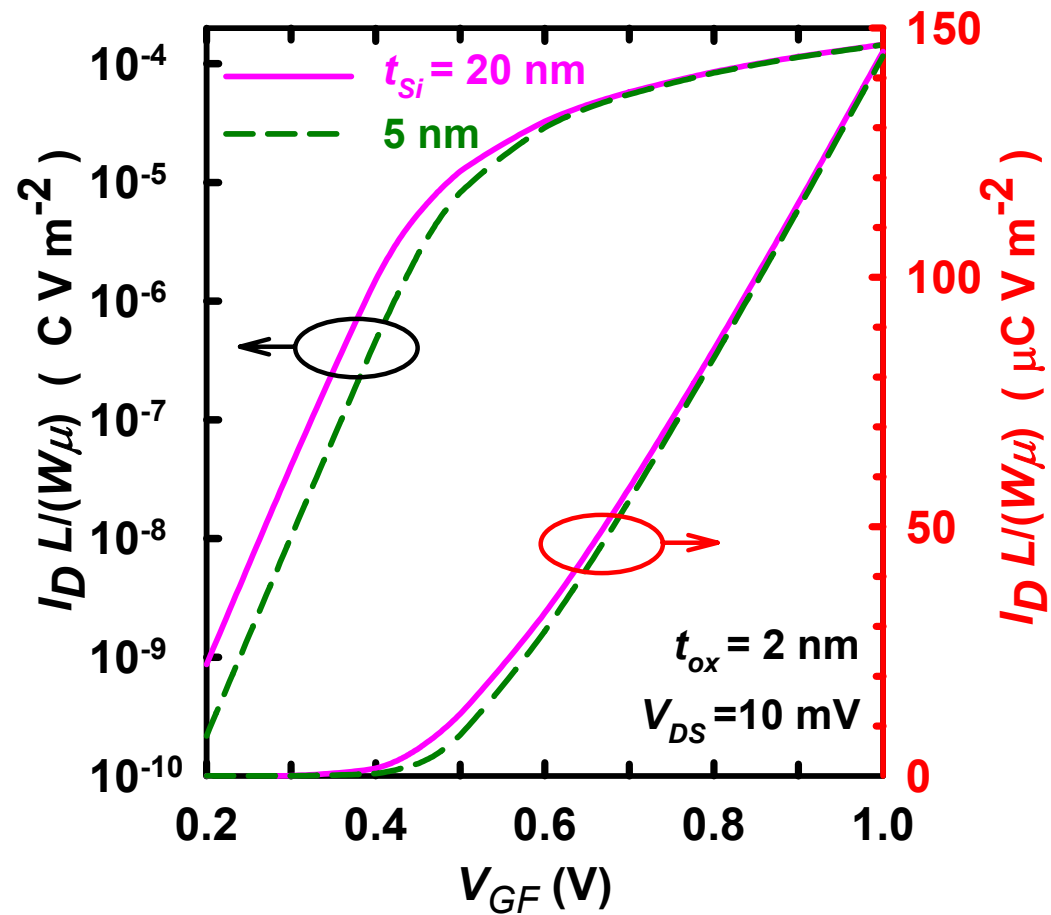
# Comparison:

Normalized drain current as a function of drain voltage



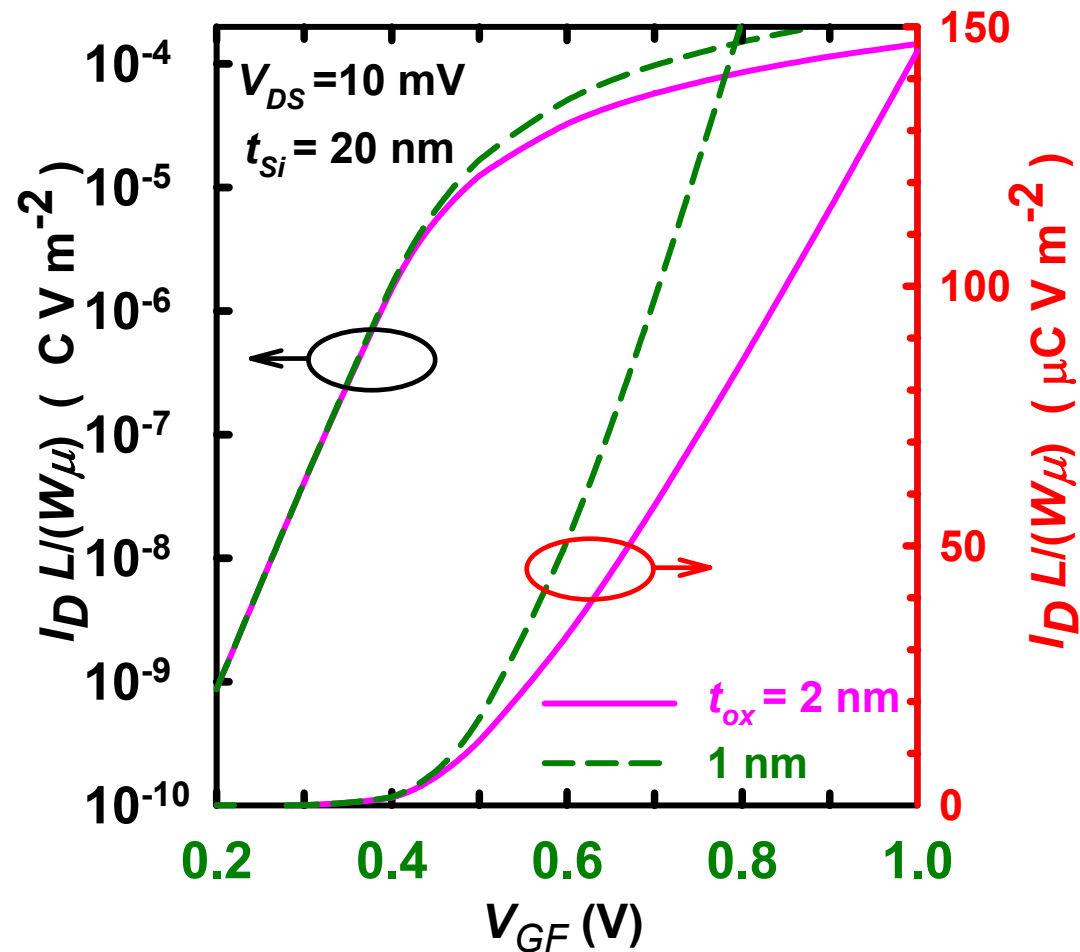
# Semiconductor film thickness:

Effect of semiconductor film thickness on drain current at low drain voltage.



# Gate oxide film thickness:

Effect of gate oxide thickness on drain current at low drain voltage.



# Current components:

In **weak conduction**: the first term is negligible, the **second term** is positive, the **third term** is negative.

The combination of the **second** and **third** term gives a positive value approximated by:

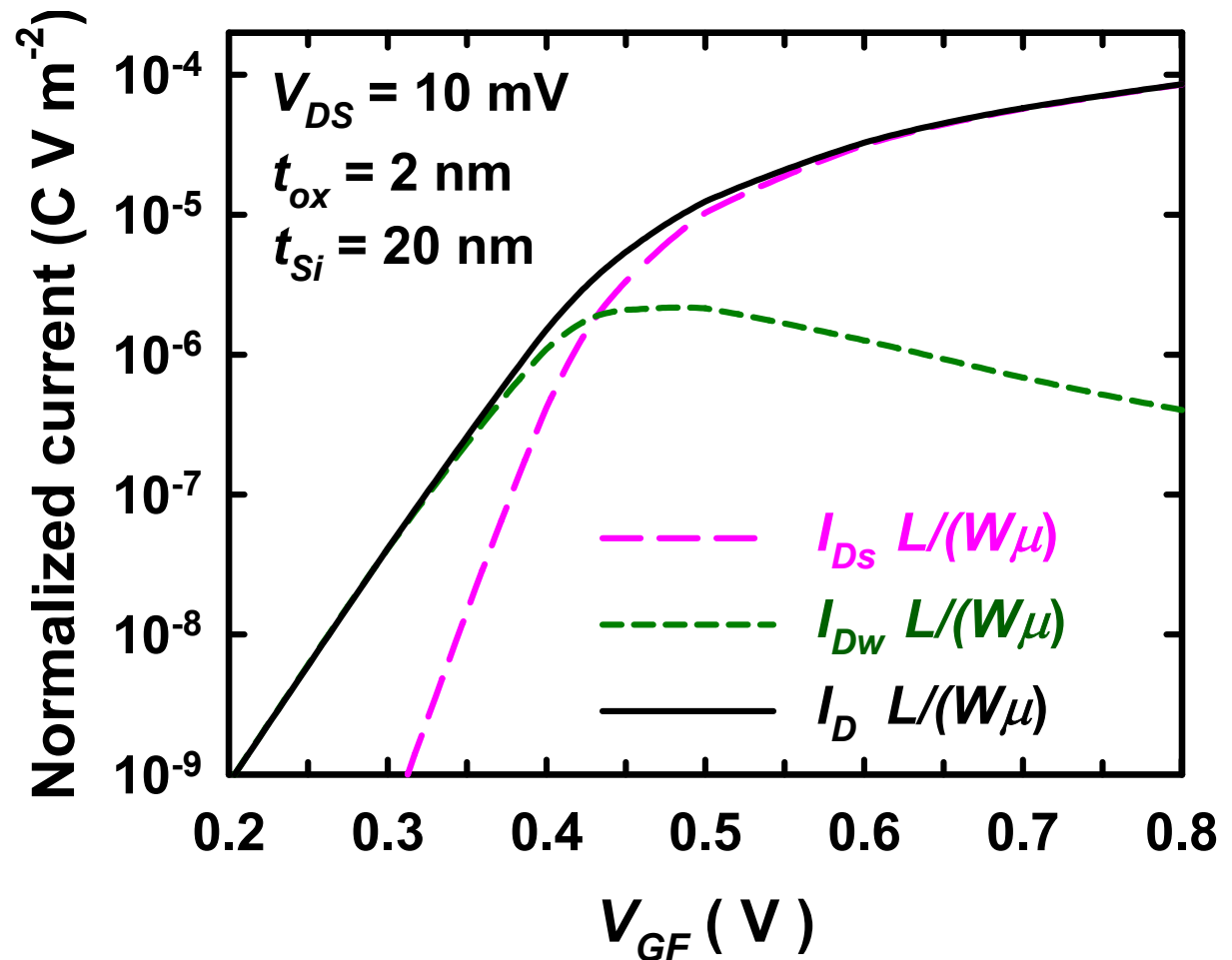
$$I_{Dw} \approx \mu \frac{W}{L} t_{Si} k T n_i \left[ e^{\beta \psi_{o0}} - e^{\beta (\psi_{oL} - V_{DS})} \right]$$

The strong conduction term can be obtained by subtracting the weak component from the total current:

$$I_{Ds} = I_D - I_{Dw}$$

# Current components:

Current equation and its two terms  $I_{Dw}$  and  $I_{Ds}$ , vs gate voltage.



# Threshold:

The intersection of the two current components ( $I_{Dw}$  and  $I_{Ds}$ ) may be understood as the transition threshold from weak to strong conduction.

Below threshold:  $\psi_{SL} \approx \psi_{S0} \approx \psi_{oL} \approx \psi_{o0} \approx V_{GF}$   $I_{Dw} = \mu \frac{W}{L} t_{Si} q n_i e^{\beta V_{GF}} V_{DS}$

Above threshold:  $I_{Ds} = \mu \frac{W}{L} Q_{bulk} V_{DS}$   $Q_{bulk} = \frac{4 C_o}{\beta} W \left( \frac{q\sqrt{2}}{2 C_o} \sqrt{\frac{\epsilon_s n_i}{kT}} e^{\frac{\beta V_{GF}}{2}} \right)$

where  $W$  is the principal branch of the Lambert Function and  $Q_{bulk}$  is the solution [11] for bulk devices. Equating  $I_{Dw}$  and  $I_{Ds}$  gives:

$$V_{Tx} = \frac{kT}{q} \left[ \ln \left( \frac{8 kT \epsilon_s}{q^2 n_i t_{Si}^2} \right) - W \left( \frac{4 \epsilon_s}{C_o t_{Si}} \right) \right] \approx \frac{kT}{q} \ln \left( \frac{8 kT \epsilon_s}{q^2 n_i t_{Si}^2} \right)$$

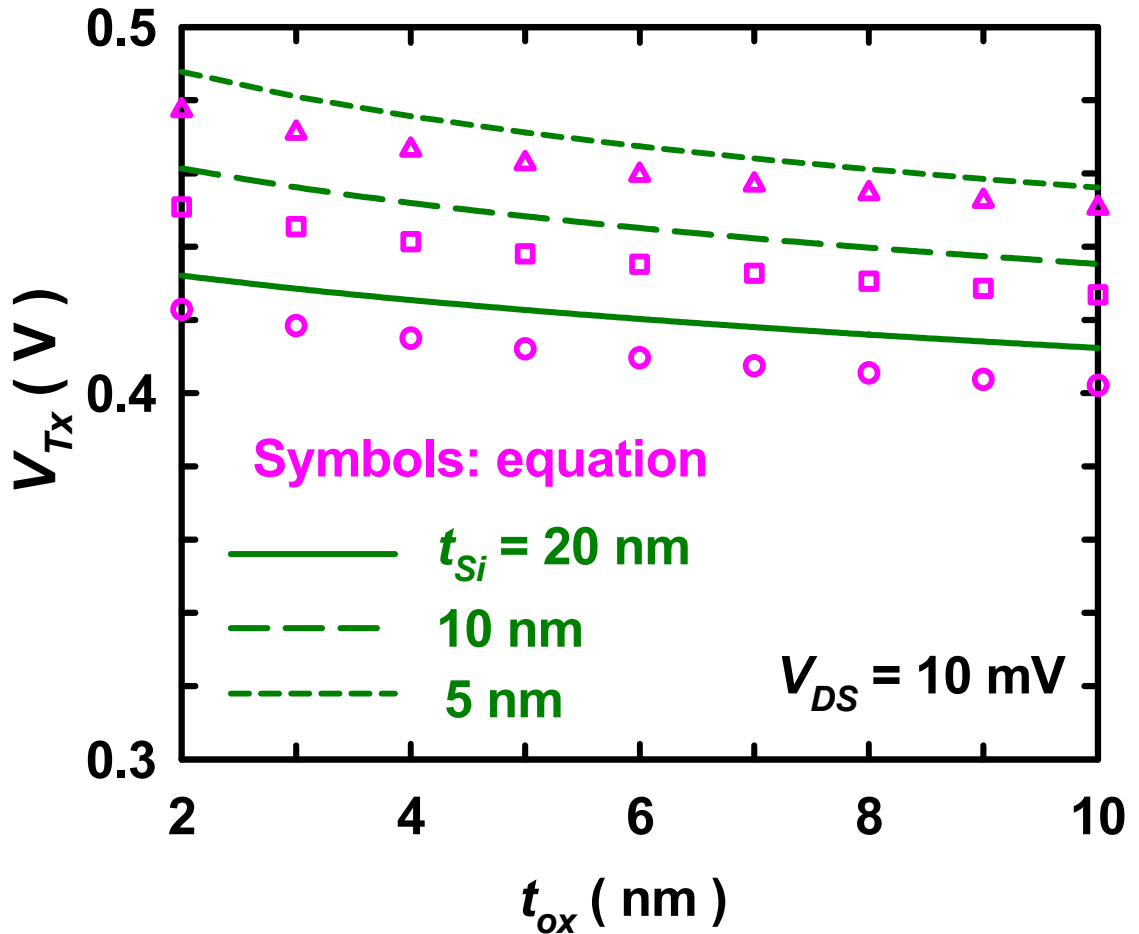
[11] Ortiz-Conde et al, "Exact analytical solution of channel surface potential as an explicit function of gate voltage in undoped-body MOSFETs using the Lambert W function and a threshold voltage definition therefrom", Solid-State Electronics, 47, 2067-2074, 2003.

# Threshold:

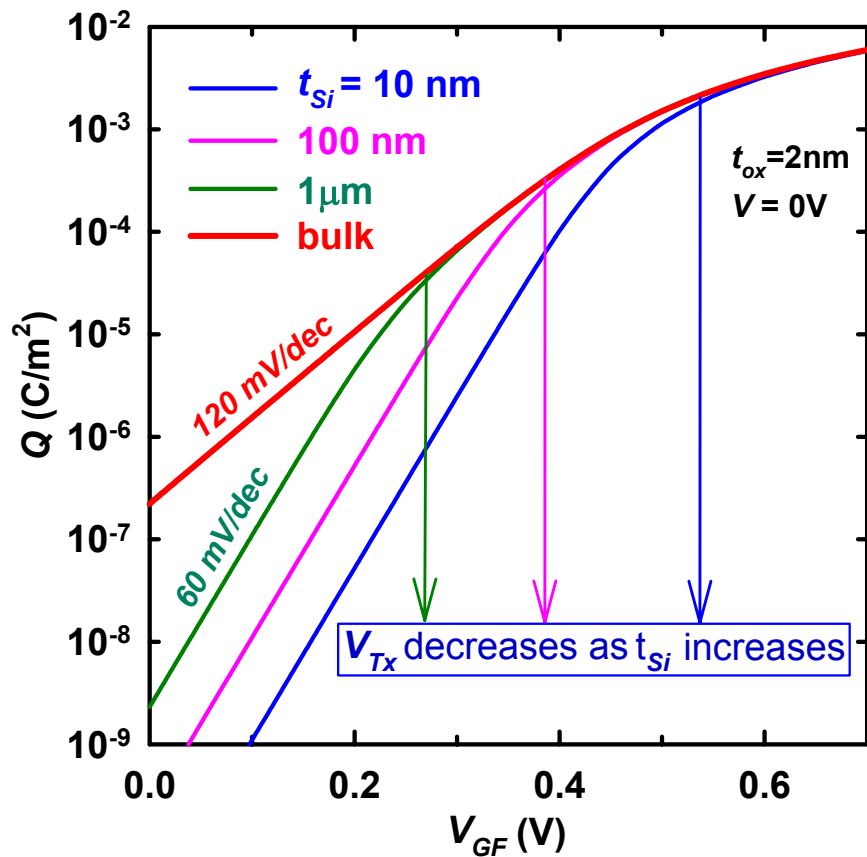
$$V_{Tx} = \frac{kT}{q} \left[ \ln \left( \frac{8 kT \epsilon_s}{q^2 n_i t_{Si}^2} \right) - W \left( \frac{4 \epsilon_s}{C_o t_{Si}} \right) \right] \approx \frac{kT}{q} \ln \left( \frac{8 kT \epsilon_s}{q^2 n_i t_{Si}^2} \right)$$

Intersection of the two current components: lines from plots and symbols from equation.

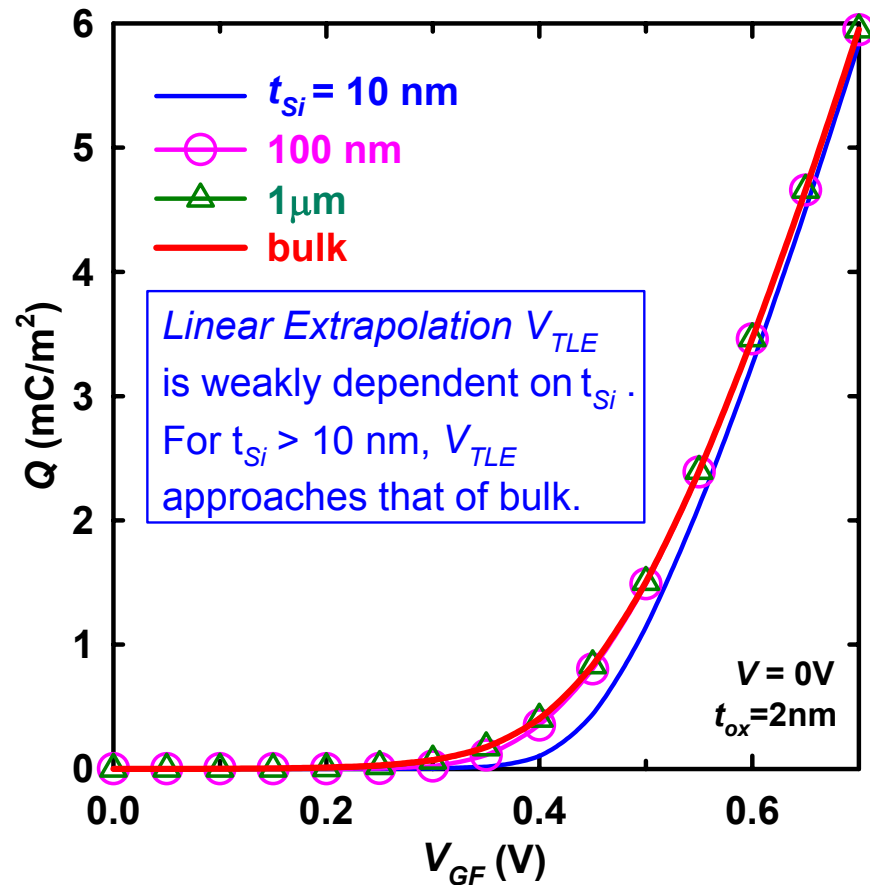
The difference between the two results is approximately  $V_{DS}$ .



For large  $t_{Si}$ , there are two sub-threshold regions ( 60 and 120 mV/dec) and  $V_{Tx}$  is the threshold between these two regions.



For strong conduction, all cases approaches bulk. Therefore, the threshold between strong and weak conduction approaches that of bulk devices.



## Conclusions:

- Exact potential-based analytic description of the drain current in the undoped-body symmetric DG MOSFET.
- Threshold voltage increases with decreasing  $t_{ox}$  (and  $t_{Si}$  in DG).
- For large  $t_{Si}$ , there are two sub-threshold regions ( 60 and 120 mV/dec).

# END of presentation

Thank you for your attention

