Quasi-2D Compact Modeling for Double-Gate MOSFET

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Outline

- A generic Surface Potential Plus framework for charge calculation in symmetric and asymmetric Double-Gate MOSFET
- Derivation of self-consistent Mobility model
- Quasi-2D Modeling of the Velocity Saturation Region
Initial assumptions

- Quasi-Fermi potential is constant along the vertical direction
- No current exchange in the vertical direction

Do not rely on Charge-Sheet approximation to allow a quasi-2D current distribution
Basic Current Equations

Similar to BSIM-SPP bulk model

- Starts with drift-diffusion equation in term of quasi-fermi potential

\[ I_{DS} = \mu_{eff} W Q_I \frac{dV_{ch}}{dy} \]

- Following the SPP derivation, we have

\[ I_{DS} = \frac{\mu_{eff}}{L_{eff}} W_{eff} C_{ox} \sqrt{V_{th}} \left[ \frac{Q_S^2 - Q_D^2}{2} + (Q_S - Q_D) \right] \]

- need to solve the charge at the source and drain, not only at the surface, but entire silicon film
Charge Calculation

Formulation

- solving the Poisson’s equation using the minimum potential as reference

\[
\frac{d^2 \phi}{dx^2} = \frac{q}{\varepsilon_{si}} n_i e^{q(\phi - \phi_{ch})/kT}
\]
Charge Calculation

- integrating the Poisson’s Equation, we have

\[
\frac{d\phi}{dx} = -\sqrt{\frac{2kT n_i}{\varepsilon_{si}}} \left( \frac{q\phi}{e^{kT}} - e^{-kT} \right) + \left( \frac{d\phi}{dx} \big|_{x=x_0} \right)^2
\]

- defining

\[
A = \frac{2kT n_i}{\varepsilon_{si}}, \quad B = A \exp\left( \frac{q\phi_{\text{min}}}{kT} \right) - \left( \frac{d\phi}{dx} \big|_{x=x_0} \right)^2
\]

- solutions for potential and carriers are

\[
\phi(x) = \frac{2kT}{q} \ln \left[ \sqrt{\frac{A}{B}} \cos \left( \frac{q\sqrt{B}}{2kT} (x_0 - x) \right) + \cos^{-1} \left( \sqrt{\frac{B}{A}} \frac{q\phi_{\text{min}}}{kT} \right) \right]
\]

\[
n(x) = \frac{n_i \left( \frac{B}{A} \right)}{\cos^2 \left( \frac{q\sqrt{B}}{2kT} (x_0 - x) \right) + \cos^{-1} \left( \sqrt{\frac{B}{A}} \frac{q\phi_{\text{min}}}{kT} \right)}
\]
Charge Calculation

- the charge is calculated by integrating the carrier along the vertical direction

\[
Q_{inv} = \frac{2kTn_i}{\sqrt{B}} \left[ \tan \left( \frac{q\sqrt{B}}{2kT} (x - x_0) + \cos^{-1}\left( \sqrt{\frac{B}{A} e^{-\frac{q\phi_{min}}{2kT}}} \right) \right) - \tan \left( \cos^{-1}\left( \sqrt{\frac{B}{A} e^{-\frac{q\phi_{min}}{2kT}}} \right) \right) \right]
\]

- the charge can be evaluated by relating \( \phi_{min} \) to the gate voltage and quasi-fermi potential

- limitation: current formulation assumes we know the location of \( \phi_{min} \)

- can only be applied to SDG or n+/p+ polysilicon ADG MOSFET at a \( V_{DD} < 1V \)
Relating to External Bias

- Relating to charge to gate voltage

\[ V_G - V_{FB} = \phi_S + E_{ox}t_{ox} = \phi_s + \frac{Q_{inv}}{\varepsilon_{ox}}t_{ox} \]

- using previous expression for potential and charge

\[ V_G - V_{FB} = \frac{kT}{q} \ln \left[ \frac{n_0}{n_i} \cos^{-2} \left( \frac{q^2n_0}{2\varepsilon_{Si}kT} \right)^{1/2} \frac{T_{Si}}{2} \right] \]

\[ + \frac{\varepsilon_{Si}}{\varepsilon_{ox}} t_{ox} \left[ 2n_0\varepsilon_{Si}kT \right]^{1/2} \tan \left[ \left( \frac{q^2n_0}{2\varepsilon_{Si}kT} \right)^{1/2} \frac{T_{Si}}{2} \right] \]
For Symmetric DG MOSFET

- Further simplification in SDG case

\[ Q_I = \left[ \frac{8n_0 kT}{\varepsilon_{Si}} \right]^{1/2} \sin \left( \frac{\left( \frac{q^2 n_0}{2\varepsilon_{Si} kT} \right)^{1/2} T_{Si}}{2} \right) \cos \left( \frac{\left( \frac{q^2 n_0}{2\varepsilon_{Si} kT} \right)^{1/2} T_{Si}}{2} \right) \approx \left[ \frac{8n_0 kT}{\varepsilon_{Si}} \right]^{1/2} \sin \left( \frac{\left( \frac{q^2 n_i}{2\varepsilon_{Si} kT} \right)^{1/2} T_{Si}}{2} \right) \cos \left( \frac{\left( \frac{q^2 n_0}{2\varepsilon_{Si} kT} \right)^{1/2} T_{Si}}{2} \right) \]

- It can be shown that

\[ Q_I = 2C_{ox} V_{th} W_0 \left\{ \left( \frac{q n_i \varepsilon_{Si}}{2V_{th} C_{ox}} \right)^{1/2} \sin \left( \frac{\left( \frac{q n_i}{2\varepsilon_{Si} V_{th}} \right)^{1/2} T_{Si}}{2} \right) \exp \left[ \frac{V_G - V_{FB} - \phi_{fn}}{2V_{th}} \right] \right\} \]
Output Characteristics

1. For **intrinsic** channel doping, volume inversion is valid and the potential through the Si film is **flat** in the subthreshold region.
2. The inversion charge (current) in the subthreshold region is proportional to $T_{si}$. 
Mobility Calculation

Assumptions

• First deal with the low-field mobility
• The dependent of mobility with gate bias can be formulated in term of effective vertical $E$-field
• The impacts of other scattering mechanisms on mobility are bias independent
• without loss of generality, the universal mobility will be used for now

$$\mu_{\text{eff}} = \frac{\mu_0}{1 + \left(\frac{\vec{E}_{\text{eff}}}{\vec{E}_0}\right)^\nu}$$
Calculating Effective $E$-Field

- Task: calculate the Effective $E$-field

$$E_{\text{eff}} = \frac{\int_0^{x_0} |E(x)| \cdot n(x) \, dx}{\int_0^{x_0} n(x) \, dx}$$

- $E$-field expression by solving the Poisson’s equation

$$E(x) = \frac{2kT n_i \left( \frac{B}{A} \right)}{\varepsilon_{si} \sqrt{B}} \left[ \tan \left( \frac{q \sqrt{B}}{2kT} (x_0 - x) + \cos^{-1} \left( \frac{B}{A} e^{-q\phi_{\text{min}}/2kT} \right) \right) - \tan \left( \cos^{-1} \left( \frac{B}{A} e^{-q\phi_{\text{min}}/2kT} \right) \right) \right] - \frac{d\phi}{dx} \bigg|_{x=x_0}$$

- Final expression for $E_{\text{eff}}$

$$E_{\text{eff}} = \frac{Q_{\text{inv}}}{2\varepsilon_{si}} - \frac{d\phi}{dx} \bigg|_{x=x_0}$$
Calculating Effective $E$-Field

**Observations**

- In SDG MOSFET, the inversion charge is doubled but effective E-field is not increased, and in term of total inversion charge, it should be

$$E_{\text{eff}} = \frac{Q_{\text{inv}}}{4\varepsilon_{si}}$$

- In ADG MOSFET, an extra non-zero electric field at the back interface contribute to the effective Electric field

$$E_{\text{eff}} = \frac{Q_{\text{inv}}}{2\varepsilon_{si}} - \frac{d\phi}{dx}\bigg|_{x=x_0}$$
1. The additional $E$-field mainly affect the mobility at the on-set of inversion when $Q_{\text{inv}}$ is small
2. Quantum effect has very little impact on the Effective $E$-field
Characteristics of Backside Field

Relatively invariant with bias and can be approximated once the geometry is known.
Velocity Saturation Region

Objectives

- Have to account for channel length modulation
- As a gateway to account for a number of high-field effects such as velocity overshoot
- Have to account for non-uniform vertical potential distribution

We start with SDG MOSFET due to simpler boundary conditions
Saturation Region Model

- Simplified boundary conditions of the Velocity Saturation Region

\[ \varepsilon_{si} \int_{\frac{T_{si}}{2}} E_{sat} \cdot dx - \varepsilon_{si} \int_{\frac{T_{si}}{2}} \frac{\partial V}{\partial y} \cdot dx - 2 \varepsilon_{gate} \int_{0}^{y} E_{gate}(k) \cdot dk = -q \int_{\frac{T_{si}}{2}}^{y} Q(x, k) \cdot dkdx \]

- Applying Guass’s Law
Saturation Region Model

$L_g = 100 \text{nm}, T_{si} = 10 \text{nm}, N_{si} = 10^{15} \text{cm}^{-3}$

$L_g = 25 \text{nm}, T_{si} = 10 \text{nm}, N_{si} = 10^{15} \text{cm}^{-3}$
Saturation Region Model

- Assume averaging is possible based on surface potential, we have

\[
\int_{-\frac{T_{si}}{2}}^{\frac{T_{si}}{2}} \frac{\partial V}{\partial y} \cdot dx = A \cdot T_{si} \cdot \frac{dV_s}{dy}
\]

- Approximating the exact solution of the vertical potential at boundary 1 with quadratic fit

\[
V(x) = -\frac{2kT}{q} \ln \left[ \cos \left( \sqrt{\frac{q^2 n_i}{2e_{si} kT}} \cdot e^{\frac{qV_o}{2kT}} \cdot x \right) \right] + V_o \approx Kx^2 + V_0
\]

- After integrating, \( A \) can be found as

\[
A = \frac{1}{3} + \frac{2}{3} \cdot \frac{\partial V_o}{\partial V_s}
\]
Saturation Region Model

- result of quadratic approximation

- only need to fit with \( \phi_s = V_{Dsat} \)
- can achieve an error < 1%

- final value of \( A \) is given by

\[
A = \frac{1}{3} + \frac{2}{3} \left( 1 + \sqrt{\frac{q^2 n_i}{2 \varepsilon_s kT} \cdot \frac{T_{si}}{2}} \right) \tan \left( \sqrt{\frac{q^2 n_i}{2 \varepsilon_s kT} \cdot \frac{T_{si}}{2}} \right)
\]
Output Characteristics of Model

Final result

\[ \frac{d^2 V_s}{dy^2} = \left[ V_s(y) - V_s(0) \right] \]

where

\[ \lambda = \sqrt{\frac{A \cdot \varepsilon_{sl}}{2 \varepsilon_{gate}}} \cdot (T_{sl})^\frac{1}{m} \cdot (T_{gate})^\frac{1}{n} \]

\[ E(y) = E_{sat} \cdot \cosh\left(\frac{y}{\lambda}\right) \]

\[ V(y) = V_{dsat} + \lambda \cdot E_{sat} \cdot \sinh\left(\frac{y}{\lambda}\right) \]

\[ E_m = \sqrt{\frac{(V_d - V_{dsat})^2}{\lambda^2}} + E_{sat}^2 \]

![Graph showing lateral electric field vs distance from the end of channel (micron)]
Output Characteristics of Model

- Correct prediction of $E_m$ at different with different device parameters
- Model output for devices with different geometry
Summary

- We have generalized the SPP core model to account for non-charge-sheet I-V characteristics in DG MOSFET.
- A self-consistency mobility for SDG and ADG MOSFET has been derived.
- An attempt to model the saturation region of SDG MOSFET has been presented, but the accuracy is yet to be improved.