Analytic Formulae for the Impact Ionization Rate for Use in Compact Models of Ultra-Short Semiconductor Devices

H. C. Morris, M. M De Pass and
H. Abebe

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Schöll-Quade Impact Ionization Rate Formula using heated Maxwellian Dist.

$$G_{II} = [n/(2\pi)^3]\int_{K_{th}}^{\infty} \frac{f_i(k)}{\tau_{II}(k)} d^3k$$

where

$$n = \frac{2}{(2\pi)^3}\int f_i(k) d^3k, \quad \frac{1}{\tau_{II}} = \frac{1}{2\tau_0}\left(\frac{k}{k_{th}} + \frac{k_{th}}{k} - 2\right) \text{ and } f_1(k) = \frac{h^3 \exp\left(-\frac{\hbar^2 k^2}{2m^* k_B T_e}\right)}{2(2\pi m^* k_B T_e)^{3/2}}$$

$$G_{II}(n, T_e) = \frac{n}{\tau_0}\left[\sqrt{\frac{u}{\pi}} \exp\left(-\frac{1}{u}\right) - erfc\left(\frac{1}{\sqrt{u}}\right)\right] = \frac{n}{\tau_0} G_{SQ}(u)$$

$$u = K_B T_e / E_{th}$$

An Integration Technique

$$G_{SQ}(u) = \sqrt{u}\, I_3(u) + \frac{1}{\sqrt{u}} I_1(u) - 2I_2(u)$$

$$where,\ I_n(u) = \frac{1}{\sqrt{\pi}} e^{-\frac{1}{u}} \int_0^\infty (x + \frac{1}{u})^{\frac{n-1}{2}} e^{-x} dx$$

$$\text{Gauss - Laguerre formulae}: \int_0^\infty e^{-x} f(x) dx = \sum_{k=1}^{n} w_k \cdot f(x_k) + \frac{(n!)^2}{(2n)!} f^{(2n)}(\zeta)$$

$where, 0 < \zeta < \infty,\ x_k$'s are zeros of the Laguerre Polynomials L_n(x), and

the weighting function $w_k = \dfrac{x_k}{(n+1)^2 [L_{(n+1)}(x_k)]^2}$

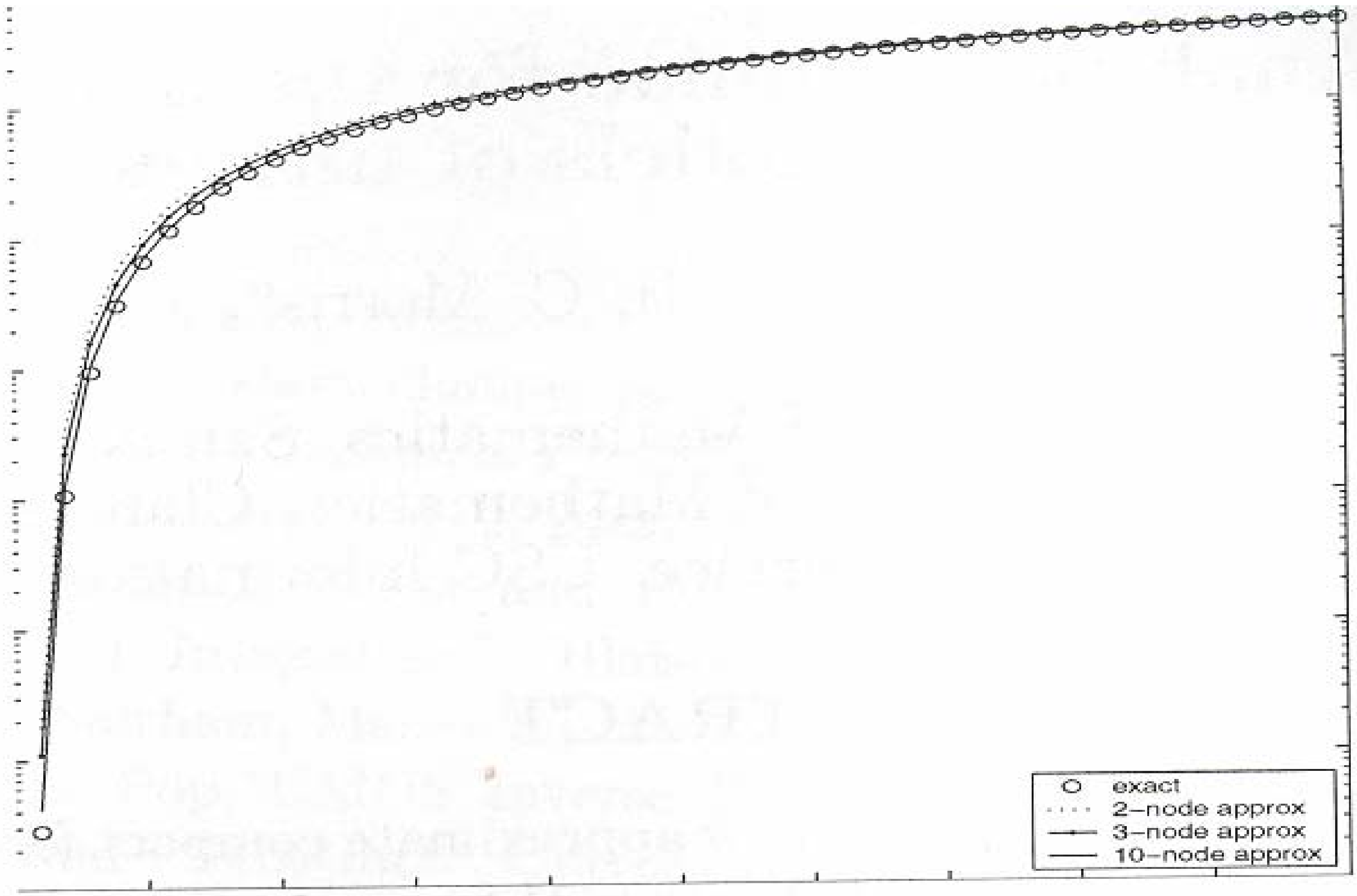

Figure 1: A fit of the exact Schöll-Quade function G_{SQ} with $u \in [0, 5]$ by Gauss-Laguerre approximation with node numbers $N = 2, 3$ and 10.

Tail Distribution Function for Small Device Application

$$f_2(k) = \frac{b\sqrt{\pi}h^3 \exp[(-\frac{\hbar^2 k^2}{2m^* k_B T_e})^b]}{4\Gamma(3/2b)(2\pi m^* k_B T_e)^{3/2}}$$

$$G_{GSQ}(u,b) = \frac{1}{2\Gamma(3/2b)}[u^{1/2}\Gamma(2/b)(1-\Gamma_{inc}(u^{-b},2/b))+$$

$$+u^{-1/2}\Gamma(1/b)(1-\Gamma_{inc}(u^{-b},1/b))-2\Gamma(3/2b)(1-\Gamma_{inc}(u^{-b},3/2b))]$$

$$where,\ \Gamma_{inc}(x,a) = \frac{1}{\Gamma(a)}\int_0^x e^{-z}z^{a-1}dz \text{ and } \Gamma(p) = \int_0^\infty e^{-z}z^{p-1}dz$$

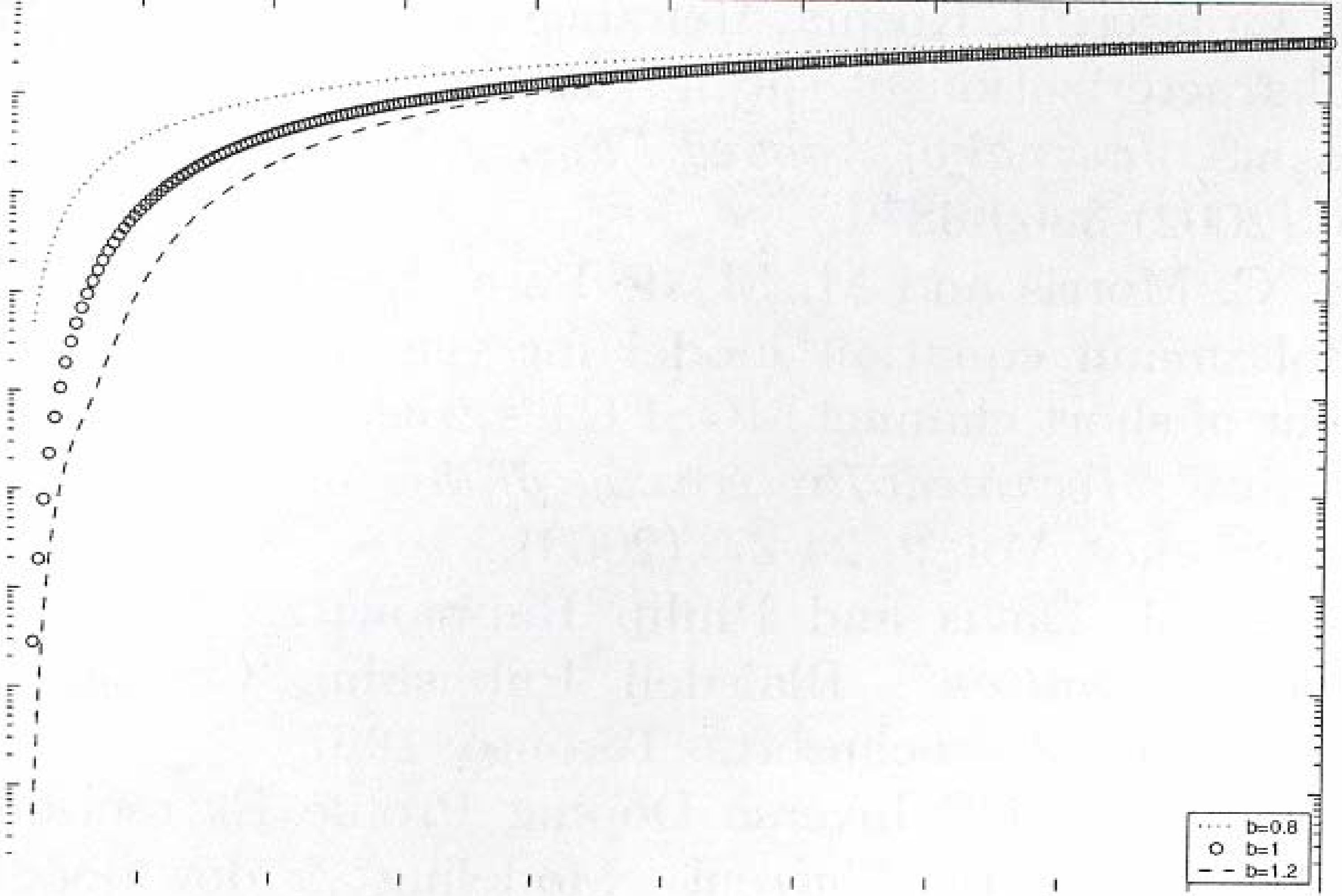

Figure 2: A semilogarithmic plot of the function G_{II}^{gen}/n with $u \in [0,5]$ for $b = 0.8, 1$ and 1.2.

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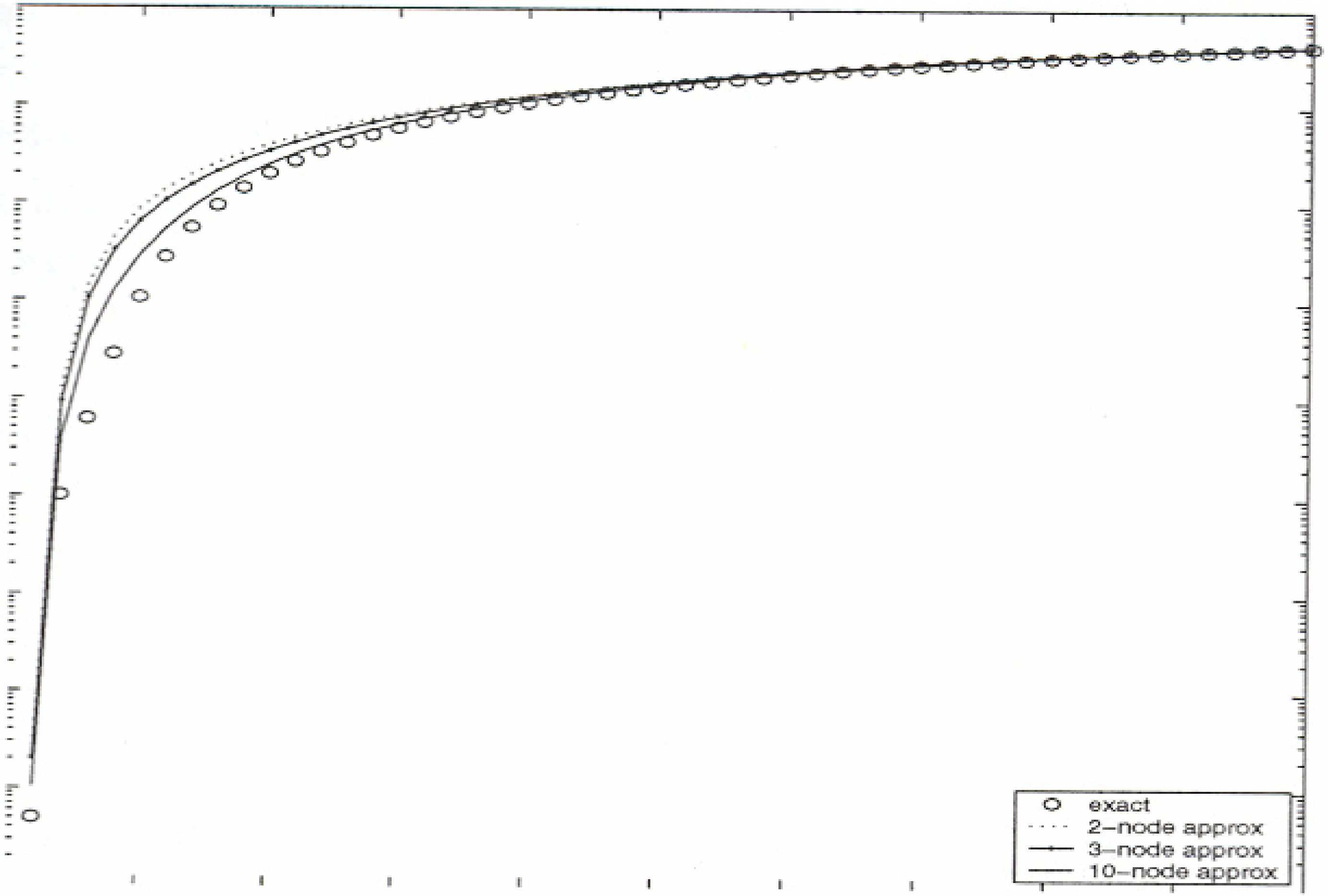

Figure 3: A fit of the function G^N_{GSQ} with $u \in [0, 5]$ with $b = 1.2$ for Gauss-Laguerre approximation with node numbers $N = 2, 3$ and 10.

A Generalized and Improved Model

$$f_3(k) = (1-c)f_1(k) + cf_2(k)$$

$$G_{II}^{N}(u,\alpha) = \frac{n}{\tau_0} G_{GSQ}^{N}(u,\alpha)$$

$$G_{GSQ}^{N}(u,\alpha) = \frac{1}{2}[G_{GSQ}(u,b) + G_{GSQ}(\alpha u,b)]$$

$$where\ ,\ \mathrm{c} = 1/2 \ \text{ and } \ 1 \le \alpha \le 2$$

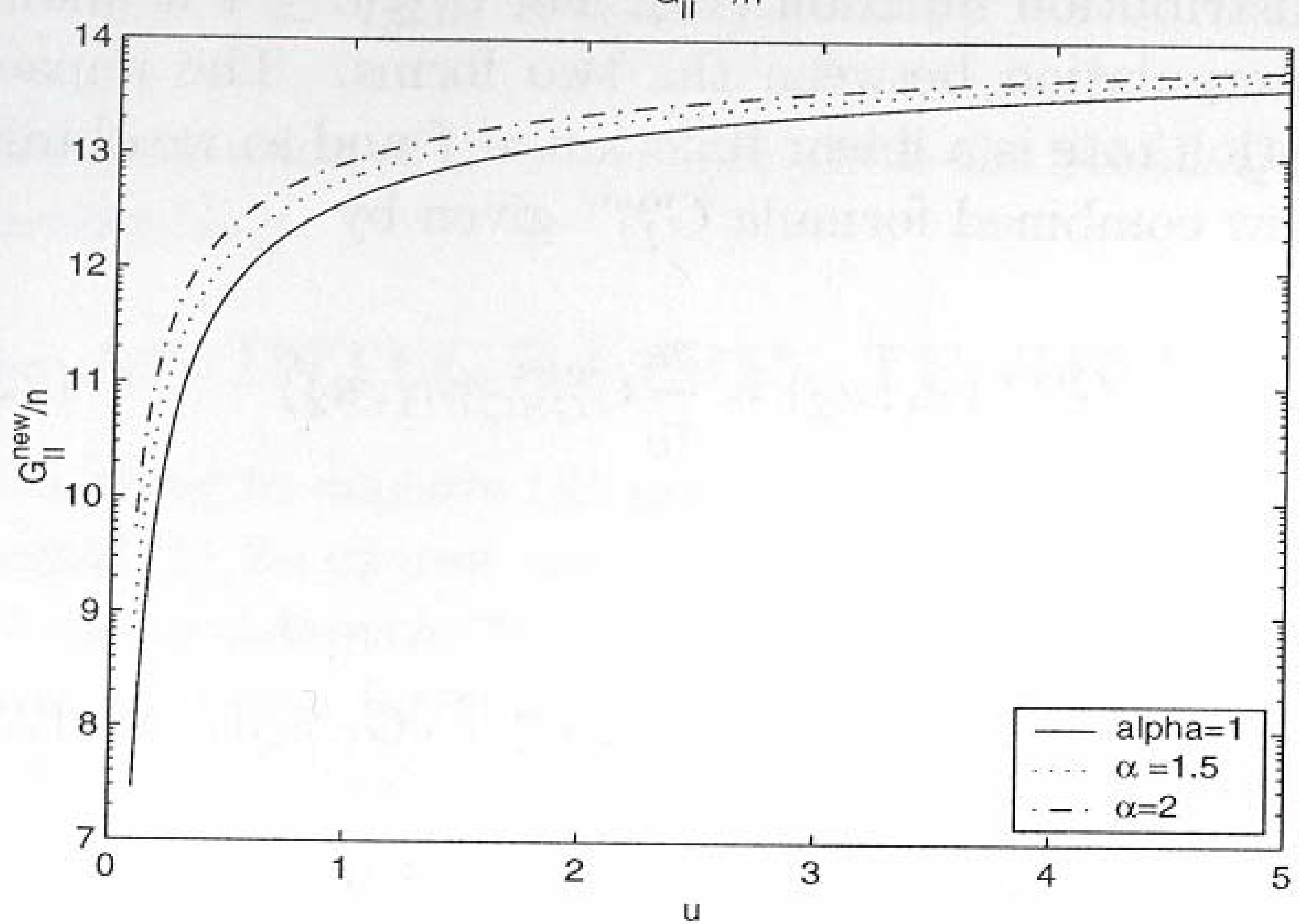

Figure 4: The generalized Schöll-Quade function $G_{II}^{\text{new}}(u,\alpha)$ with $u \in [0,5]$ for $\alpha = 1$ (Quade) and $\alpha = 1.5$ and $\alpha = 2$.

Conclusion

- New impact ionization rate analytic formulae that depends only on two parameters.
- Two node Gauss-Laguerre integration method shown to be adequate to capture the quantitative shape of the exact solution.

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