

# Physical Modeling of Substrate Resistance in RF MOSFETs

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# *Contents*

- + Introduction
- + Extraction method of  $R_{sub}$
- + Dependence of  $R_{sub}$  on bias conditions and layout geometries
- + Substrate-signal coupling through  $R_{sub}$
- + A scalable model as a function of no. of fingers
- + Conclusion

# *Introduction*

## **+ Substrate parasitics in MOSFETs**

### **● Lossy silicon substrate**

- ▶ The influence of the substrate resistance becomes significant as the operation frequency increases.**

### **● Four-terminal nature of MOSFETs**

- ▶ Signals in RF MOSFETs are coupled through the substrate R-C network.**

**▶ Careful modeling and accurate extraction of the substrate parasitics are very important for RF CMOS modeling**

**▶ Careful analysis of substrate signal coupling is needed**

# *Introduction*

## Previous works

- Curve fitting or optimization technique was used to extract

$R_{\text{sub}}$

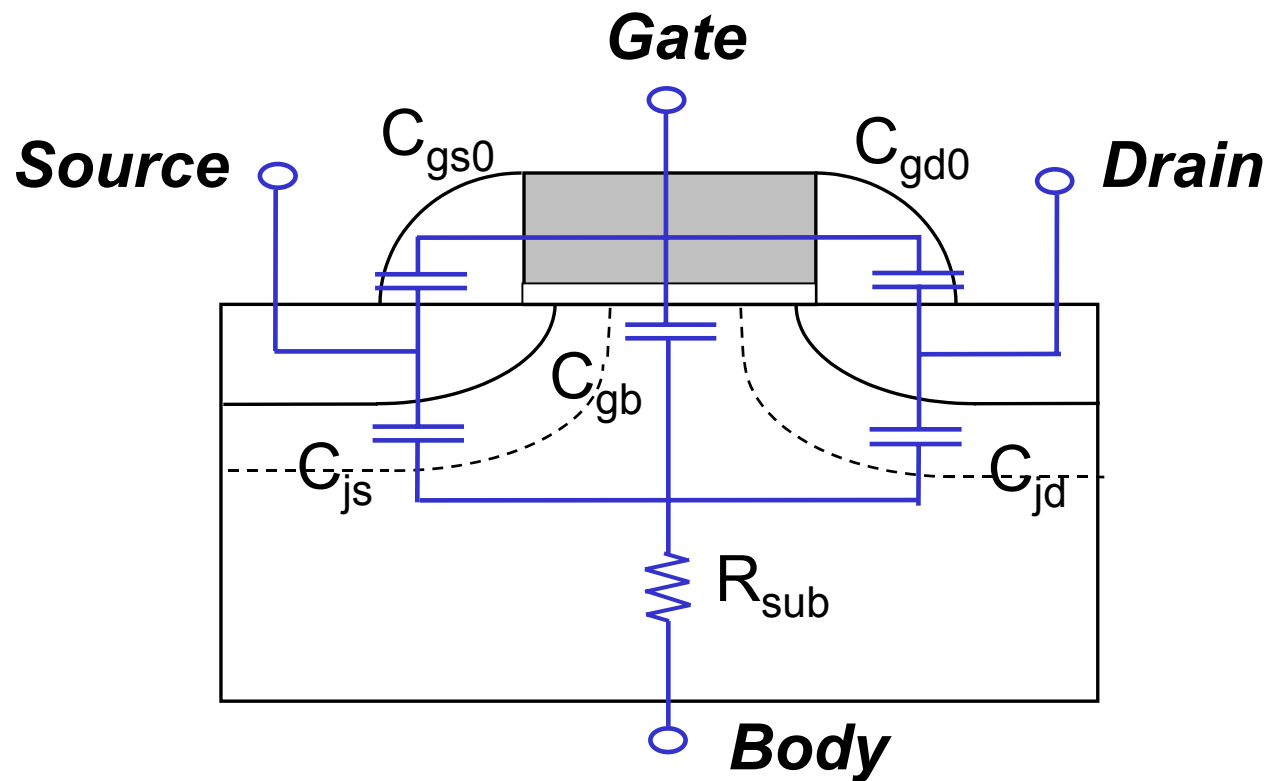
- ▶ Unphysical value can be extracted.
- ▶ It is complicated.

## Our research

- Simple and efficient method to extract the accurate substrate resistance of RF MOSFETs

# *Extraction Method*

- Equivalent circuit of an RF MOSFET, when it is **off** :  $V_{gs} < V_{th}$



# *Y-parameters*

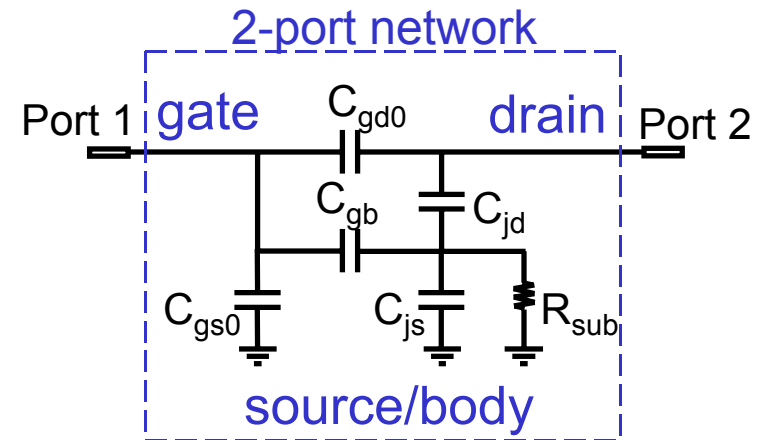
## Some of the y-parameters related to $R_{sub}$

$$\text{Im}[Y_{11}] = \omega(C_{gs0} + C_{gd0}) + \frac{\omega C_{gb} + \omega^3 R_{sub}^2 C_{gb} (C_{js} + C_{jd})(C_{gb} + C_{js} + C_{jd})}{1 + \omega^2 R_{sub}^2 (C_{gb} + C_{js} + C_{jd})^2}$$

$$\text{Im}[Y_{12}] = -\omega C_{gd0} - \frac{\omega^3 R_{sub}^2 C_{jd} C_{gb} (C_{gb} + C_{js} + C_{jd})}{1 + \omega^2 R_{sub}^2 (C_{gb} + C_{js} + C_{jd})^2}$$

$$\text{Re}[Y_{22}] = \frac{\omega^2 R_{sub} C_{jd}^2}{1 + \omega^2 R_{sub}^2 (C_{gb} + C_{js} + C_{jd})^2}$$

$$\text{Im}[Y_{22}] = \omega C_{gd0} + \frac{\omega C_{jd} + \omega^3 R_{sub}^2 C_{jd} (C_{gb} + C_{js})(C_{gb} + C_{js} + C_{jd})}{1 + \omega^2 R_{sub}^2 (C_{gb} + C_{js} + C_{jd})^2}$$



# Equations for Extracting $R_{sub}$

## ✚ At low frequencies up to a few GHz

Assuming that  $\omega^2 R_{sub}^2 (C_{gb} + C_{js} + C_{jd})^2 \ll 1$   
and that  $\omega^3$ -terms  $\ll$   $\omega$ -terms.

$$\text{Im}[Y_{11}] \approx \omega (C_{gs0} + C_{gd0} + C_{gb})$$

$$\text{Im}[Y_{12}] \approx -\omega C_{gd0}$$

$$\text{Re}[Y_{22}] \approx \omega^2 R_{sub} C_{jd}^2$$

$$\text{Im}[Y_{22}] \approx \omega (C_{gd0} + C_{jd})$$



$$C_{gd0} \approx -\frac{\text{Im}[Y_{12}]}{\omega}$$

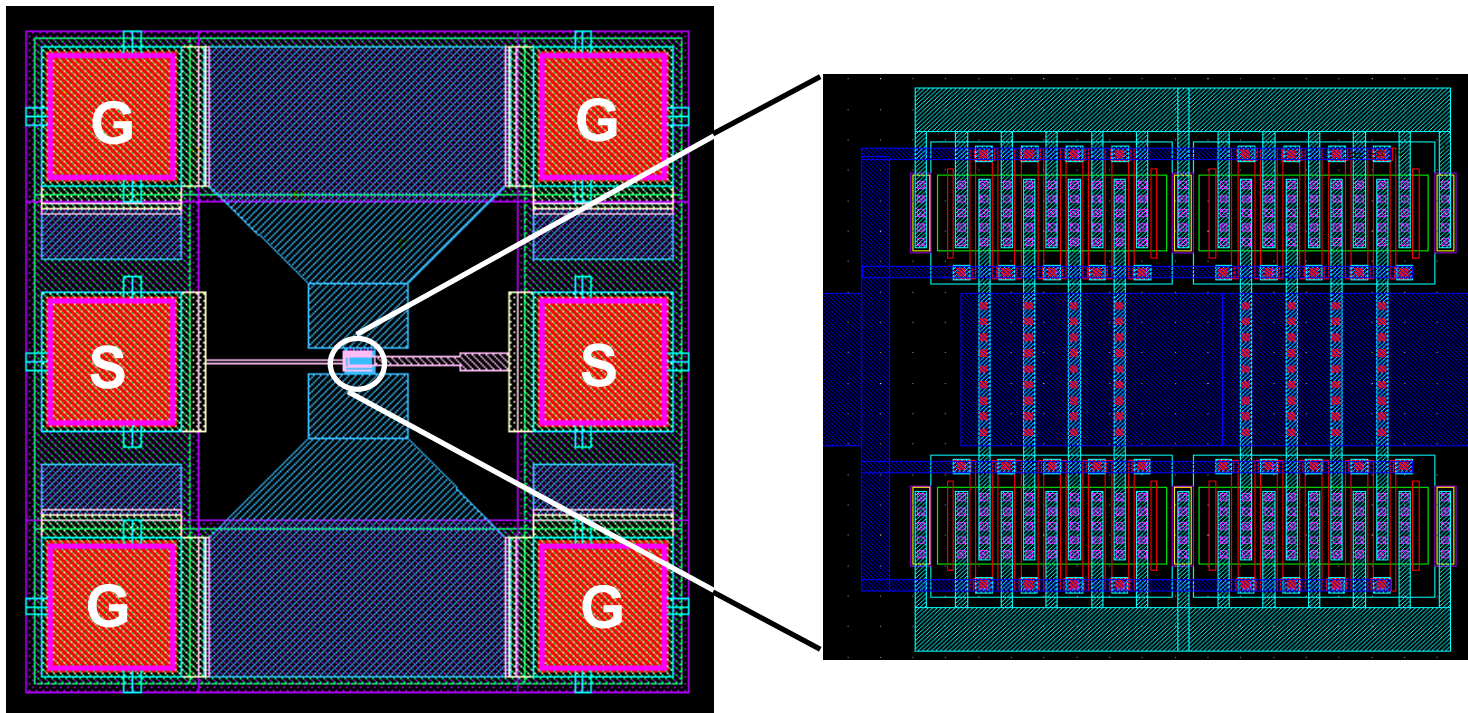
$$C_{gb} \approx \frac{\text{Im}[Y_{11}] + 2 \text{Im}[Y_{12}]}{\omega}$$

$$C_{jd} \approx \frac{\text{Im}[Y_{22}] + \text{Im}[Y_{12}]}{\omega}$$

$$R_{sub} \approx \frac{\text{Re}[Y_{22}]}{(\text{Im}[Y_{22}] + \text{Im}[Y_{12}])^2}$$

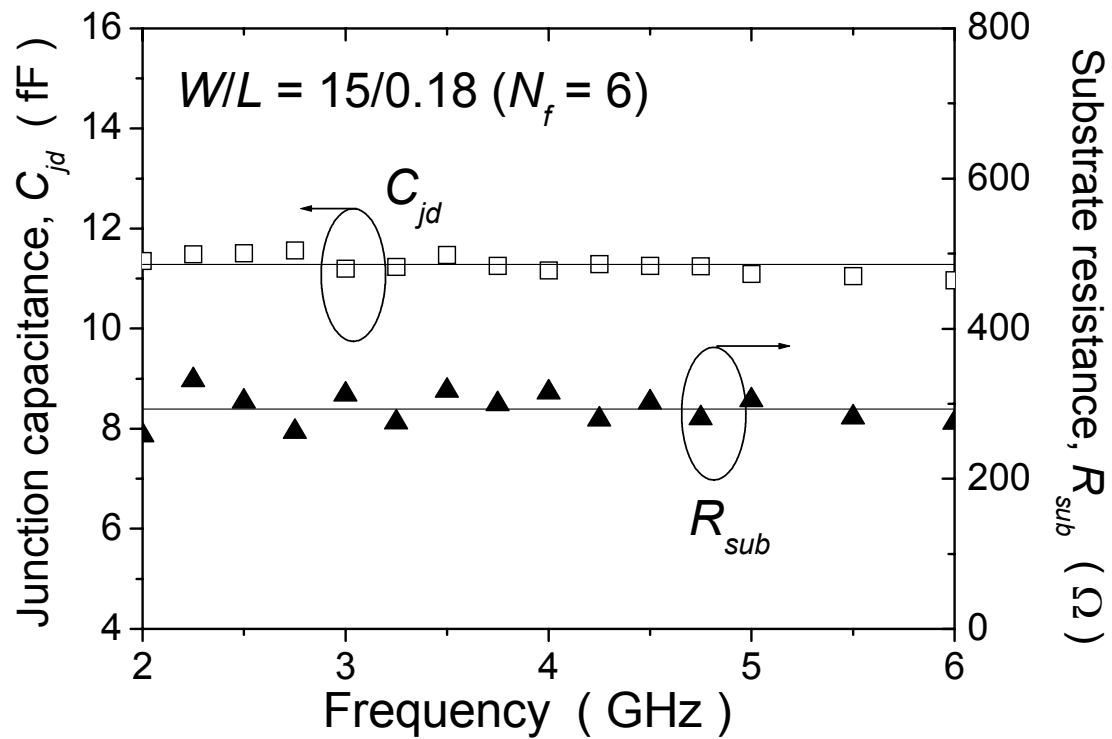
# *S-parameter Measurement*

- ✚ MOSFET test patterns : 0.18  $\mu\text{m}$  CMOS technology
- ✚ S-parameter measurement
  - Vector network analyzer
  - Probe station



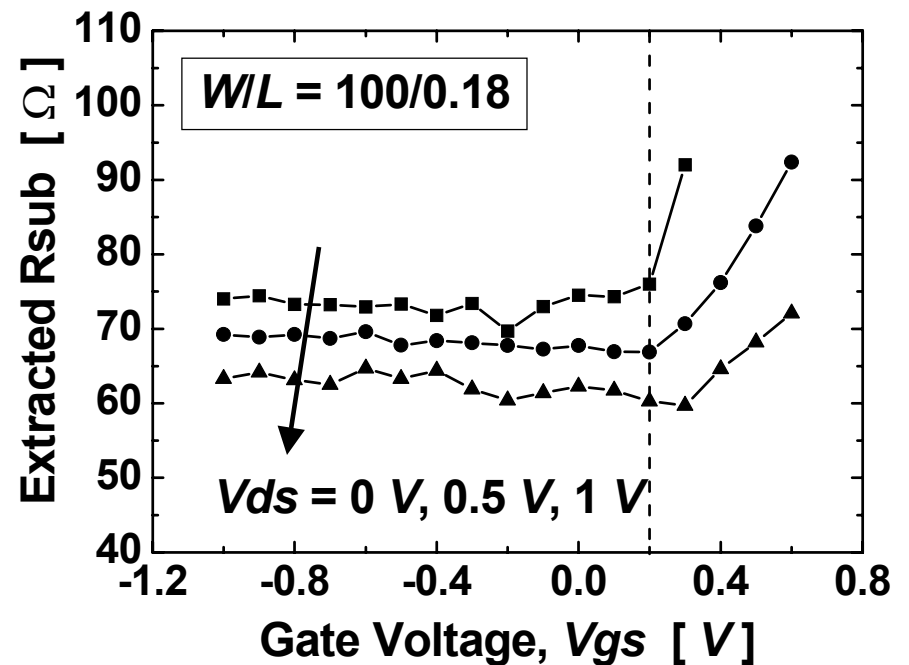
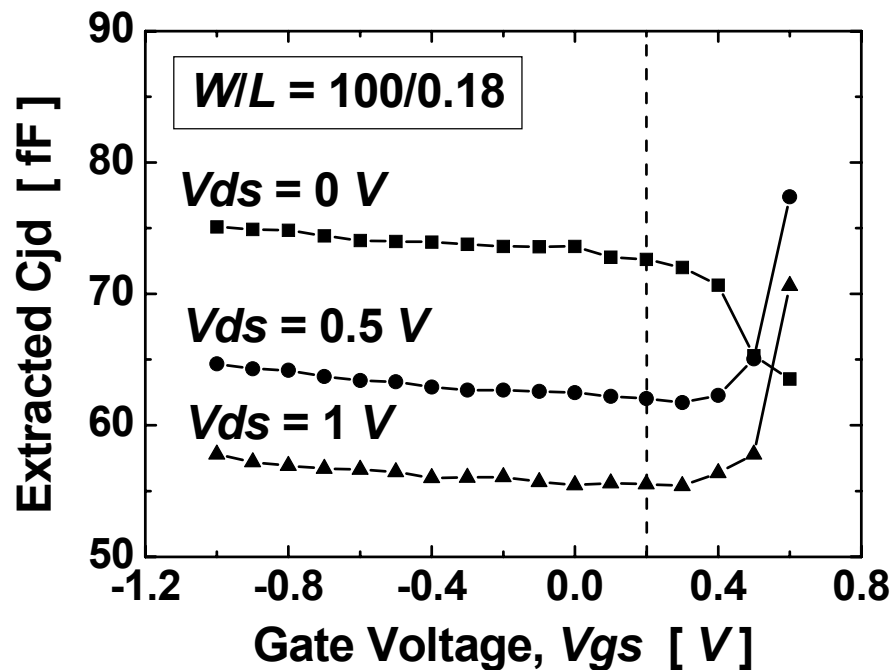
# Extraction Results

✚  $C_{jd}$  and  $R_{sub}$



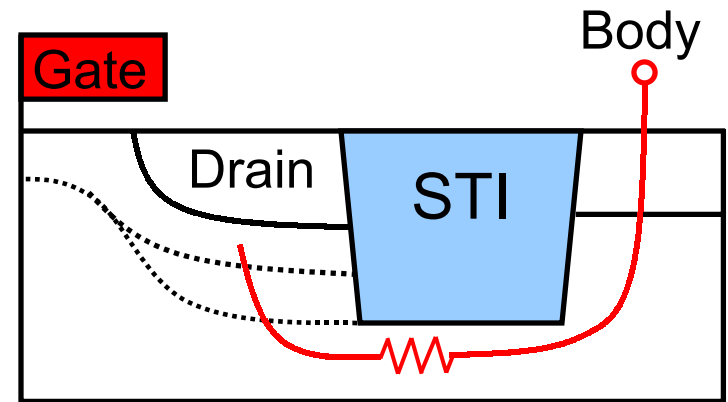
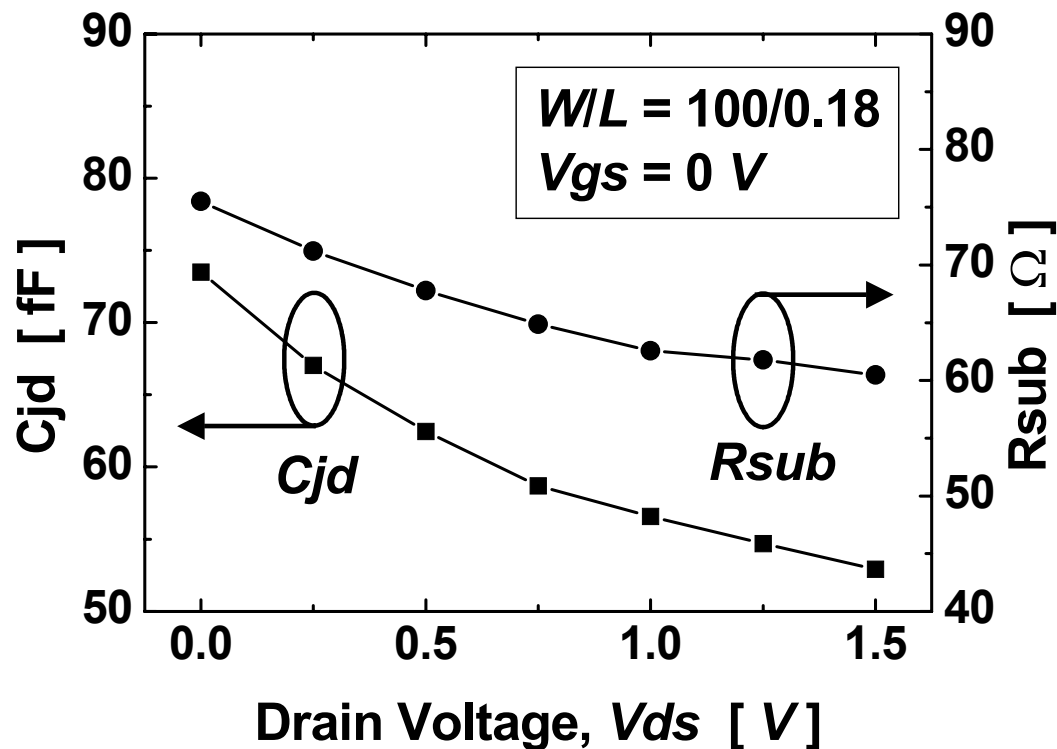
# Extraction Results ( $C_{jd}$ , $R_{sub}$ ) - Gate Voltage Dependency

- $V_{gs} \approx V_{th} \Rightarrow$  the intrinsic components of the MOSFET become significant.



# Drain Voltage Dependency

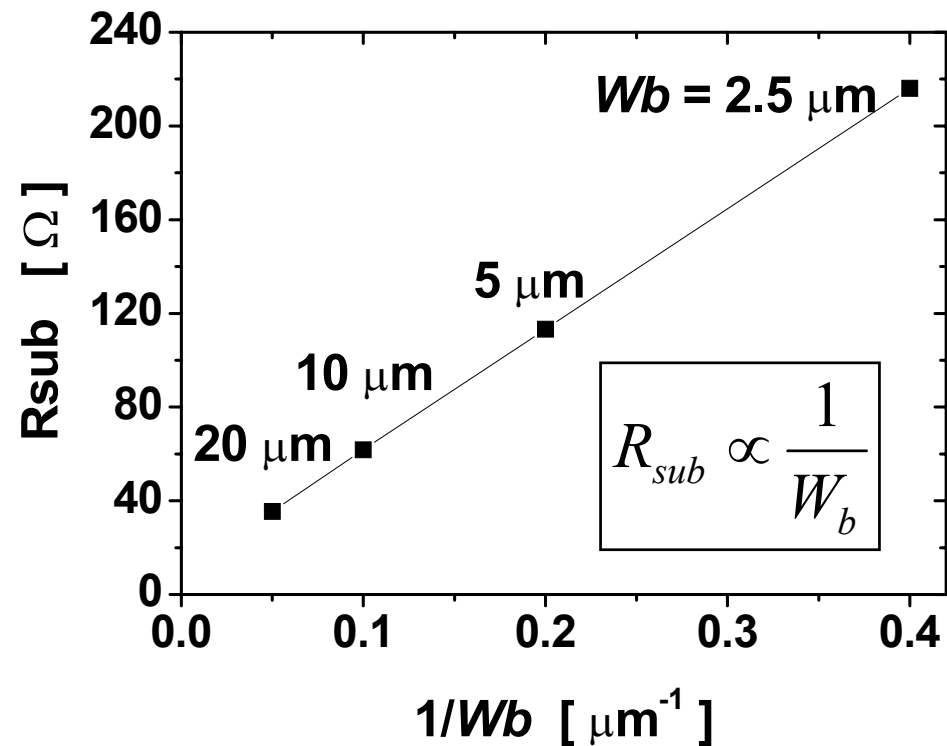
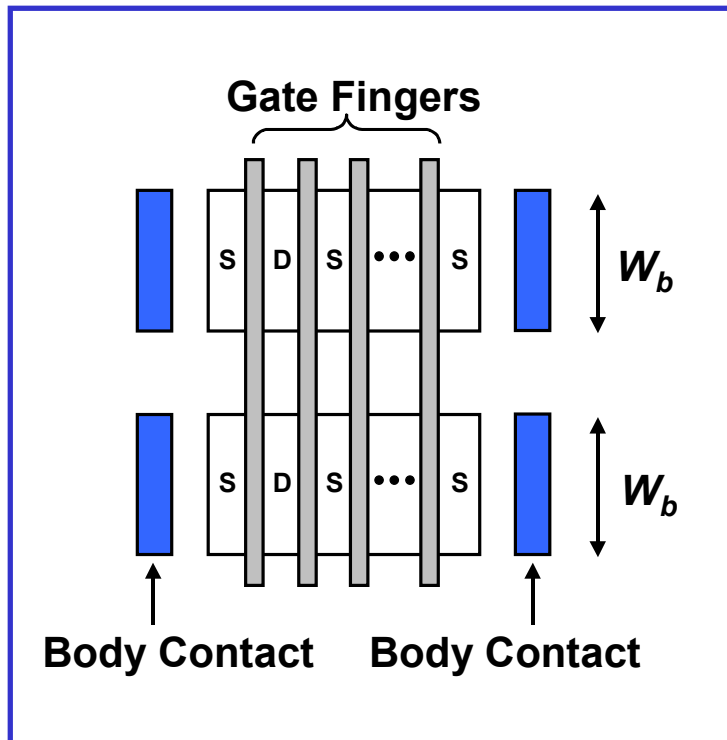
- As the  $V_{ds}$  increases,  $R_{sub}$  decreases, because the path between the intrinsic body and substrate contact becomes shorter with the depletion region widening.



# Effect of Geometric Parameters

## ✚ The widths of resistive path

- $W_b = 2.5, 5, 10, \text{ and } 20 \mu\text{m}$  for each device.



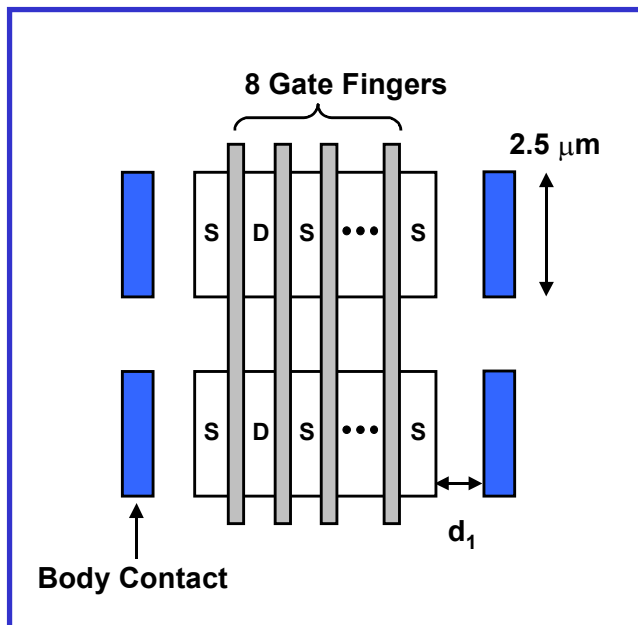
# Effect of Geometric Parameters

## + Distance to body contacts

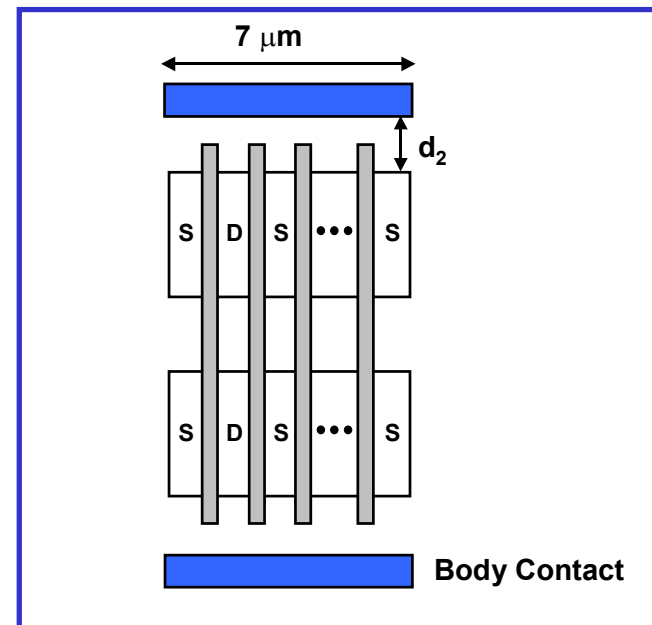
### ● Vertical-type, horizontal-type body contacts

▶  $d_1 = 1.25\mu\text{m}$  ( $\times 1$ ),  $2.5\mu\text{m}$  ( $\times 2$ ),  $3.75\mu\text{m}$  ( $\times 3$ ).

▶  $d_2 = 6.825\mu\text{m}$  ( $\times 1$ ),  $13.65\mu\text{m}$  ( $\times 2$ ),  $20.475\mu\text{m}$  ( $\times 3$ )



Vertical-type



Horizontal-type

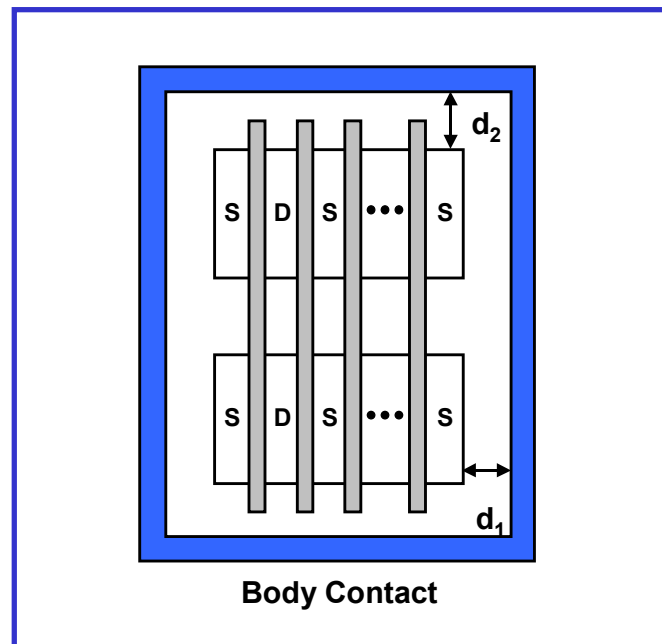
# *Effect of Geometric Parameters*

## + Distance to body contacts

### ● Ring-type body contacts

▶  $d_1 = 1.25\mu\text{m}$  ( $\times 1$ ),  $2.5\mu\text{m}$  ( $\times 2$ ),  $3.75\mu\text{m}$  ( $\times 3$ ).

▶  $d_2 = 6.825\mu\text{m}$  ( $\times 1$ ),  $13.65\mu\text{m}$  ( $\times 2$ ),  $20.475\mu\text{m}$  ( $\times 3$ )

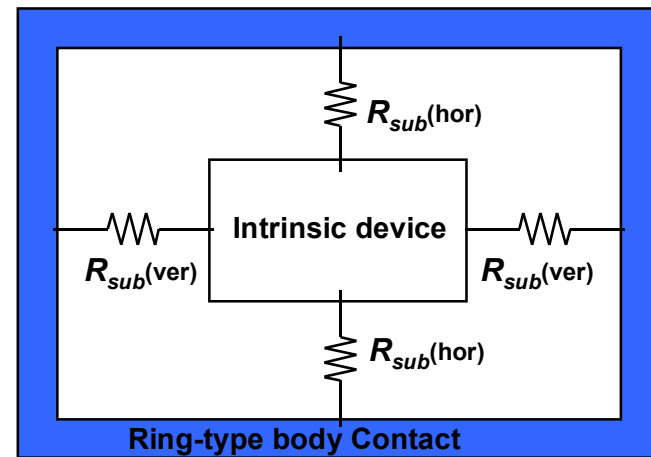
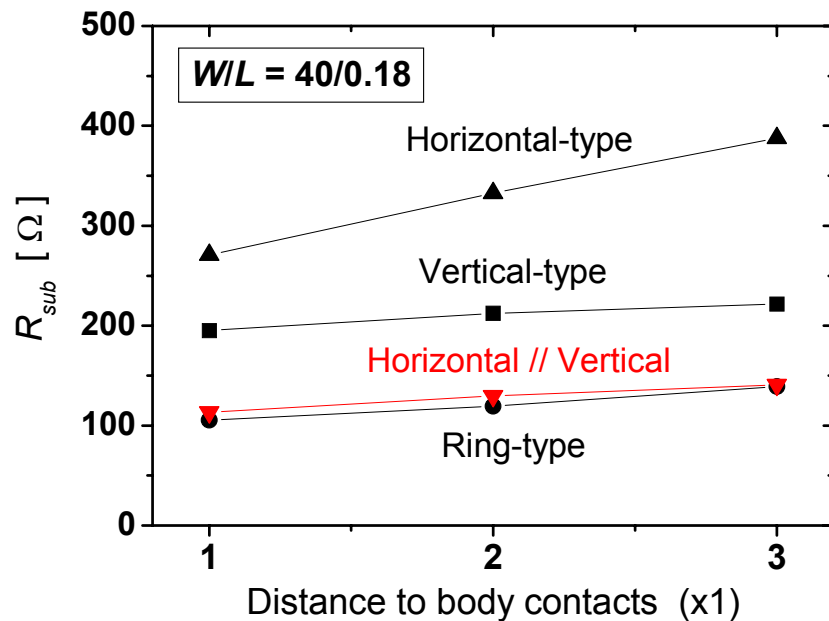


Ring-type

# Effect of Geometric Parameters

## Distance to body contacts

- $R_{sub}$  of the ring-type body contacts can be considered as a parallel combination of  $R_{sub}$  of the vertical-type body contacts and  $R_{sub}$  of the horizontal-type body contacts

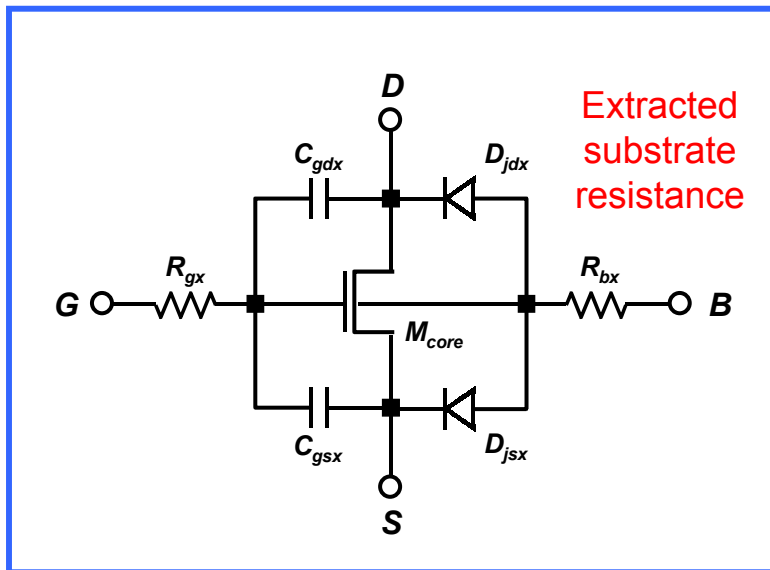


$$R_{sub}(ring) = R_{sub}(ver) // R_{sub}(hor)$$

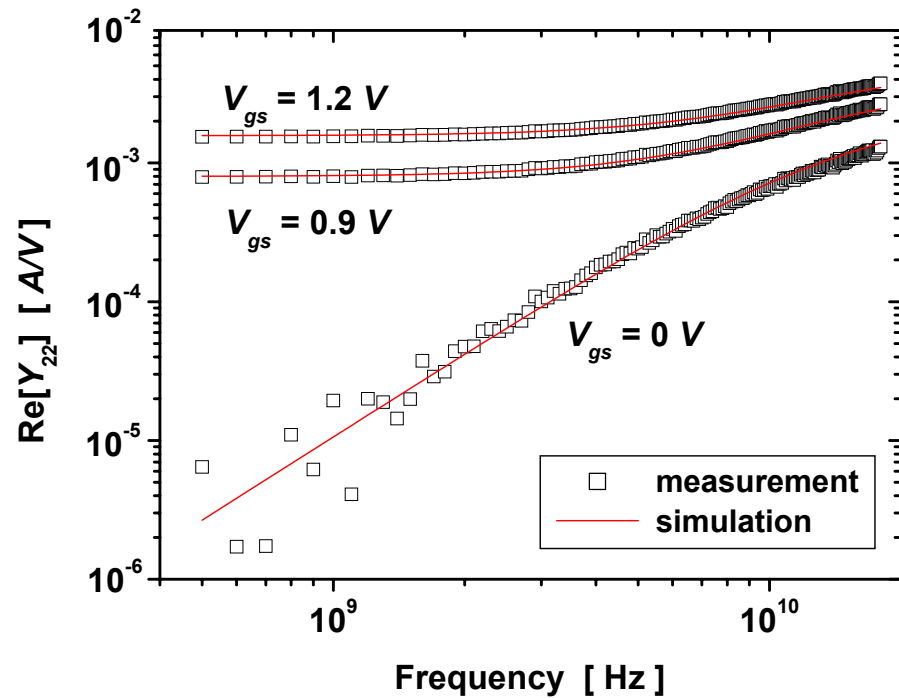
# Validity of $R_{sub}$ Values

## Macro-modeling

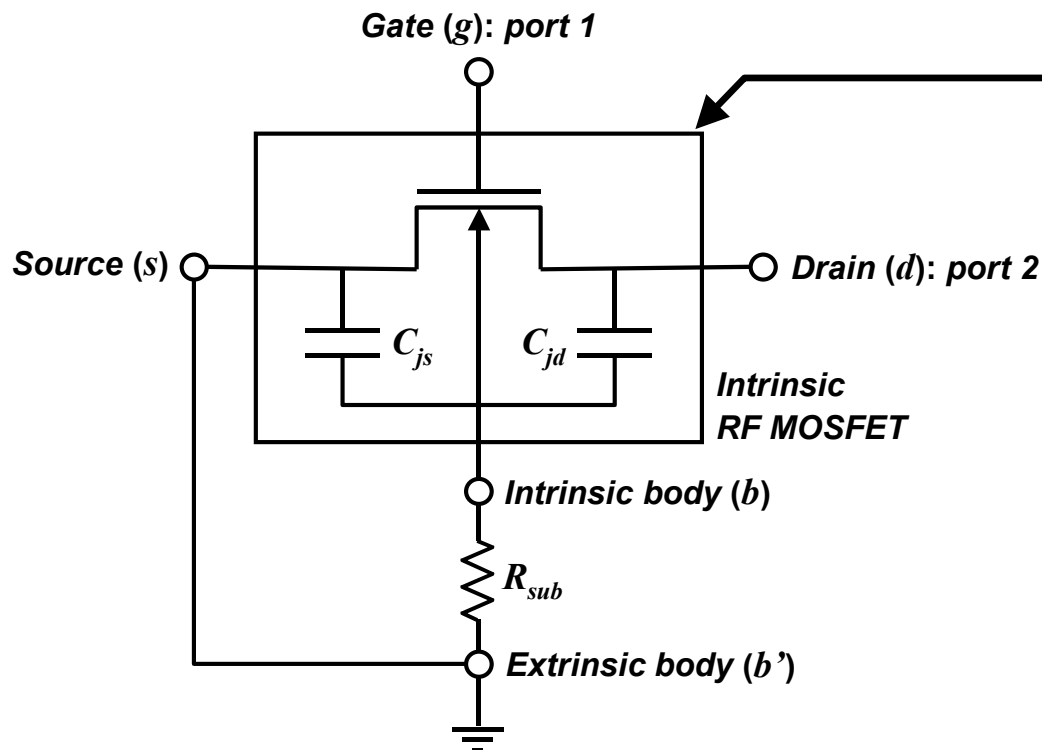
- The macro model for  $Y_{22}$  agrees very well with the measurement, indicating that the extracted  $R_{sub}$  value is valid for strong inversion.



Macro model including substrate resistance



# Substrate-signal coupling through $R_{sub}$



- ✚ One-substrate-resistor model
- ✚ After deembedding the parasitic series resistances

$$y_{gs} \approx \frac{-j\omega C_{gs}}{1+j\omega\tau} - j\omega C_{gs0}$$

$$y_{gd} \approx \frac{-j\omega C_{gd}}{1+j\omega\tau} - j\omega C_{gd0}$$

$$y_{gb} \approx -j\omega(C_{gb} + C_{gb0})$$

$$y_{bs} \approx \frac{-j\omega C_{bs}}{1+j\omega\tau} - j\omega C_{js}$$

$$y_{bd} \approx \frac{-j\omega C_{bd}}{1+j\omega\tau} - j\omega C_{jd}$$

$$y_m \approx \frac{g_m}{1+j\omega\tau_m} \quad y_{mb} \approx \frac{g_{mb}}{1+j\omega\tau_m}$$

$$y_{sd} \approx \frac{-g_{sd}}{1+j\omega\tau_m} \quad y_{mx} \approx 0$$

# *Y-Parameter Analysis*

## + Substrate coupling voltage due to the drain signal

$$i_b = y_{bd}v_{ds} + y_{bb}v_{bs}, \quad v_{bs} = -R_{sub}i_b$$

$$i_b = y_{bd}v_{ds} - y_{bb}R_{sub}i_b$$

$$v_{bs} = -\frac{y_{bd}R_{sub}}{1 + y_{bb}R_{sub}}v_{ds}$$

## ● $Y_{22}$ derivation

$$i_d = y_{dd}v_{ds} + y_{db}v_{bs} = y_{dd}v_{ds} - \frac{y_{db}y_{bd}R_{sub}}{1 + y_{bb}R_{sub}}v_{ds}$$

$$Y_{22} = y_{dd} - \frac{y_{db}y_{bd}R_{sub}}{1 + y_{bb}R_{sub}}$$

# $Y_{22}$ Equations

## + Re( $Y_{22}$ )

$$g_{sd} + \omega^2 \left\{ \tau (C_{gd} + C_{bd}) - \tau_m^2 g_{sd} \right\} + H.O.T_{int}$$

$$+ \frac{1}{1 + \omega^2 \tau_{sub}^2} \left[ \omega^2 \left\{ \underbrace{g_{mb} R_{sub}^2 (C_{bd} + C_{jd}) C_B}_{\text{Term A}} + \underbrace{R_{sub} (C_{db} + C_{jd}) (C_{bd} + C_{jd})}_{\text{Term B}} \right\} + H.O.T_{ssc} \right]$$

$$\tau_{sub} = \sqrt{2\tau R_{sub} (C_{bs} + C_{bd}) + R_{sub}^2 C_B^2} \approx R_{sub} C_B$$

$$C_B = C_{gb} + C_{bs} + C_{bd} + C_{gb0} + C_{js} + C_{jd}$$

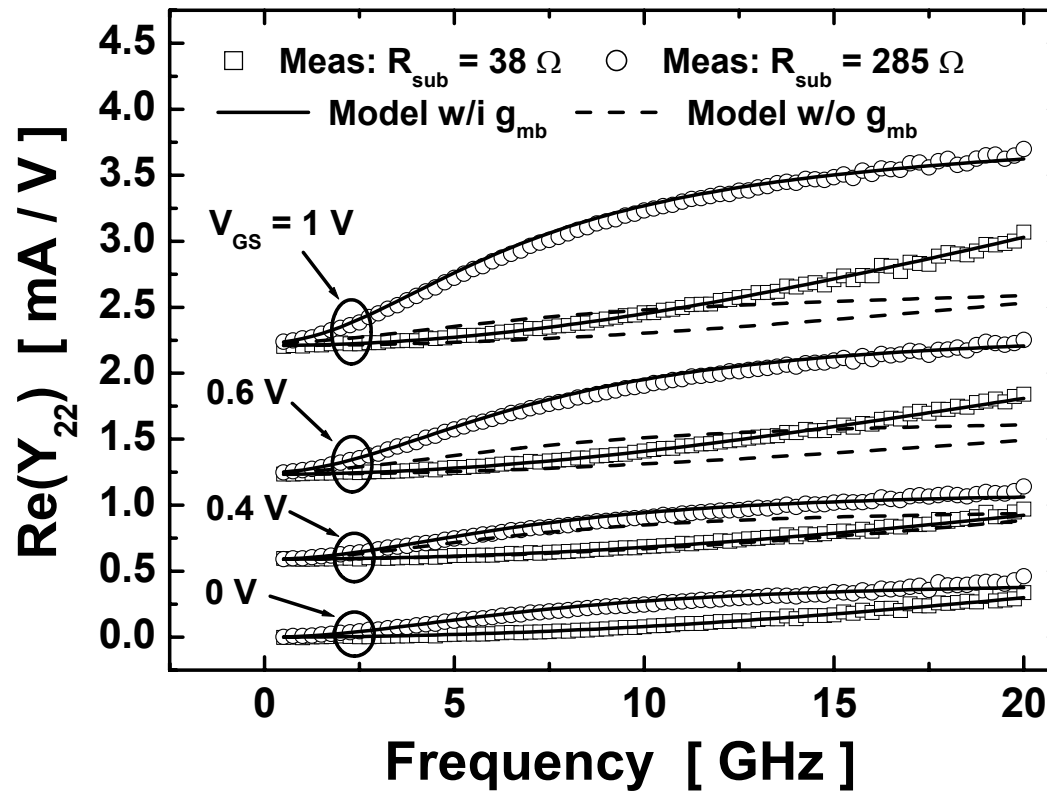
## ● Im( $Y_{22}$ )

$$\omega (C_{gd} + C_{bd} + C_{gd0} + C_{jd} + C_{sd}) + H.O.T_{int}$$

$$+ \frac{1}{1 + \omega^2 \tau_{sub}^2} \left[ \omega \underbrace{g_{mb} R_{sub} (C_{bd} + C_{jd})}_{\text{Term C}} - \omega^3 \underbrace{R_{sub}^2 (C_{bd} + C_{jd}) (C_{db} + C_{jd}) C_B}_{\text{Term D}} + H.O.T_{ssc} \right]$$

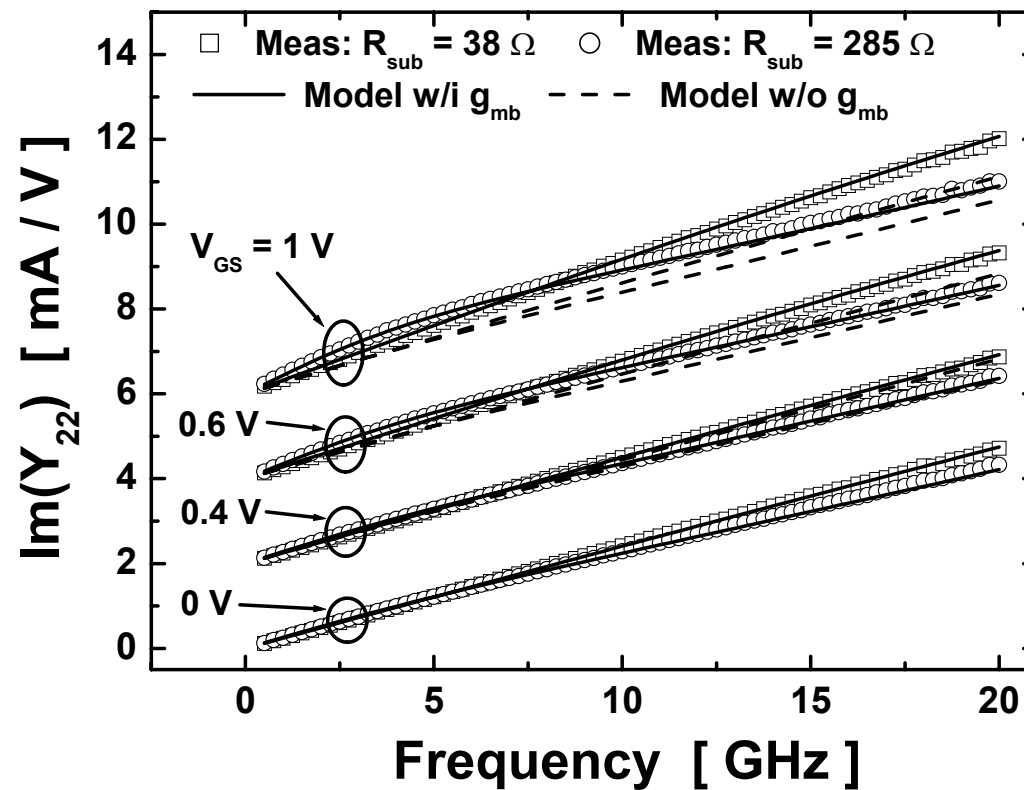
# Results: $Re(Y_{22})$

$$\frac{1}{1 + \omega^2 R_{sub}^2 C_B^2} \left[ \underbrace{\omega^2 g_{mb} R_{sub}^2 (C_{bd} + C_{jd}) C_B}_{\text{Term A}} + \underbrace{\omega^2 R_{sub} (C_{db} + C_{jd}) (C_{bd} + C_{jd})}_{\text{Term B}} \right]$$



# Results: $Im(Y_{22})$

$$\frac{1}{1 + \omega^2 R_{sub}^2 C_B^2} \left[ \underbrace{\omega g_{mb} R_{sub} (C_{bd} + C_{jd})}_{\text{Term C}} - \underbrace{\omega^3 R_{sub}^2 (C_{bd} + C_{jd})(C_{db} + C_{jd}) C_B}_{\text{Term D}} \right]$$



# *Conclusions*

- A simple and efficient method to extract the accurate substrate resistance of an RF MOSFET was presented.
- The extracted results have been presented for various bias conditions and various layout geometries.
- The amount of substrate-signal coupling was predicted and verified.