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# A BASIC PROPERTY OF MOS TRANSISTORS AND ITS CIRCUIT IMPLICATIONS

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# INTRODUCTION

- Goals of transistor modeling:
  - simulation by quantitative calculation on computer
  - highlighting properties to facilitate
    - understanding circuits
    - synthesis of robust circuits

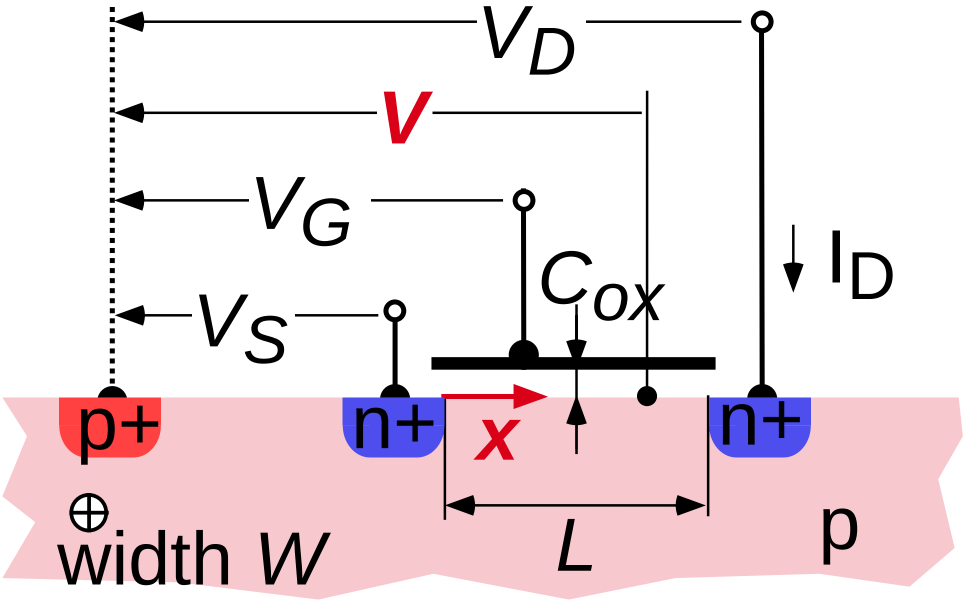
- Best models: combine both goals by hierarchical structure

  example: EKV model.

- EKV approach will be used to explicit a basic property.

# DEFINITIONS

for EKV model



- Substrate referred-voltages

  $V_S$, $V_D$, $V_G$

- Local "channel voltage" $V$

  splitting of quasi-Fermi levels due non-0 $V_S$ and/or $V_D$

  $V = V_S$ at source

  $V = V_D$ at drain

  n-channel: holes at equilibrium

  thus $V$ = electron quasi-Fermi level + constant.

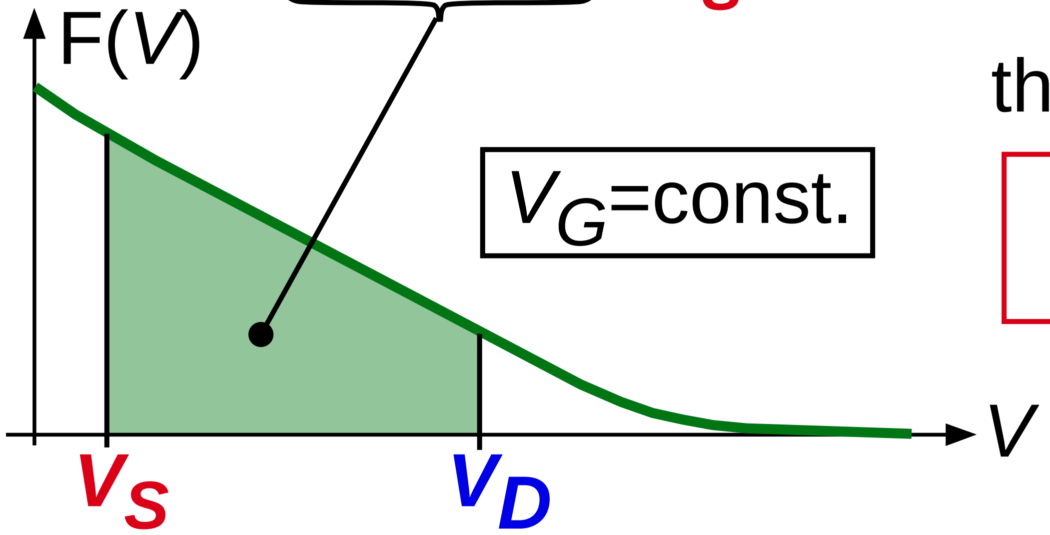
# BASIC PROPERTY (1)

- For a long and wide channel:

$$I_D = \mu W(-Q_i)\,\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{\mathrm{F}(V,V_G)\,\mathrm{d}V}{\mathrm{G}(x,V_G)\,\mathrm{d}x}$$

- Condition: separable in $V$ and $x$

$$\text{then: } I_D\underbrace{\int_0^L \mathrm{G}\,\mathrm{d}x}_{} = \int_{V_S}^{V_D} \mathrm{F}\,\mathrm{d}V \equiv \int_{V_S}^{\infty} \mathrm{F}\,\mathrm{d}V - \int_{V_D}^{\infty} \mathrm{F}\,\mathrm{d}V$$



thus:

$$\boxed{I_D = \mathrm{I}(V_S,V_G) - \mathrm{I}(V_D,V_G)}$$

# BASIC PROPERTY (2)

$$\boxed{I_D = \mathrm{I}(V_S, V_G) - \mathrm{I}(V_D, V_G)}$$

- The drain current is the superposition of independent and symmetrical effects of source and drain voltages.

- Definitions:
  - **Forward** current $I_F = \mathrm{I}(V_S, V_G)$, independent of $V_D$
  - **Reverse** current $I_R = \mathrm{I}(V_D, V_G)$, independent of $V_S$

$$\boxed{\text{then } I_D = I_F - I_R}$$

# DOMAIN OF VALIDITY (1)

- Condition:

$$\boxed{\mu W(-Q_i) = \frac{F(V,V_G)}{G(x,V_G)}}$$

with: $-Q_i = C_{ox}(V_G - V_{FB} - \Psi_s) - \sqrt{2qN_b\varepsilon_{si}\Psi_s}$

total charge depletion charge $Q_b$

- Mobile charge $Q_i$ depends on surface potential $\Psi_s$, and $\Psi_s = f(V)$, thus $Q_i$ should not be a (direct) function of $x$ to be part of F. Therefore:

  - $V_G - V_{FB}$
  - $C_{ox}$
  - $N_b$

- must be independent of position $x$ along the channel : homogeneous channel.
- but may depend on $\Psi_s$ or $V$.

(*e.g.*: $C_{ox}(\Psi_s)$: polydepletion)

# DOMAIN OF VALIDITY (2)

- Condition: $\boxed{\mu W(-Q_i) = \frac{F(V,V_G)}{G(x,V_G)}}$

- $W$ is independent of $V$; thus:
  - part of G, may depend on $x$: ⇒ any shape of channel.

- Mobility $\mu$ depends on vertical field thus on $\Psi_S$ , thus
  - included in F, provided velocity $v \ll v_{sat}$ (otherwise depends on $I_D$ itself)

- Furthermore, the effective value of $L$ along which $G(x,V_G)$ is integrated must be independent of $I_D$, $V_S$ and $V_D$.

# EFFECT OF NARROW CHANNEL

- Increased importance of side effects.
- Equivalent to parallel connection of several transistors with different characteristics.
  - if each transistor *i* fulfils

$$I_{Di} = \mathrm{I}_i(V_S, V_G) - \mathrm{I}_i(V_D, V_G)$$

  - then the sum $I_D$ of $I_{Di}$ fulfils it as well.
- The property is not degraded.

# DOMAIN OF VALIDITY (SUMMARY)

The basic property is available

- For long and homogeneous channel
- Independently of channel shape
- Even if the channel is very narrow
- Even for large gate voltages reducing the mobility.

# CAUSES OF DEGRADATION (1)

- Non homogeneous channel: $Q_i$ direct function of $x$.

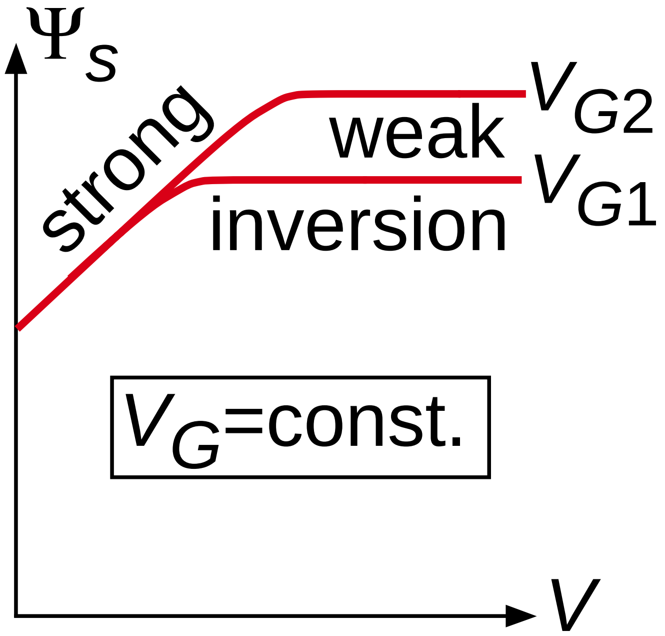
$$Q_i = -C_{ox}(V_G - V_{FB} - \Psi_s) + \sqrt{2q\, N_b\, \varepsilon_{si} \Psi_s}$$

There may be variations with position $x$ in the channel of:

- substrate doping $N_b$, which can be
  - intentional (e.g.: LDD)
  - artifact of process (gradient or piling-up )
    (always present at very ends of channel)
- flat-band voltage $V_{FB}$ , caused by
  - variation of $N_b$
  - variation of charge in oxide
- effective $C_{ox}$, always present at very ends of channel.

# SPECIAL CASE OF WEAK INVERSION

- Weak inversion characterized by $Q_i \ll Q_b$ , therefore:
  - $Q_i$ has negligible effect on potential and field
- Can be expressed as $-Q_i = G_q(\Psi_s)e^{-V/U_T}$
  - with $\Psi_s$ independent of $V$, thus:
  - $G_q$ can be any function of $x$ and is included in G, therefore:



- The property is valid even if the channel is not homogeneous.

- Mobility μ independent of $V$, thus part of G, F is reduced to $F = e^{-V/U_T}$ : independent of $V_G$.

# CAUSES OF DEGRADATION (2)

- Channel long ⇒ non-long ⇒ short
  - property progressively degraded by...
  - several independent mechanisms:

**a.** Voltage effects:

- channel length modulation
  - $I_F$ or $I_R$ becomes function of both $V_D$ and $V_S$
  - effect proportional to $1/L$
- barrier lowering and 2-D effects: further degradation.

**b.** Current effects:

- if $I_D$ is increased by reducing $L$, then

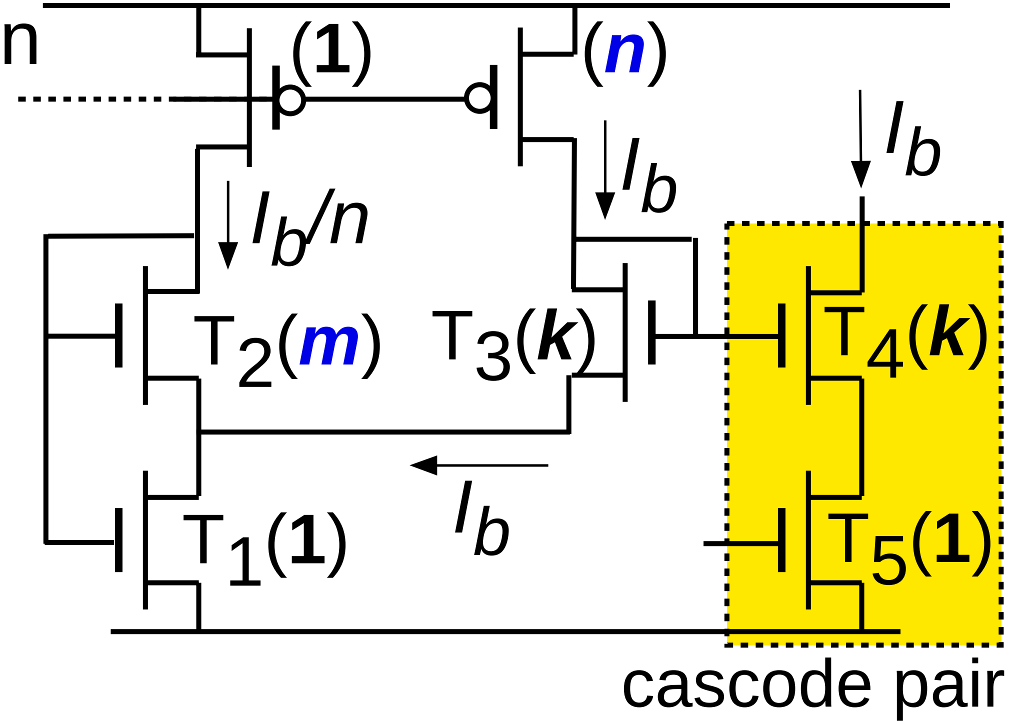
  ⇒ carrier velocity increases towards saturation

  ⇒ mobility reduced, thus function of $I_D$

**c.** Non-homogeneous channel (except in weak inversion):

- importance of end-effects proportional to $1/\boldsymbol{L}$.

# CIRCUIT EXAMPLE: LOW-VOLTAGE CASCODE



- Goal: $V_{D5}$ min. for saturation
- Means: control $\frac{I_{R5}}{I_{F5}} \ll 1$:
  - $m$ and $n \gg 1$, hence:
  - $I_{D5} \cong I_{D1}$ with $V_{D5}=V_{D1}$
  - thus $I_{R5}/I_{F5} \cong I_{R1}/I_{F1}$

$$\left.\begin{array}{l} \bullet\ I_{F2} = I_b/n = m\, I_{R1} \\ \bullet\ I_{F1} \cong I_b \end{array}\right\} \Rightarrow \frac{I_{F1}}{I_{R1}} \cong \boxed{mn \cong \frac{I_{F5}}{I_{R5}}}$$

- Large enough to ensure saturation
- Independent of $I_b$.

# CONCEPT OF PSEUDO-RESISTOR

- We have shown that:

$$I_D = \frac{1}{\int_0^L G\mathrm{d}x}\left[\int_{V_S}^{\infty} \mathrm{F}\mathrm{d}V - \int_{V_D}^{\infty} \mathrm{F}\mathrm{d}V\right]$$

- Definitions:
  - pseudo-voltage: $$\boxed{V^* = -K_0\int_V^{\infty} \mathrm{F}(V,V_G)\mathrm{d}V}$$
  - pseudo-resistor: $$\boxed{R^* = K_0\int_0^L \mathrm{G}\,(x,V_G)\mathrm{d}x}$$

(where $K_0$: any positive constant)

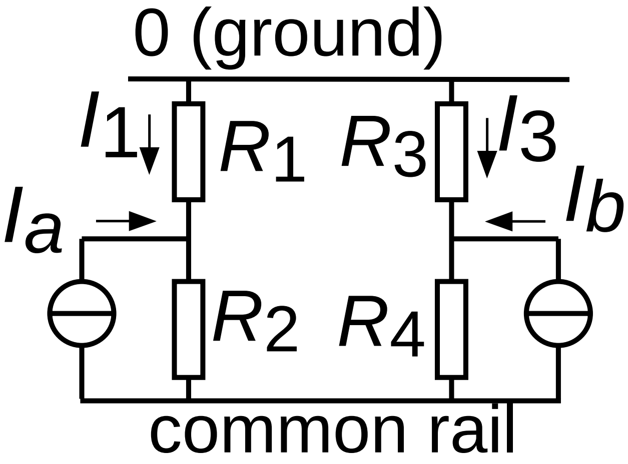
- Results in pseudo Ohm's Law: $$\boxed{I_D = (V_D^* - V_S^*)/R^*}$$

# LINEAR CURRENT-MODE CIRCUITS

- Implications of pseudo Ohm's law $I_D = (V_D^* - V_S^*)/R^*$
  - Any network interconnecting transistors with same F($V$,$V_G$) and same $V_G$ is linear with respect to currents.
  - Any circuit of linear resistors can be implemented by transistors only, provided only currents are considered.
  - A resistor connected to ground ($V$=0) in the resistive prototype corresponds to a saturated transistor that provides a pseudo-ground ($V^*$=0).
- In weak inversion:
  - F indep. of $V_G$, but $V_G$ included in function G, hence:
  - Different $V_G$ possible for each transistor
  - Each $R^*$ can be separately adjusted by its $V_G$.

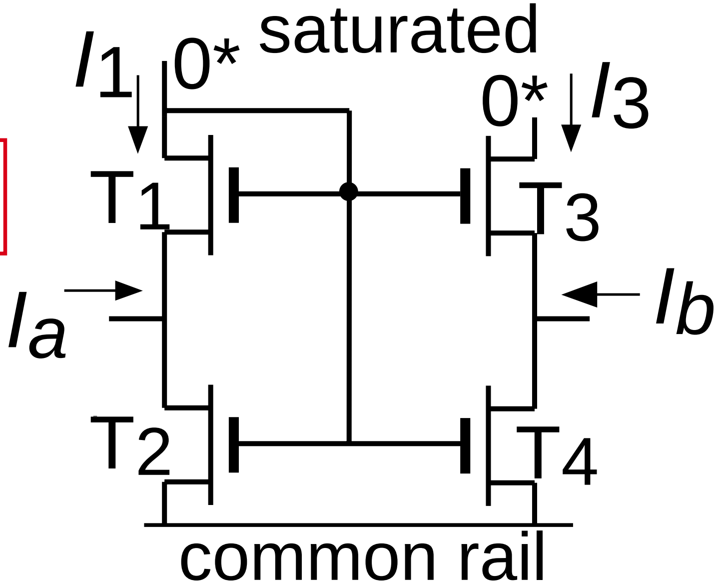
# APPLICATION OF PSEUDO-RESISTORS

(example)



$$(R_3+R_4)I_3+R_4I_b=(R_1+R_2)I_1+R_2I_a$$

$$\boxed{R_i \sim L_i\,/\,W_i}$$



thus: $$\boxed{I_3 = k_1I_1 + k_aI_a - k_bI_b}$$

- Output $I_3$ is the weighted sum of $I_1$, $I_a$ and **-**$I_b$.
- $I_a$ is low-voltage input (T$_2$ not saturated)
- For $R_4$=0 (T$_4$ in s/c) and T$_1$=T$_2$=T$_3$ ⇒ $k_1 = 2$
  - for $N$ unit trans. in series at input ⇒ $k_1 = N$
    - for $M$ units in parallel at output ⇒ $k_1 = M.N$

# FURTHER APPLICATIONS OF PSEUDO-RESISTORS

- Linear attenuators (electrical control in weak inversion)
  - R-2R network for D/A conversion.
- Spatial information processing:
  - $n^{\text{th}}$ oder moment computation
  - diffusion networks (isotropic or not)
  - 2-D emulation of physical media
  - path finding.
- In weak inversion: exploitation of current distribution in voltage- (or current-) dependent linear networks:
  - local normalisation in vision processing
  - generation of nonlinear functions
  - ...

# CONCLUSION

- Basic MOS property for long and homogeneous channels:

$$I_D = \mathrm{I}(V_S,V_G) - \mathrm{I}(V_D,V_G) = I_F - I_R$$

  - superposition of independent and symmetrical effects of S and D voltages.
  - forward and reverse components.
- Property progressively degraded when channel shortened.
- Underlies the concept of pseudoresistor:
  - linear current mode circuits
  - transistor implementation of arrays of resistors.
  - simpler analysis of transistor circuits.