Overview of an
Advanced Surface-Potential-Based Model

SP

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What is SP?

SP is a latest generic compact MOSFET model developed at The Pennsylvania State University. It is surface-potential-based, symmetric, contains no iterative loops anywhere, and has a relatively small number of parameters.

The development of SP is based on solution of several long-standing problems of compact MOSFET modeling.
Why $\Phi_s$-based models?

- Accurate description of the moderate inversion region
- Starting point is Brews’ charge-sheet model which is totally symmetric and satisfies all benchmark tests
- Today computing $\Phi_s$ is not a problem
- No matching points, no singularities, no discrepancies between I-V and C-V models
- We have learned how to introduce small-geometry effects without relying on threshold voltages.
Novel features of SP

- Analytical (non-iterative) evaluation of the surface potential $\Phi_s$
- Symmetric bulk charge linearization
- Inclusion of velocity saturation in a form consistent with Gummel symmetry
- Bias-dependent lateral field gradient
- Mobility model incorporating Coulomb scattering
- Analytical approximation for the quantum corrections and polysilicon depletion effect based on the surface potential method

The additional features of SP include simple and physical charge model which is consistent with the drain current model and is free from the unphysical behavior at $V_{ds} = 0$. 
Symmetric Linearization

Compact MOSFET models almost always use bulk and inversion charge linearization

The most popular form is

$$\sqrt{\Phi_s} = \sqrt{\Phi_{ss}} + \delta (\Phi_s - \Phi_{ss})$$

$$\delta = \frac{1}{2\sqrt{\Phi_{ss}}} \left(1 - \frac{1}{a_1 + a_2 \cdot \Phi_{ss}}\right)$$

In addition in $V_{TH}$ – based models

$$\Phi_{ss} = 2\Phi_b + V_{sb}$$

**Disadvantage:** symmetry between source and drain is lost
SP is based on “symmetric linearization” method

Define potential midpoint (subscript “m”) where the surface potential

\[ \Phi_m = \frac{1}{2} (\Phi_{ss} + \Phi_{sd}) \]

This is NOT a geometric midpoint:

\[ y_m = \frac{L}{2} \left( 1 + \frac{\Phi_{sd} - \Phi_{ss}}{4H} \right) > \frac{L}{2} \]

Set inversion charge (per unit channel area)

\[ q_i \equiv q_{im} + \left( \frac{dq_i}{d\Phi_s} \right)_{\Phi_s = \Phi_m} (\Phi_s - \Phi_m) \quad (I) \]
Advantages:

- Equations are even simpler than for the original linearization scheme.
- Symmetry (S ↔ D) is preserved.
- Accuracy can be checked by studying a special case of long-channel charge-sheet model where exact expressions are available for the drain current and terminal charges.
Comparison of the original (circles) and symmetrically linearized (solid line) charge-sheet model; \( t_{\text{ox}} = 2 \text{ nm}, N_{\text{sub}} = 5 \cdot 10^{17} \text{ cm}^{-3}, \mu = 500 \text{ cm}^{2}/\text{V} \cdot \text{s}, V_{\text{fb}} = -0.8 \text{ V}, W/L = 2. \)
$V_{b0} = 1 \text{ V}$

$V_{gb0} = 2.5 \text{ V}$

Circuit diagram and simulated results of the Gummel symmetry test for the exact (circle) and symmetrically linearized charge-sheet model (solid line)
Analytical Approximation For the Surface Potential

- Works for any voltage range (from accumulation to inversion)

- Tested for a wide range of device parameters

  \[ t_{ox} = 10 - 2000 \text{ Å} \]

  \[ N_{sub} = 10^{15} - 10^{19} \text{ cm}^{-3} \]

- Has no numerical difficulties for \( |V_{gb}| < 500kV \) (at least)

- Has been independently verified and used (Motorola)
$\psi_{SS}, \text{V}$

$V_{gb} - V_{fb}, \text{V}$

$Tox=2 \text{ nm}, \ N_{sub}=10^{18} \text{ cm}^{-3}, V_{ds}=1 \text{ V}, V_{sb}=0 \text{ V}$

- **Analytical approximation**
- **Exact solution**
$\phi_{SS}$, nV

$V_{gb} - V_{fb}$, V

$\text{Tox}=2 \text{ nm, } N_{sub}=10^{18} \text{ cm}^{-3}$, $V_{ds}=1 \text{ V, } V_{sb}=0 \text{ V}$
Tox=1 nm, N_{sub}=10^{18} \text{ cm}^{-3}, V_{ds}=1 \text{ V}, V_{sb}=2 \text{ V}

Normalized Transcapacitances

\begin{align*}
V_{gb} - V_{fb}, \text{ V} \\
C_{gg} \\
C_{sg} \\
C_{dg} \\
C_{bg}
\end{align*}
Mobility Model

The effective mobility ($\mu$) model used in SP includes both the universal dependence on the effective vertical field $E_{\text{eff}}$ and the deviation from universality associated with Coulomb scattering§)

$$\mu = \frac{MU0}{1 + (\mu_{E}E_{\text{eff}})^{\theta} + CS[q_{b}/(q_{i} + q_{b})]^{2}}$$

§) C.-L. Huang and N. D. Arora, Solid State Electronics, 37, 97 (1994)
MU0 and CS are global (unscalable) model parameters, $\mu_E$ and $\theta_\mu$ are local parameters and the last term in the denominator introduces Coulomb scattering. The inversion charge $q_i$, the bulk charge $q_b$ and $E_{\text{eff}}$ are computed at the potential midpoint in order to assure Gummel symmetry of the model. The inclusion of the Coulomb scattering term is required primarily to assure a good fit of low-temperature experimental data without sacrificing the quality of the fits at room and high temperatures.
Measured (circles) and modeled (solid lines) linear transconductance for $W/L=10/0.2 \mu m$, $T_{ox}=40 \text{Å}$, $N_{\text{sub}} = 2.4 \times 10^{17} \text{cm}^{-3}$, $V_{ds} = 0.1 \text{V}$, $V_{sb} = 0 \sim 2 \text{V}$

Experimental data provided by P. Bendix of LSI Logic
\[
\nu_d = \frac{\mu_0 \left| \frac{d\phi_s}{dy} \right|}{1 + \delta_0 (V_{ds}) \left| \frac{d\phi_s}{dy} \right| / E_c}
\]

\(\delta_0(V_{ds})\) was originally introduced in


to sharpen \(\nu_d\) vs. \(E\) dependence

Additional advantage:

removal of the singularity of \(\frac{d^2 I_d}{dV_x^2}\) for \(V_x = 0\) (Gummel symmetry)
\[ f = 1 - \frac{\varepsilon_s}{qN_{sub}} \frac{\partial^2 \phi_s}{\partial y^2} \]

\[ t_{ox} = 40 \text{ Å}, N_{sub} = 2.4 \times 10^{17} \text{ cm}^{-3}, V_{ds} = 1 \text{ V and } V_{sb} = 0 \]

![Graph showing lateral gradient factor, f, as a function of Vgs, V.](image)

- L = 20 µm
- L = 0.2 µm
Intrinsic Charges

\[
\frac{Q_G}{WLC_{ox}} = V_{gb} - V_{fb} - \phi_m + \frac{\eta_p \phi}{2} \left( \frac{r_L \phi}{6H} - 1 + r_L \right)
\]

\[
\frac{|Q_I|}{WLC_{ox}} = q_{im} + \frac{\alpha \phi}{2} \left( \frac{r_L \phi}{6H} - 1 + r_L \right)
\]

\[
\frac{|Q_D|}{WLC_{ox}} = q_{im} - \frac{\alpha \phi}{4} \left( 1 - r_L^2 \right) - \frac{r_L(1-\eta_p)\phi^2}{24H}
\]

\[
- \frac{r_L^2 \phi(\alpha + 1 - \eta_p)}{12} \left( 1 - \frac{\phi}{2H} - \frac{\phi^2}{20H^2} \right)
\]

\[
|Q_S| = |Q_I| - |Q_D|; \quad |Q_B| = |Q_G| - |Q_I|
\]

where \( H = \frac{q_{im}/\alpha + \phi_t}{1 + 80r_LV_c} \); \( r_L = \frac{1}{1 + \Delta L_{CLM} / L} \)

\[
\eta_p = \left\{ 1 + \left( \frac{2C_{ox}^2}{q\varepsilon_{si}N_p} \right) \left[ V_{gb} - V_{fb} - \phi_m^{(0)} \right] \right\}^{-1/2}
\]
W/L=20/10 µm, \( t_{ox} = 20 \) Å, \( N_{sub} = 2\times10^{17} \) cm\(^{-3}\)

\( V_{gs} = 2 \) V, \( V_{sb} = 0 \)

\( V_{sb} = 0 \)

All capacitances are normalized to \( WLC_{ox} \)
W/L=20/20 μm, $t_{ox} = 40$ Å, $N_{sub} = 8 \cdot 10^{17}$ cm$^{-3}$
W/L=20/0.2 μm, $t_{ox} = 40$ Å, $N_{sub} = 8 \cdot 10^{17}$ cm$^{-3}$
Tox = 2 nm, N_{sub} = 5 \times 10^{17} \text{cm}^{-3}

$\log_{10}(\phi_p, V) \quad \phi_s = 2\phi_f + V_{sb}$

$V_{gs}, \text{V}$
Poly Depletion Correction to Surface Potential

W/L=20/10 μm, $t_{ox} = 20$ Å, $N_{sub} = 2 \cdot 10^{17}$ cm$^{-3}$, $V_{ds} = 1$ V, $V_{sb} = 0$

$N_p = 5 \cdot 10^{19}$ cm$^{-3}$
SP with Poly Depletion

$W/L = 20/10 \, \mu m$, $t_{ox} = 20 \, \text{Å}$, $N_{sub} = 2 \times 10^{17} \, \text{cm}^{-3}$, $V_{ds} = 1 \, \text{V}$, $V_{sb} = 0$

![Graph showing normalized $C_{gg}$ vs. $V_{gb} - V_{fb}$]
Quantum Corrections to Surface Potential

$T_{ox} = 2.5 \text{ nm}$

$N_{sub} = 5 \times 10^{17} \text{cm}^{-2}$

$V_{fb} = -0.8 \text{ V}$

$f = 0.7$

$V_{ds} = 0.1 \text{ V}$

$V_{sb} = 1.0 \text{ V}$

Surface potential computed numerically (solid line) and analytically (solid circles). Broken lines represents surface potential without quantum correction.
GUMMEL SYMMETRY TEST: 0.25/0.2

Experimental data provided by S. Veeraraghavan of Motorola
TREETOP TEST

W/L=20/10 μm, \( t_{ox} = 40 \text{ Å} \), \( N_{sub} = 2.4 \times 10^{17} \text{ cm}^{-3} \)

\( V_{ds} = 0.1 \text{ V} \)

\( V_{ds} = 1.8 \text{ V} \)
TREETOP TEST

W/L=20/0.2 µm, $t_{ox} = 40$ Å, $N_{sub} = 2.4\cdot10^{17}$ cm$^{-3}$

$V_{ds} = 0.1$ V

$V_{ds} = 1.8$ V

$V_{gb} - V_{fb}$, V

$V_{sb} = 2$ V

$G_m / I_d$
GLOBAL FIT EXAMPLE

Experimental data provided by P. Bendix of LSI Logic
GLOBAL FIT EXAMPLE

Experimental data provided by P. Bendix of LSI Logic
GLOBAL FIT EXAMPLE

\[ t_{ox} = 30 \, \text{Å}, \, N_{sub} = 2.5 \times 10^{17} \, \text{cm}^{-3}, \, W/L=10/0.13 \, \mu\text{m} \]

Experimental data provided by P. Bendix of LSI Logic
Simulation Example: R2R circuit

Background

This circuit is sensitive to model symmetry. It presents certain difficulties for some widely used $V_{TH}$-based MOSFET models.
R2R Circuit Diagram
Simulation results for R2R Circuit

W/L = 8/30
and
16/30 µm

Different curves correspond to $I_1/I_{fs}$, $2I_2/I_{fs}$, ..., $2^6I_6/I_{fs}$
Conclusions

- Surface-potential-based models form a valid alternative to threshold-voltage-based models
- Iterative computations of $\Phi_s$ are not necessary
- Symmetric linearization leads to Gummel symmetry for both currents and charges/capacitances
- SP solves several long-standing problems retarding the development of surface-potential-based models
- SP includes major small-geometry effects, QM and poly corrections
- SP is developed in modular form and other models can (and do) use SP elements
- SP development was initiated by the industry and proceeds with significant industry input.
- SP is ready for implementation in circuit simulators