Unified Statistical Modeling for Circuit Simulation

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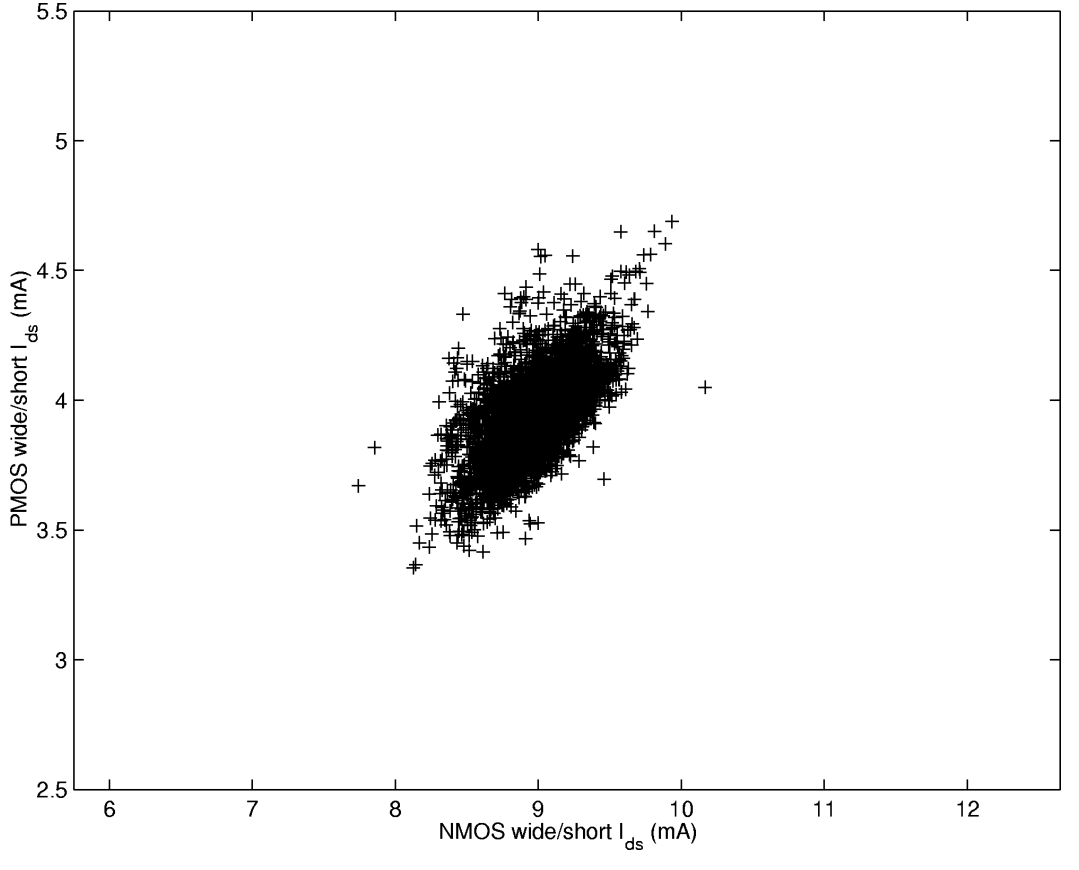
Statistical Modeling

- IC design flows can embody different types of statistical design practices
- The types of statistical simulation techniques used dictates the types of statistical models that are required
  - Corner simulations require case files
  - MC simulations need distributional models
  - Mismatch analysis needs mismatch models
  - Sensitivity analysis needs physical models

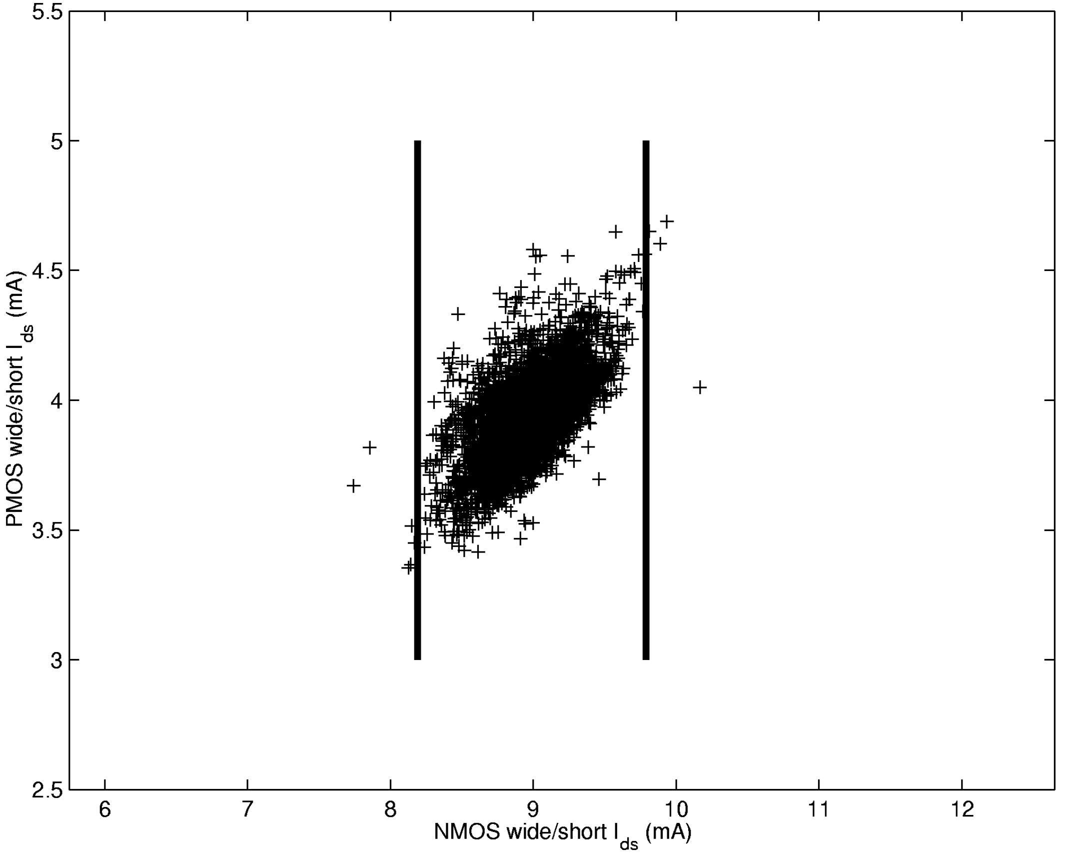
Goal of (Statistical) Modeling

- **To accurately represent the (mean and standard deviation of) electrical performances E_i of devices (circuits)**
- **The goal is NOT to accurately represent the statistics of "parameters"**
  - **Effect of Δ_L differs between models**
  - **Different techniques to measure Δ_L give different values**

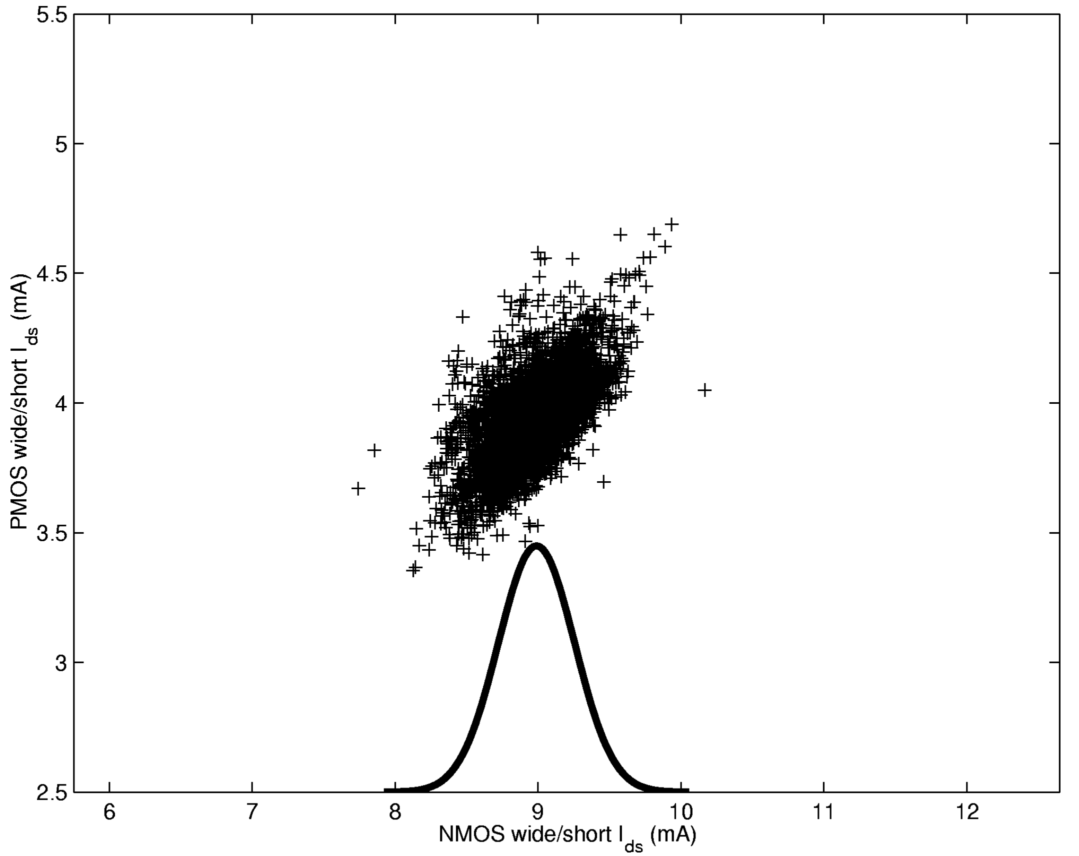
Goal: Model Fab Variations



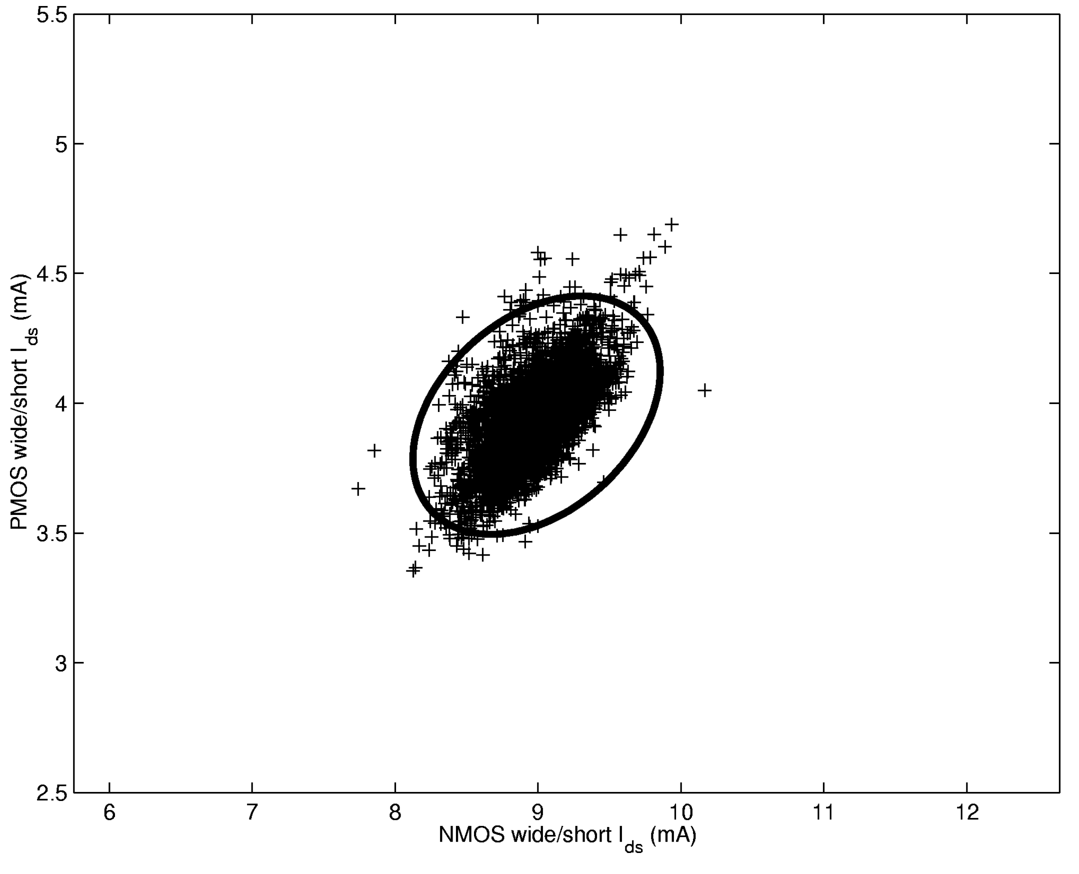
Goal: Model Fab Limits



Goal: Model Fab Distributions



Goal: Model Fab Correlations



Physical Modeling Basis

- Variations in physical, independent process parameters P_k cause variations in electrical performances E_i
  - E.g. T_{OX}, Δ_L, ρ_{SH}, N_B, V_{FB}, x_J
  - V_{TH}, β are NOT basic process parameters
- SPICE model parameters are characterized as functions of P_k
  - And geometry g if necessary
- Allows effective and efficient modeling of correlations with uncorrelated P_k

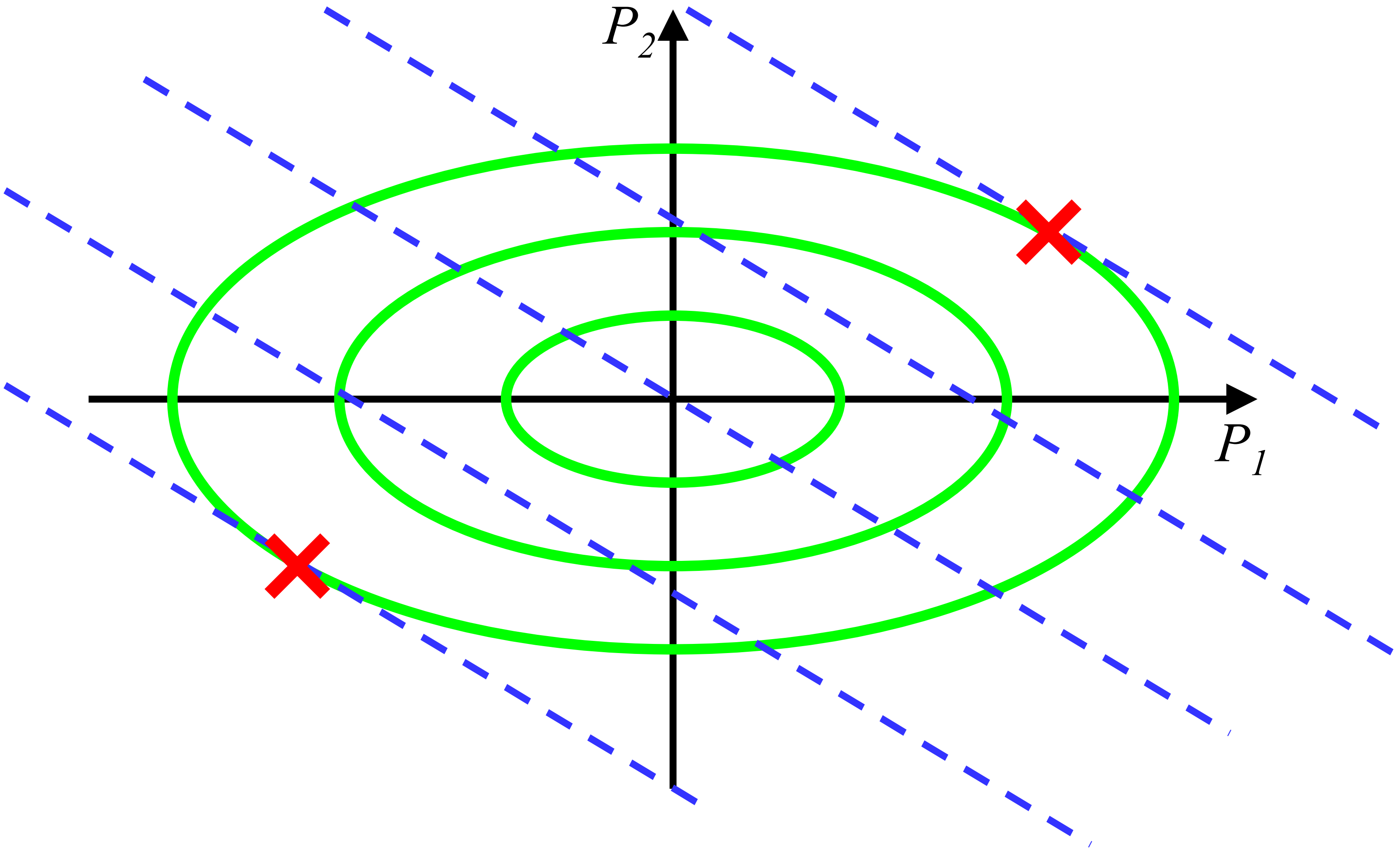
Statistical Modeling in a Nutshell

$$\delta E_i = \sum_k \frac{\partial E_i}{\partial P_k} \delta P_k$$

$$\delta P_j = \pm n \frac{\sigma_{P_j}^2 \left(\partial E_i / \partial P_j \right)}{\sqrt{\sum_k \sigma_{P_k}^2 \left(\partial E_i / \partial P_k \right)^2}}$$

$$\sigma_{E_i}^2 = \sum_k \left(\frac{\partial E_i}{\partial P_k} \right)^2 \sigma_{P_k}^2$$

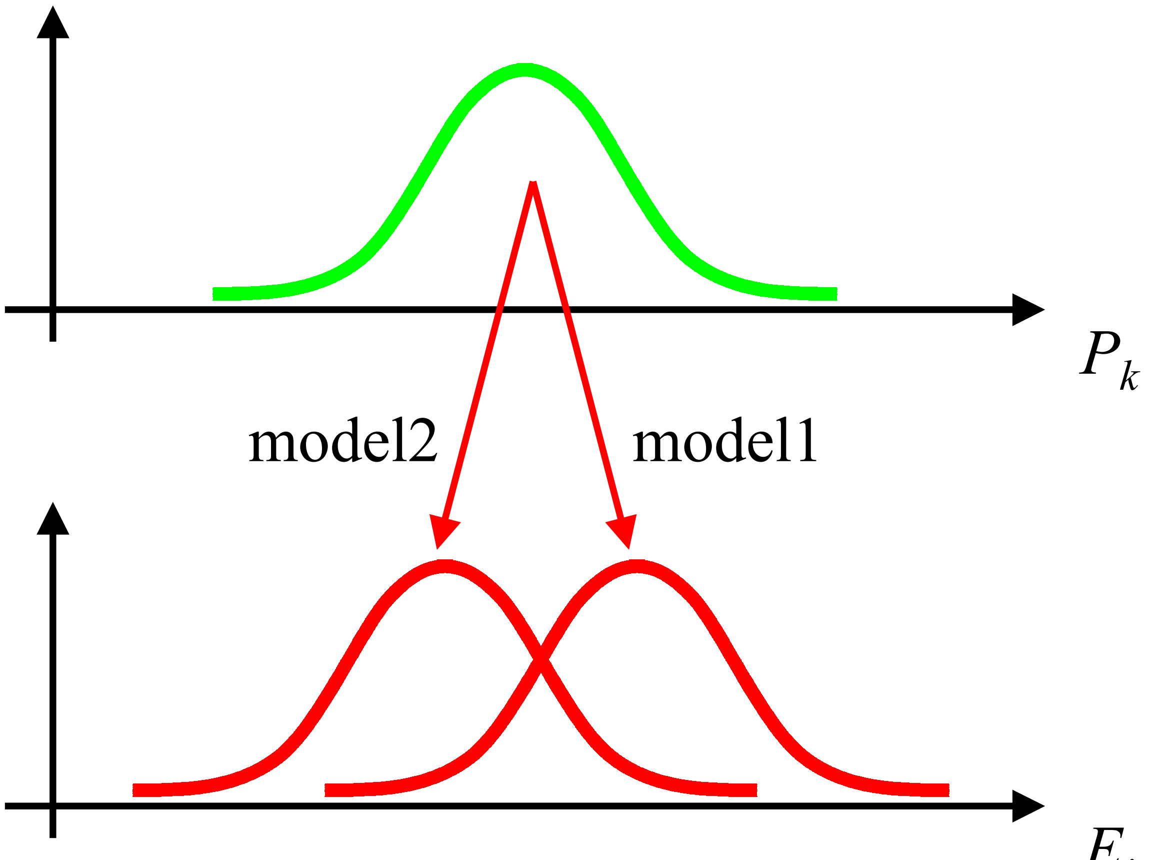
Constant E_i in P-Space



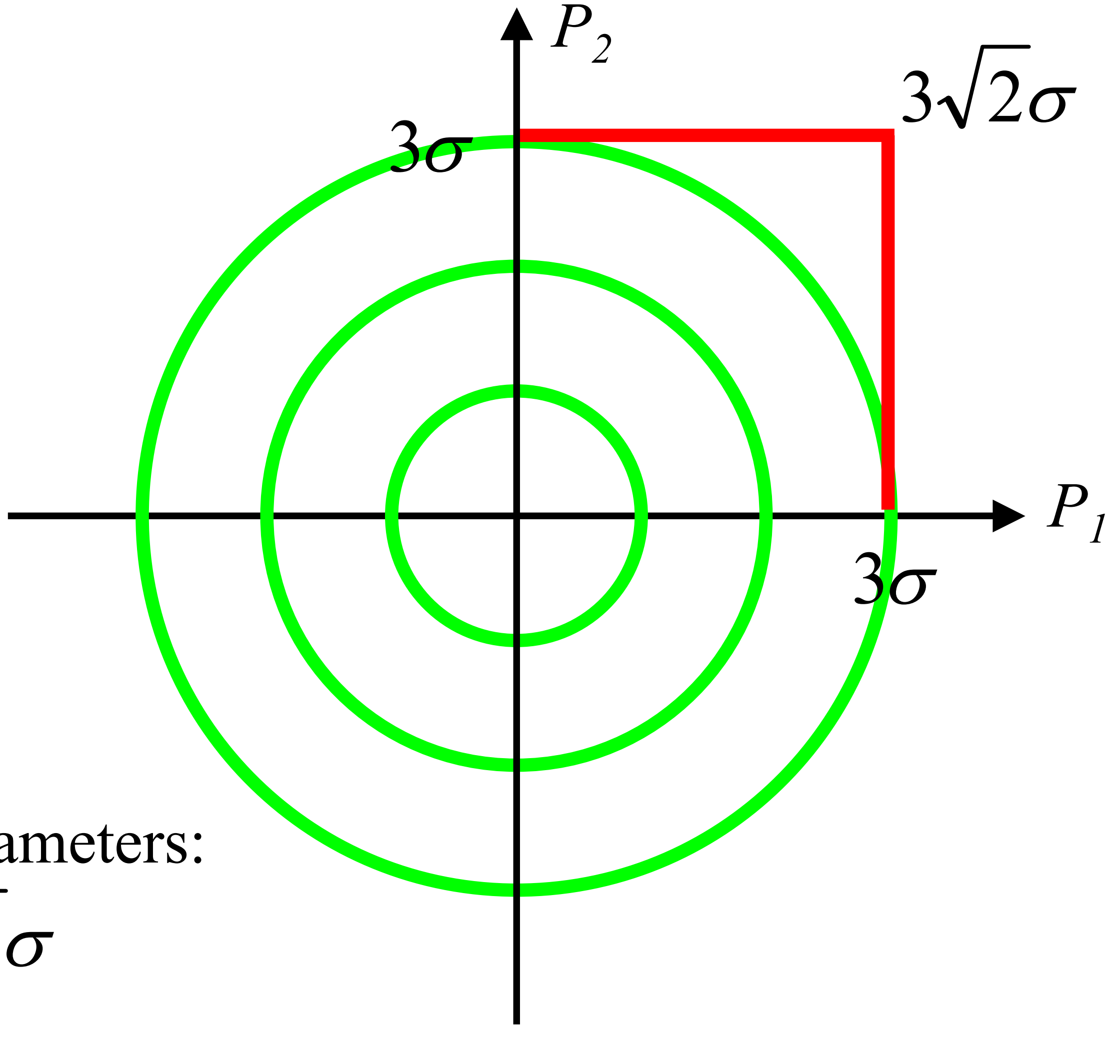
Backward Propagation of Variance

- **Obtain statistics for E_i**
- **Calculate sensitivities from SPICE models**
  - **To process, not SPICE, parameters**
- **Solve linear equations to get σ_{P_k}**
- **Mathematical constraints**
  - **SPICE models must be reasonable**
  - **P must be observable in E**
  - **σ_E must be self-consistent**

FPV Error in Modeling E



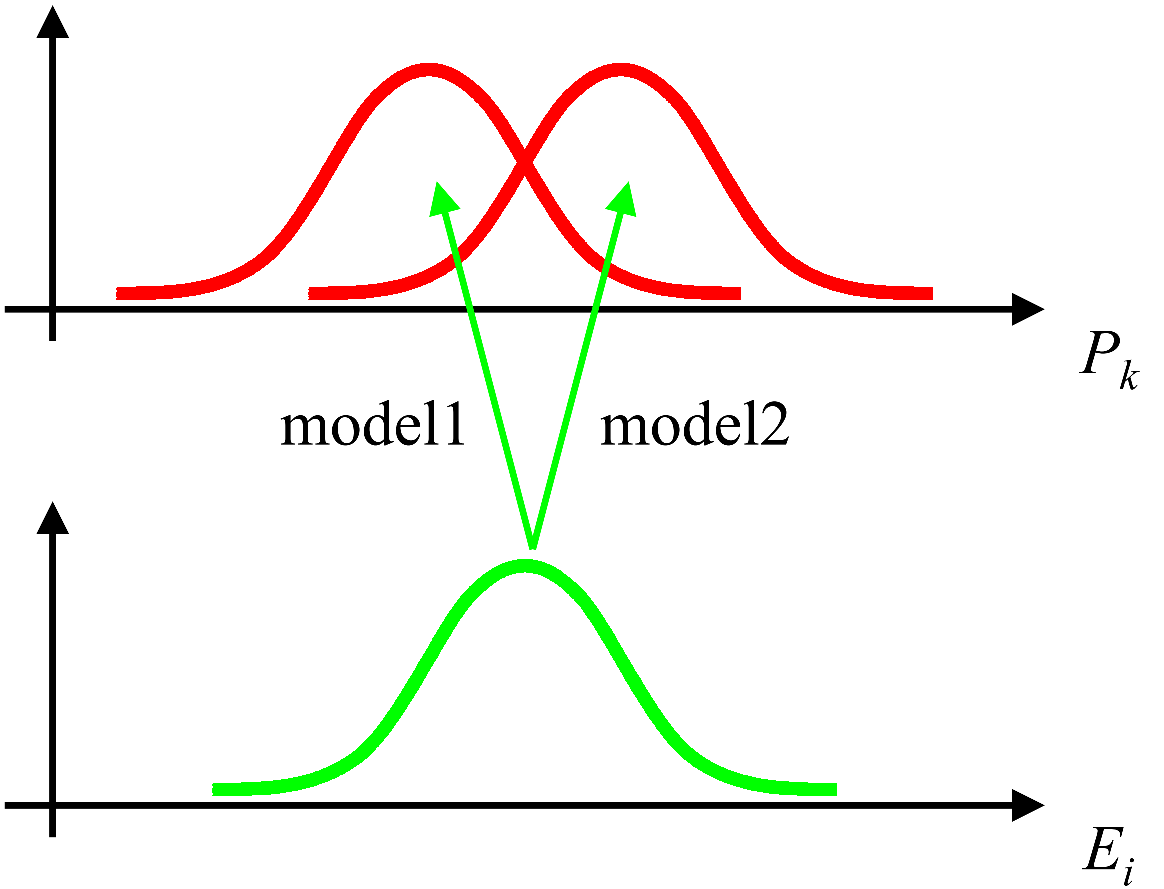
FPV Low Probability



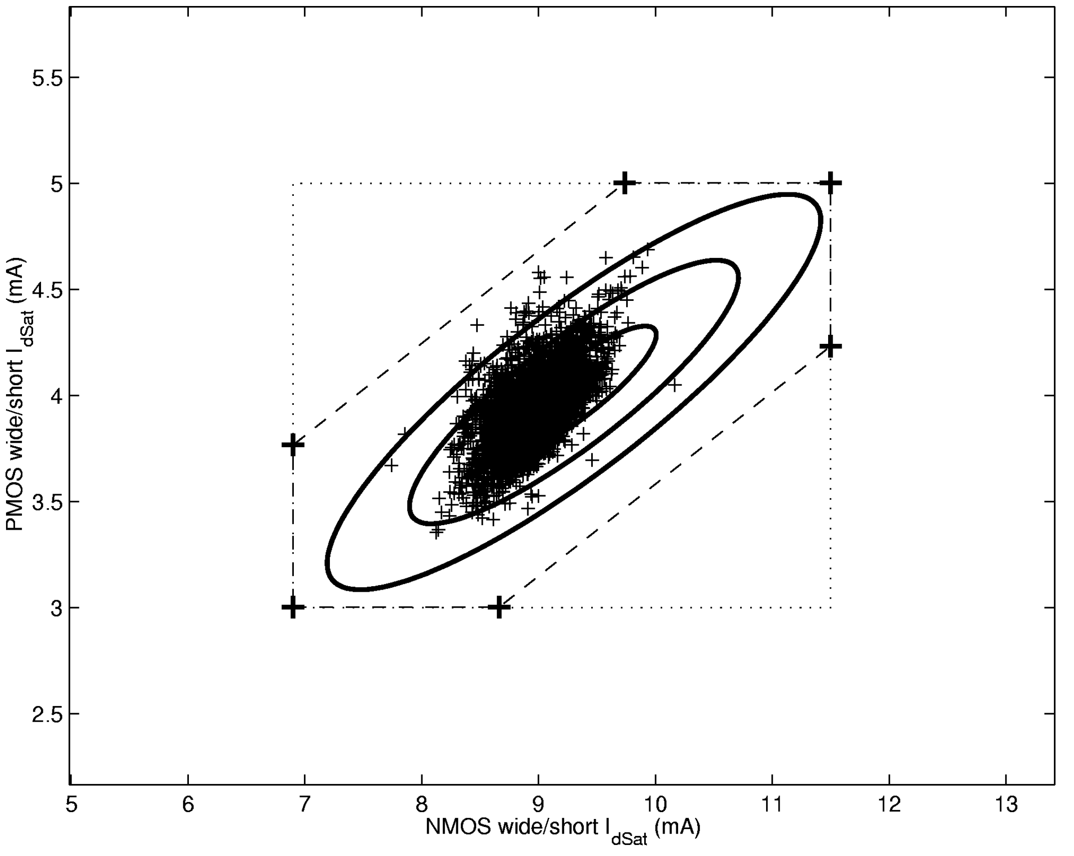
For n parameters:

$$3\sqrt{n}\sigma$$

BPV Process



Statistical Models



Statistics

- **There are two sources of variation in semiconductor manufacturing**
  - Global variation
  - Local variation
- **The BPV analysis does NOT distinguish between them**
  - Applies to both, target E_i may be different
  - Identical modeling/extraction formalism

Global and Local Variation

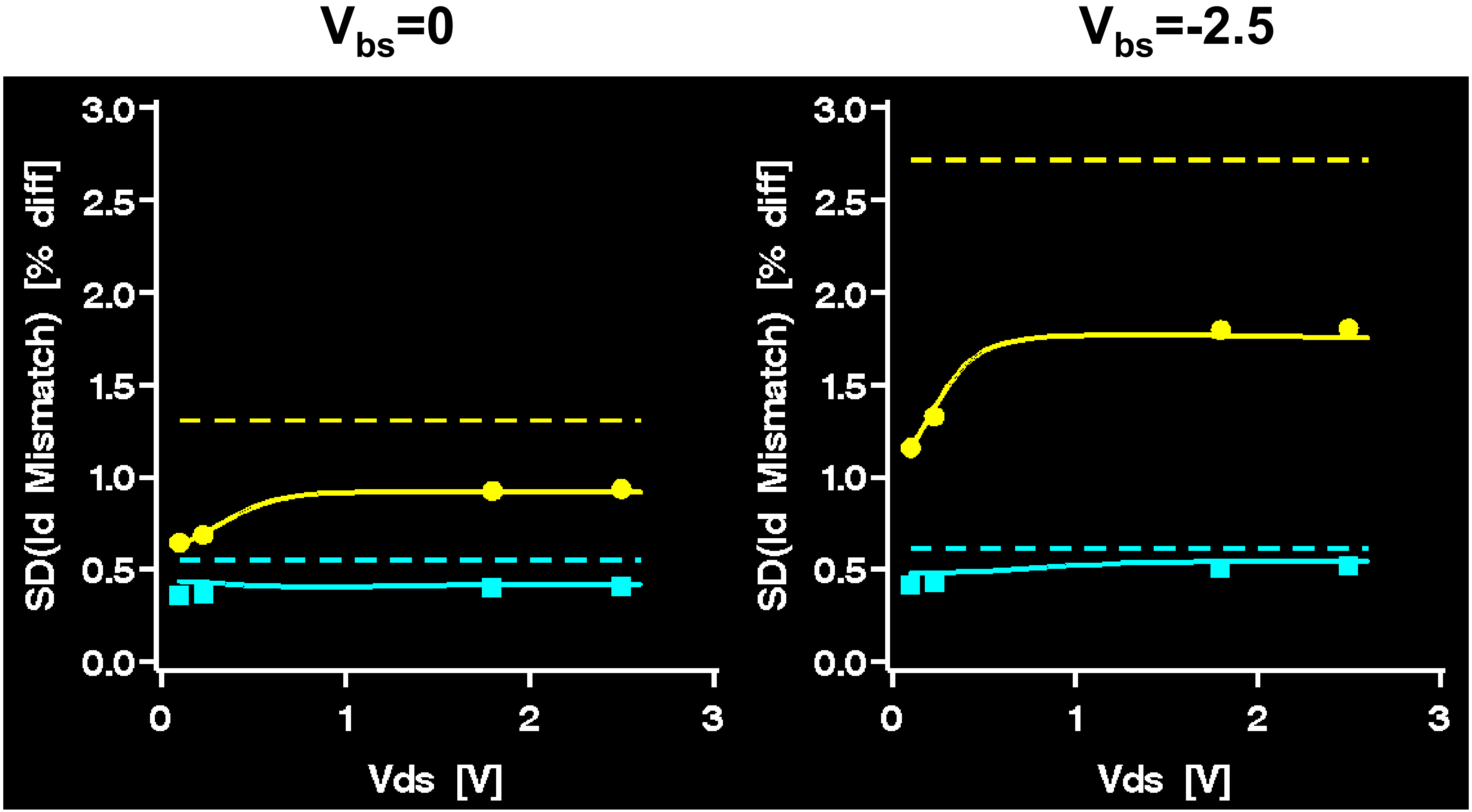
δP_G	δP_L
Global	**Local (mismatch)**
Inter-die	**Intra-die**
Correlated across die	**Uncorrelated**
Geometry independent	**Geometry dependent**

$$\sigma_P^2 = \sigma_{P_G}^2 + \sigma_{P_L}^2 (L, W, \ldots)$$

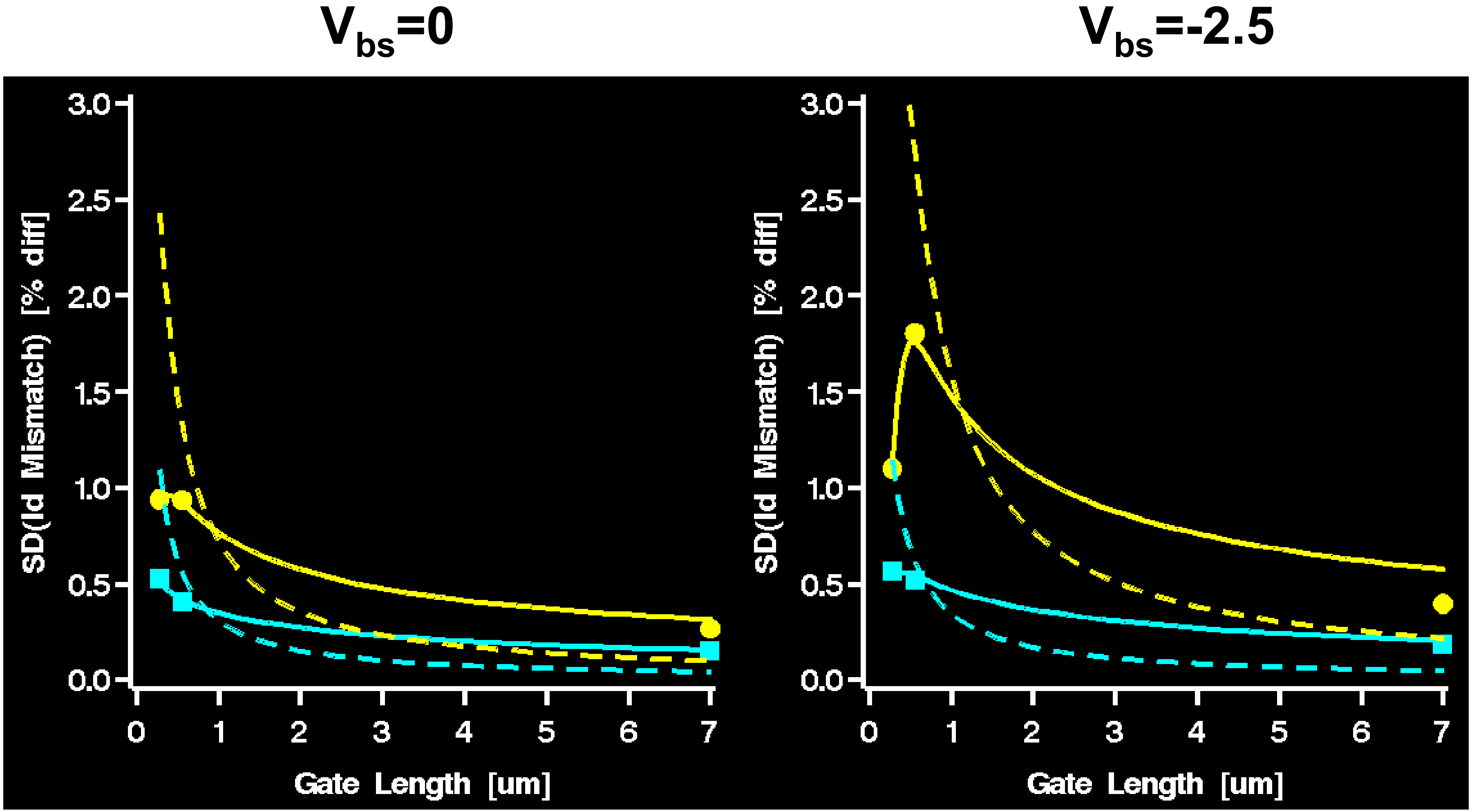
Common Inaccuracies

- V_{TH} and β are considered to be mismatch parameters for MOSFETs
  - Neither is a process parameter
  - T_{OX} variation is double counted
  - Misses effects like body effect mismatch
  - Measured in linear region, but analog circuits mostly operate in saturation
  - No guarantee I_d mismatch is modeled well
  - Inverse area fixation has precluded testing of wide/short and narrow/long devices
  - Some P_L vary as $1/L$ or $1/W$

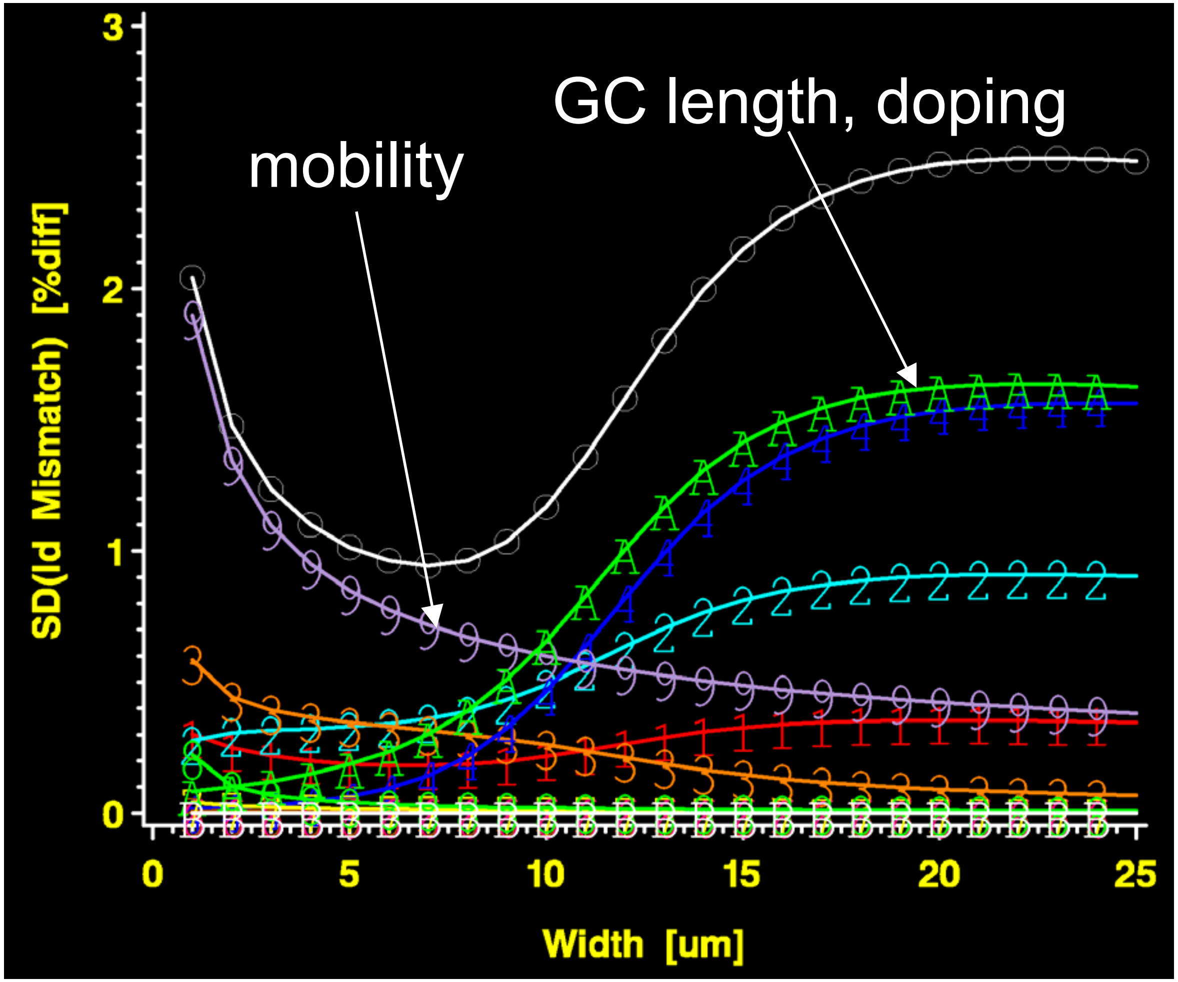
0.25μm NMOS Example



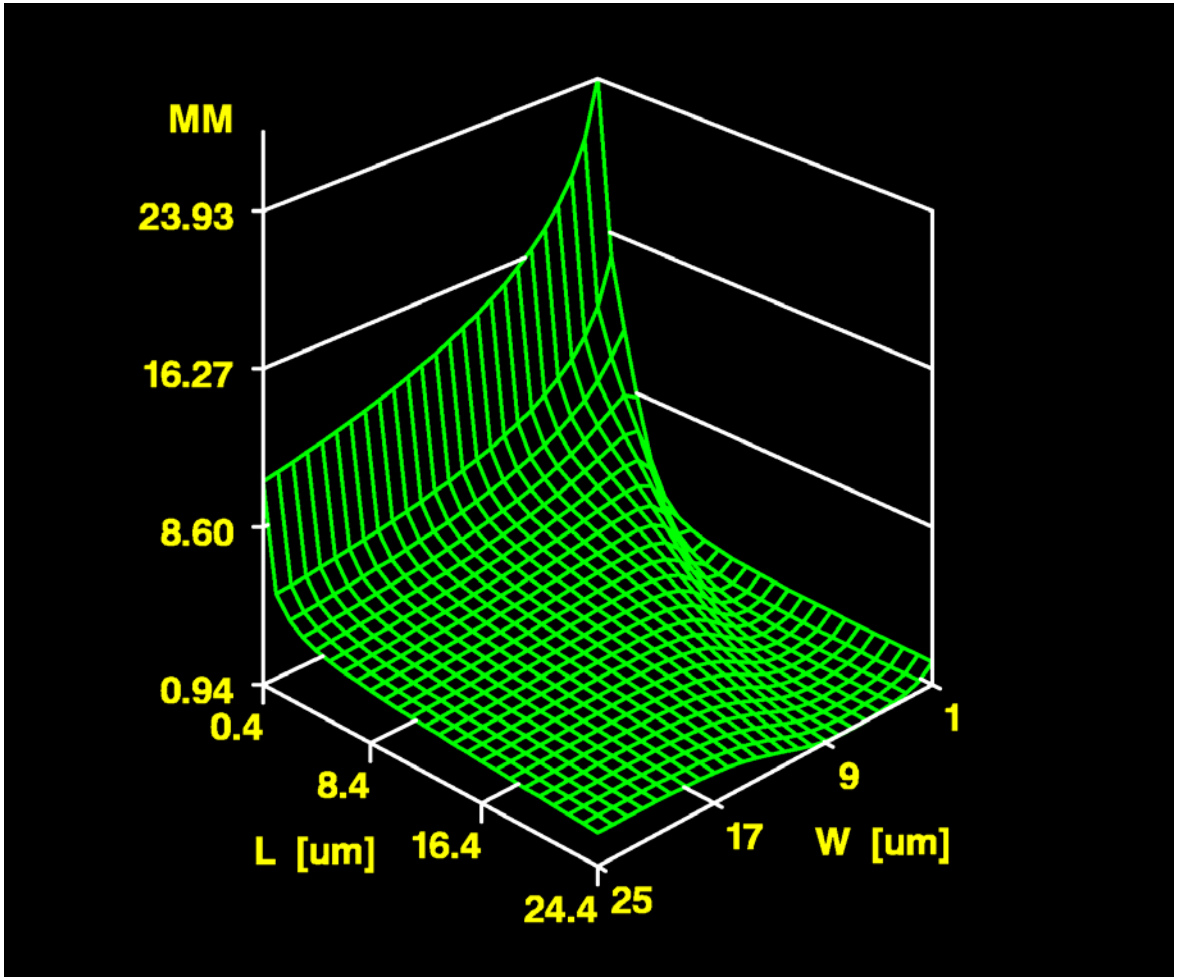
0.25µm NMOS Example



Mismatch Components



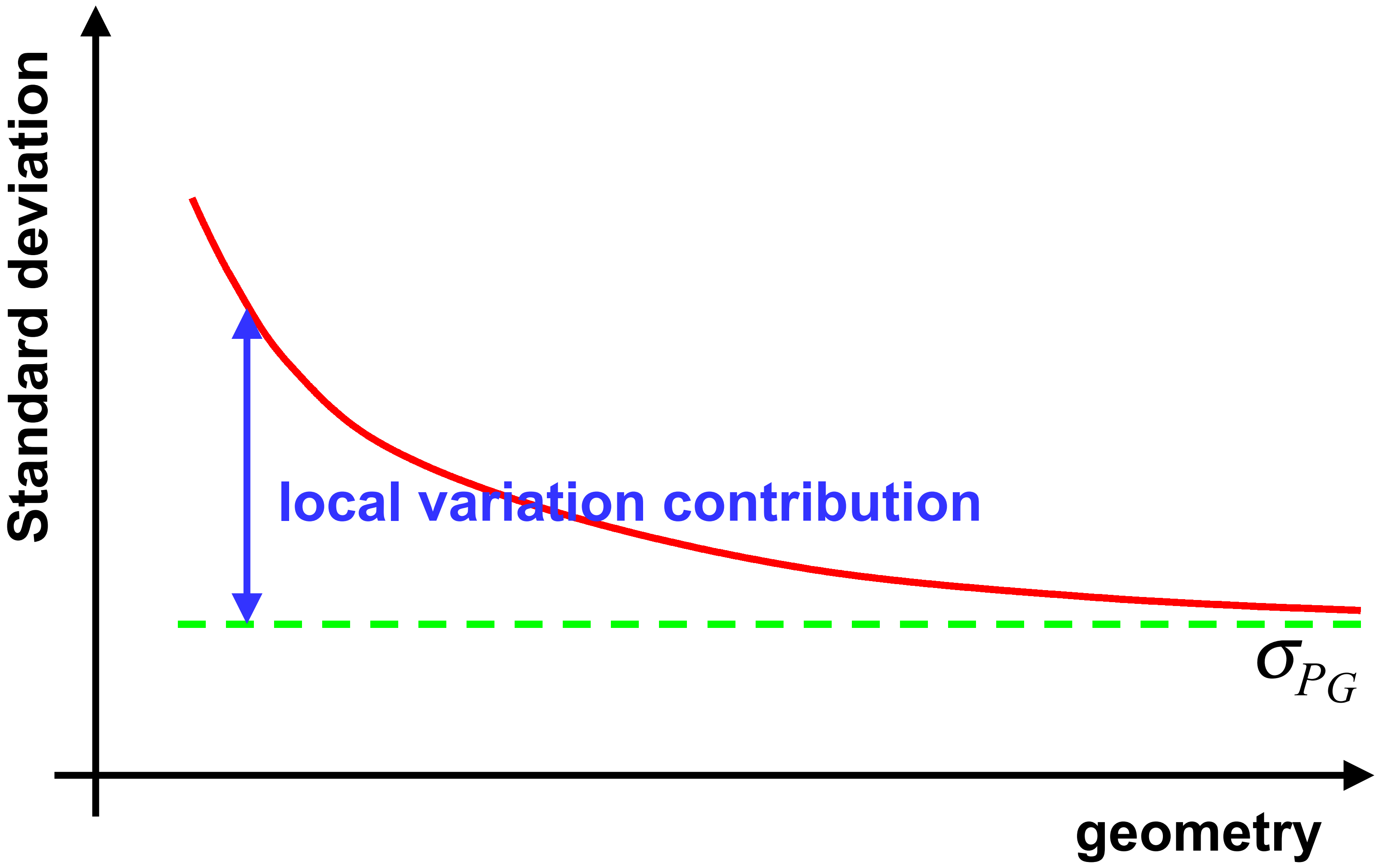
Current Mirror, 10μA



Issues when σ_P is Assumed to be σ_{P_G}

- **ET dependence on geometry ignored**
- **Global statistical simulations do not account for the geometry variation of σ_P**
- **Local variation can be double counted**
- **Standard "corner" simulation methodology fails**
  - **Assumes devices are correlated**
  - **Becomes very pessimistic**

The Wall


Standard deviation
local variation contribution
σ_{PG}
geometry

Issues with Correlation Viewpoint

- Physically incorrect
- Can miss need to handle >2 devices
- Can miss need for different geometries
- N devices require N(N+1)/2 correlations
- Can miss geometry dependence of σ_P
- Cannot handle σ_{P_G} being zero

$$\rho = \frac{\sigma_{P_G}^2}{\sqrt{(\sigma_{P_G}^2 + \sigma_{P_L}^2(g_1))(\sigma_{P_G}^2 + \sigma_{P_L}^2(g_2))}}$$

Summary

- Statistical modeling should be based on physical process parameters
- Process parameters can always be formulated to be uncorrelated
  - Yet still model correlated E_i accurately
- This plus BPV process gives a unified methodology that is identical for global and local variation
- Substantially improves accuracy