

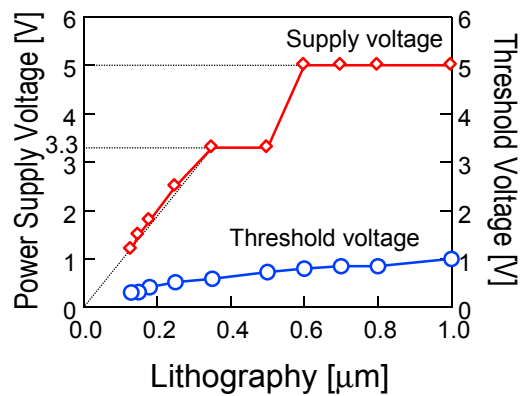
THE FOUNDATIONS OF THE EKV MOS TRANSISTOR CHARGE-BASED MODEL

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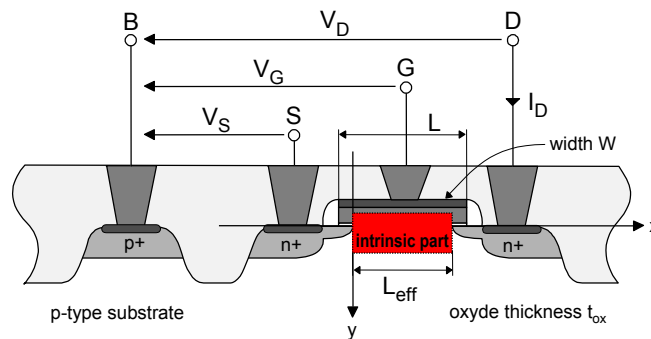
TECHNOLOGY AND VOLTAGE SCALING



INTRODUCTION

- ❖ Technology and voltage scaling result in **reduction** of the **overdrive voltage**
- ❖ This reduction is compensated by an increase of C'_{ox} and of the W/L ratio, which progressively moves the operating points from strong inversion to **moderate** and even **weak inversion**
- ❖ Circuit design in deep-submicron technologies requires compact models **accurate** and **continuous** over **all regions of operations**
- ❖ Main features of the EKV compact MOST model are
 - ▶ **Design-oriented** and **physics based**
 - ▶ **Hierarchical** structure allowing for a **coherent** approach going from a simple hand calculation model up to the full circuit simulation model
 - ▶ **Low number of parameters** (EKV v2.6 < 20 intrinsic parameters)
 - ▶ **Available** in major commercial circuit simulators

DEVICE SYMMETRY



- ❖ **Symmetrical** with respect to source (S) and drain (D)
- ❖ Terminal voltages are referred to the local substrate
- ❖ Leads to symmetrical model

DRAIN CURRENT

❖ Expressed with **surface potential** Ψ_s

$$I_D = \mu_n \cdot W \cdot \left(\underbrace{-Q'_i \cdot \frac{d\Psi_s}{dx}}_{\text{drift}} + \underbrace{U_T \cdot \frac{dQ'_i}{dx}}_{\text{diffusion}} \right)$$

❖ Expressed with **channel voltage** V_{ch} (quasi-Fermi potential):

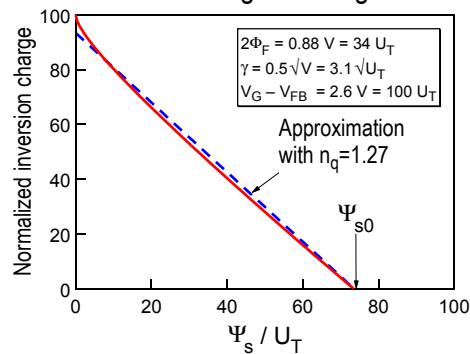
$$I_D = \mu_n \cdot W \cdot (-Q'_i) \cdot \frac{dV_{ch}}{dx}$$

$$I_D = \beta \cdot \int_{V_S}^{V_D} \frac{-Q'_i}{C'_{ox}} \cdot dV_{ch} \quad \beta \equiv \mu_n \cdot C'_{ox} \cdot \frac{W}{L}$$

INVERSION CHARGE LINEARIZATION

$$-Q'_i = C'_{ox} \cdot (V_G - V_{FB} - \Psi_s - \gamma_s \cdot \sqrt{\Psi_s}) \equiv n_q \cdot C'_{ox} \cdot (\Psi_{s0} - \Psi_s)$$

Constant gate voltage



$$\gamma_s = \frac{\sqrt{2 \cdot q \cdot \epsilon_{si} \cdot N_s}}{C'_{ox}}$$

$$n_q = 1 + \frac{\gamma_s}{2 \cdot \sqrt{\Psi_s}}$$

CURRENT & CHARGE NORMALIZATION

Inversion charge linearization $\Rightarrow \frac{d\Psi_s}{dx} = \frac{1}{n_q \cdot C'_{ox}} \cdot \frac{dQ'_i}{dx}$

$$I_D = \mu_n \cdot W \cdot \left(-Q'_i \cdot \frac{d\Psi_s}{dx} + U_T \cdot \frac{dQ'_i}{dx} \right) = \mu_n \cdot W \cdot \left(\frac{-Q'_i}{n_q \cdot C'_{ox}} + U_T \right) \cdot \frac{dQ'_i}{dx}$$

Normalization $\Rightarrow \boxed{i_d = \frac{I_D}{I_S} = -(2q'_i + 1) \cdot \frac{dq'_i}{d\xi}}$

$$\begin{aligned} q'_i &= Q'_i / Q_S & I_S &= 2 \cdot n_q \cdot \beta \cdot U_T^2 & \beta &= \mu_n \cdot C'_{ox} \cdot \frac{W}{L} \\ i_d &= I_D / I_S & Q_S &= -2 \cdot n_q \cdot C'_{ox} \cdot U_T & \xi &= \frac{x}{L} \end{aligned}$$

FORWARD & REVERSE MODES

$$i_d = \int_{q_r}^{q_f} (2q'_i + 1) \cdot dq'_i = \left[q_i^2 + q_i \right]_{q_r}^{q_f} = \underbrace{(q_f^2 + q_f)}_{=i_f} - \underbrace{(q_r^2 + q_r)}_{=i_r} = i_f - i_r$$

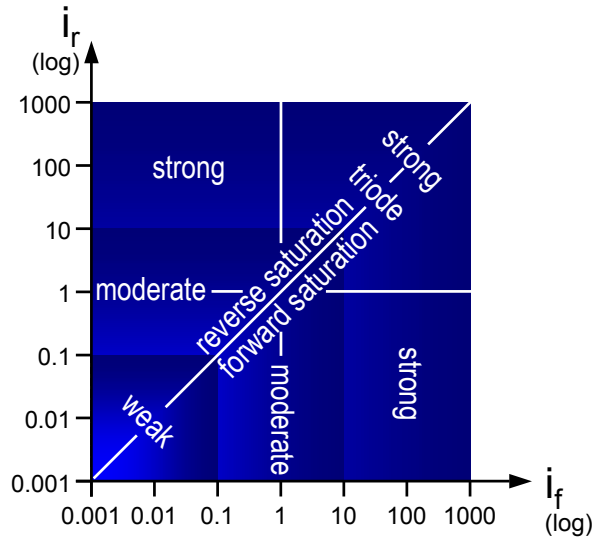
- ❖ Drain current depends only on **charge densities** at the **source** q_f and at the **drain** q_r

$$q_f = q'_i(\xi = 0) \quad \text{and} \quad q_r = q'_i(\xi = 1)$$

- ❖ Drain current split into difference between **forward current** i_f and **reverse current** i_r

$$\boxed{i_f = \frac{I_F}{I_S} = q_f^2 + q_f \quad \text{and} \quad i_r = \frac{I_R}{I_S} = q_r^2 + q_r}$$

MODES OF OPERATION



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CHARGES vs. CURRENTS

- ❖ The forward and reverse charges q_f and q_r can be expressed as a function of the forward and reverse currents i_f and i_r .

$$i_f = q_f^2 + q_f$$

$$i_r = q_r^2 + q_r$$

- ❖ solving for q_f and q_r leads to

$$q_f = \frac{2i_f}{\sqrt{4i_f + 1} + 1} = \frac{1}{2} \cdot \sqrt{4i_f + 1} - 1$$

$$q_r = \frac{2i_r}{\sqrt{4i_r + 1} + 1} = \frac{1}{2} \cdot \sqrt{4i_r + 1} - 1$$

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TRANSCONDUCTANCES vs. CHARGES

- ❖ The **source** and **drain transconductances** g_{ms} and g_{md} are directly related to the charges at source and drain according to

$$I_D = \beta \cdot \int_{V_S}^{V_D} \frac{-Q'_i}{C'_{ox}} \cdot dV_{ch} \quad g_{ms} = -\frac{\partial I_D}{\partial V_S} = \beta \cdot \frac{-Q'_i}{C'_{ox}} \Big|_{V_{ch}=V_S} = Y_0 \cdot q_f$$

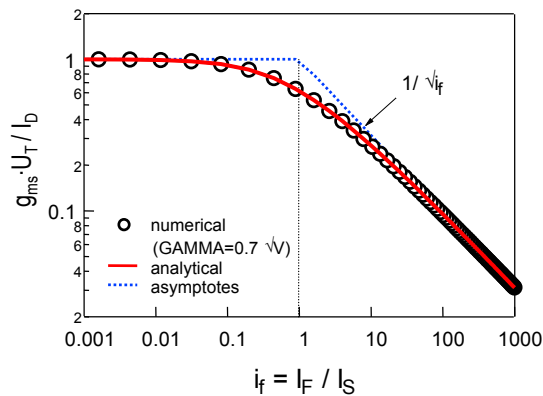
$$Y_0 = \frac{I_S}{U_T} \quad g_{md} = \frac{\partial I_D}{\partial V_D} = \beta \cdot \frac{-Q'_i}{C'_{ox}} \Big|_{V_{ch}=V_D} = Y_0 \cdot q_r$$

- ❖ and therefore to the current

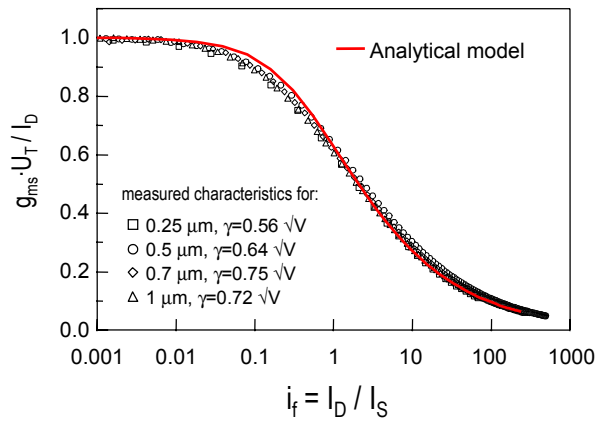
$$g_{ms(d)} = Y_0 \cdot q_{f(r)} = Y_0 \cdot \frac{2i_f(r)}{\sqrt{4i_f(r)+1}+1}$$

THE g_m / I_D CHARACTERISTIC

$$\frac{g_{ms}}{Y_0 \cdot i_f} = \frac{g_{ms} \cdot U_T}{I_S \cdot i_f} = \frac{g_{ms} \cdot U_T}{I_F} = \frac{q_f}{i_f} = \frac{2}{\sqrt{4i_f+1}+1} = \begin{cases} 1 & WI \\ 1/\sqrt{i_f} & SI \end{cases}$$



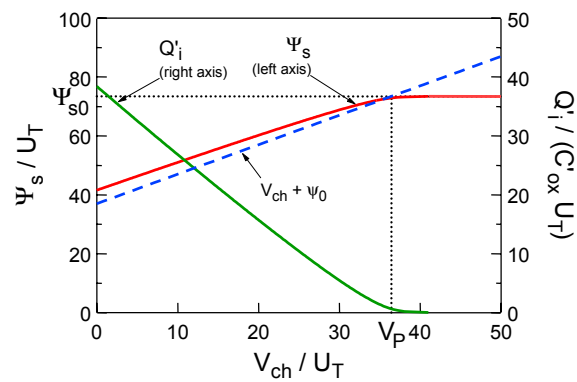
MEASURED g_m / I_D CHARACTERISTICS



PINCH-OFF VOLTAGE

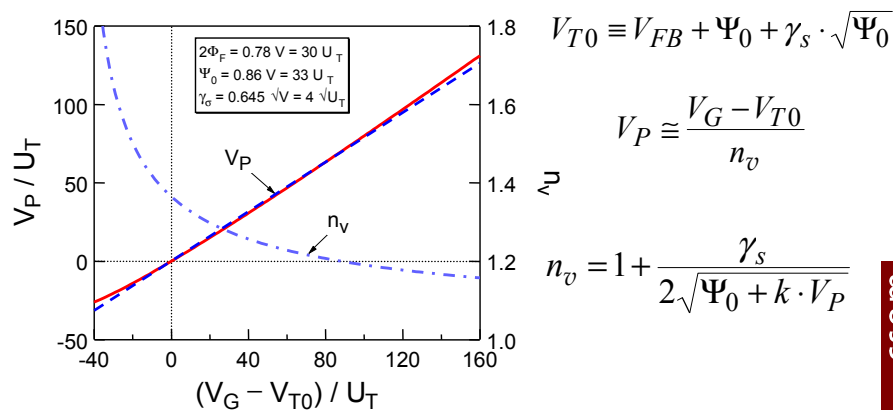
$$\Psi_s \cong V_{ch} + \Psi_0 \quad \text{where} \quad \Psi_0 \cong 2\Phi_F + m \cdot U_T \quad \text{with} \quad m \cong 2 \dots 4$$

$$\Psi_{s0} \cong V_P + \Psi_0$$



PINCH-OFF vs. GATE VOLTAGE

$$V_P = V_G - V_{T0} - \gamma_s \cdot \left[\sqrt{V_G - V_{T0} + \left(\sqrt{\Psi_0} + \frac{\gamma_s}{2} \right)^2} - \left(\sqrt{\Psi_0} + \frac{\gamma_s}{2} \right) \right]$$



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CHARGES vs. VOLTAGES AND CURRENTS

- ❖ **General relation** between inversion charge density Q'_i and channel voltage V_{ch} **valid all along the channel**

$$v_p - v_{ch}(\xi) = \ln[q'_i(\xi)] + 2q'_i(\xi) \quad \text{for } 0 \leq \xi \leq 1$$

- ❖ in particular at the **source** ($\xi = 0$) and **drain** ($\xi = 1$) ends

$$q'_i(\xi = 0) = q_f \Rightarrow v_p - v_s = \ln(q_f) + 2q_f$$

$$q'_i(\xi = 1) = q_r \Rightarrow v_p - v_d = \ln(q_r) + 2q_r$$

- ❖ which can be extended to forward and reverse currents

$$v_p - v_s = \ln\left(\sqrt{i_f + \frac{1}{4}} - \frac{1}{2}\right) + \sqrt{4i_f + 1} - 1$$

$$v_p - v_d = \ln\left(\sqrt{i_r + \frac{1}{4}} - \frac{1}{2}\right) + \sqrt{4i_r + 1} - 1$$

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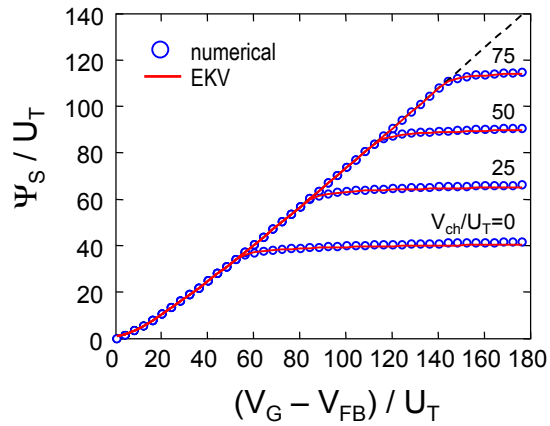
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EKV SURFACE POTENTIAL APPROXIMATION

$$\Psi_s \cong \Psi_0 + V_P + \frac{Q'_i}{n_q \cdot C'_{ox}}$$



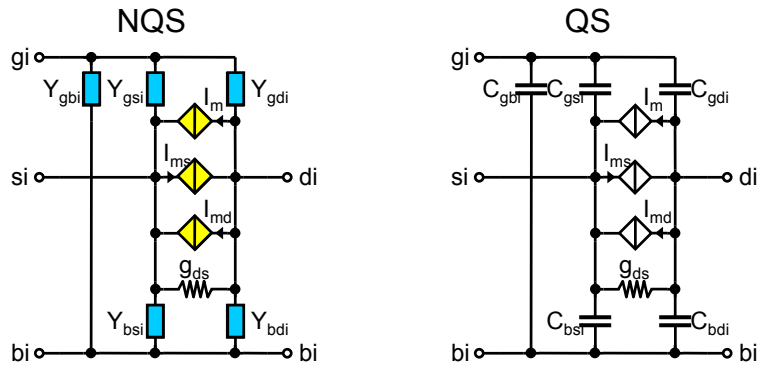
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GENERAL INTRINSIC SMALL-SIGNAL MODEL



5 admittances and
3 transadmittances

$$I_m = Y_m \cdot (V(gi) - V(bi))$$

$$I_{ms} = Y_{ms} \cdot (V(si) - V(bi))$$

$$I_{md} = Y_{md} \cdot (V(di) - V(bi))$$

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NON-QUASI-STATIC (NQS) MODEL

- ❖ General relations **valid in all modes of operations**

$$Y_m = \frac{1}{n_v} \cdot (Y_{ms} - Y_{md})$$

$$Y_{gbi} = \frac{n_v - 1}{n_v} \cdot (j\omega C_{ox} - Y_{gsi} - Y_{gdi})$$

- ❖ Thanks to the source/drain and gate/bulk "symetry" the NQS model only requires **two independent y-parameters**, for example Y_{ms} and Y_{gsi}

$$\text{Source/drain symetry} \quad \Rightarrow \quad \begin{cases} Y_{ms} & \Rightarrow & Y_{md} \\ Y_{gsi} & \Rightarrow & Y_{gdi} \end{cases}$$

$$\text{Gate/bulk "symetry"} \quad \Rightarrow \quad \begin{cases} Y_{bsi} = (n_v - 1) \cdot Y_{gsi} \\ Y_{bdi} = (n_v - 1) \cdot Y_{gdi} \end{cases}$$

NQS TRANSADMITTANCES (1/2)

- ❖ Normalized **source transadmittance** Y_{ms} given by

$$\xi_m \equiv \frac{Y_{ms}}{g_{ms}} = \frac{\lambda}{\sinh(\lambda)} \cong \frac{1}{1 + j\omega \cdot \tau_{qs}} \quad \text{for : } \omega \cdot \tau_{qs} \ll 1$$

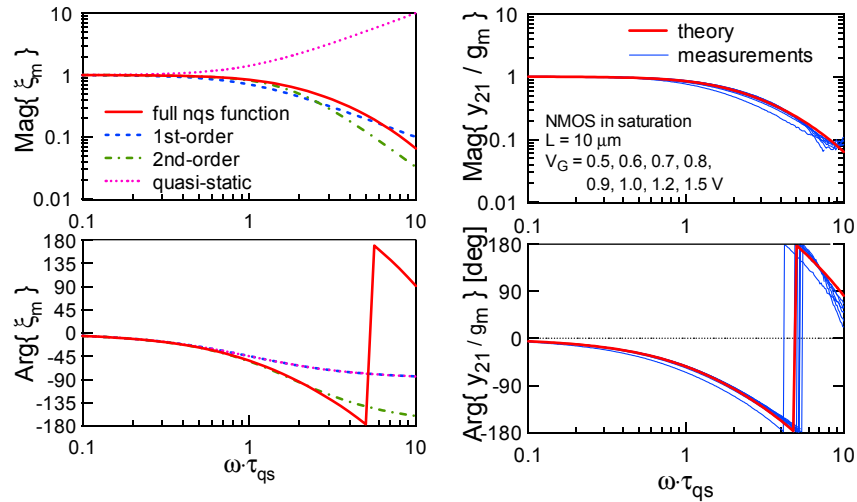
$$\text{with} \quad \lambda \equiv (1 + j) \cdot \sqrt{3\omega\tau_{qs}}$$

- ❖ In **saturation**

$$\frac{\tau_{qs}}{\tau_0} = \frac{1}{30} \cdot \frac{4q_f^2 + 10q_f + 5}{(q_f + 1)^3} = \begin{cases} \frac{2}{15} \cdot \frac{1}{\sqrt{i_f}} \cong \frac{4}{15} \cdot \frac{n_q \cdot U_T}{V_G - V_{TO} - n_q \cdot V_S} & SI \\ \frac{1}{6} & WI \end{cases}$$

$$\text{with} \quad \tau_0 \equiv L_{eff}^2 / (\bar{\mu}_{eff} \cdot U_T)$$

NQS TRANSADMITTANCES (2/2)



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NQS ADMITTANCES (1/2)

❖ Admittance Y_{gsi} given by

$$Y_{gsi} = j\omega C_{ox} \cdot c_c \cdot \xi_c$$

↑ geometry ↑ bias ↑ frequency (bias)

where

$$\xi_c \equiv 2 \cdot \frac{\cosh(\lambda) - 1}{\lambda \cdot \sinh(\lambda)} \quad \lambda \equiv (1 + j) \cdot \sqrt{3\omega\tau_{qs}}$$

$$C_{ox} = W_{eff} \cdot L_f \cdot C'_{ox}$$

$$c_c = \frac{1}{3} \cdot \frac{q_f \cdot (2q_f + 4q_r + 3)}{(q_f + q_r + 1)^2} = \begin{cases} \frac{2}{3} & SI \\ q_f & WI \end{cases}$$

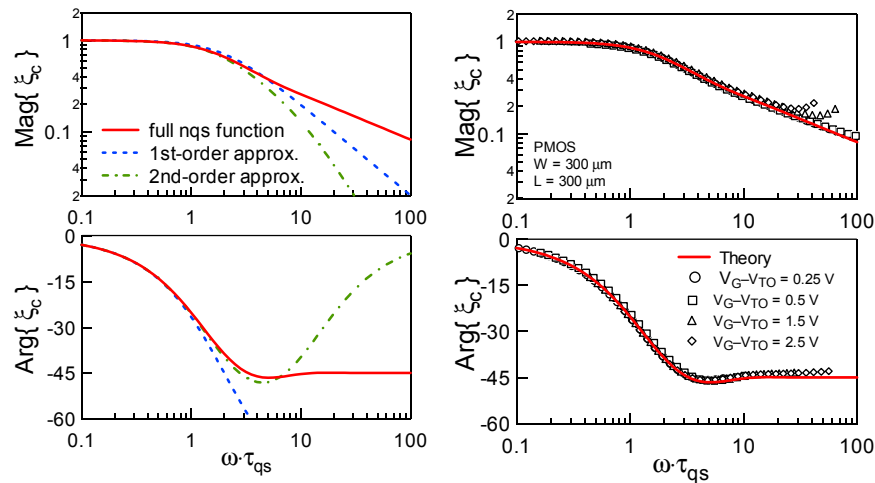
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NQS ADMITTANCES (2/2)



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CONCLUSION

- ❖ EKV charge-based model based on **fundamental physical considerations**
- ❖ Relations between the current, the charges, the voltages and the small-signal parameters **valid in all modes of operations**
- ❖ Model exploits
 - ▶ Device **symmetry**
 - ▶ Inversion charge **linearization**
 - ▶ **Reverse** and **forward** currents
 - ▶ Charge, current and voltage **normalization**
 - ▶ **Pinch-off** vs. gate **voltage**
- ❖ Basic **long-channel** model can be extended to include important effects related to the reduction of the device geometry (EKV v3.0) (see next paper by Matthias Bucher)

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